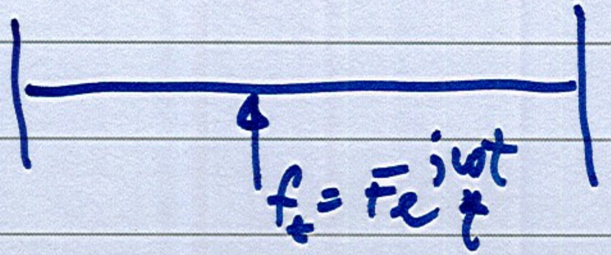
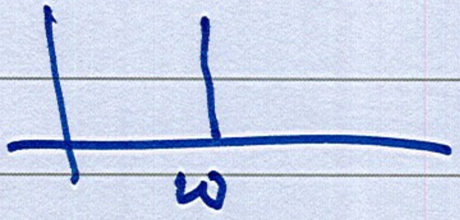
ME 513 Session 14 9/20/19 1

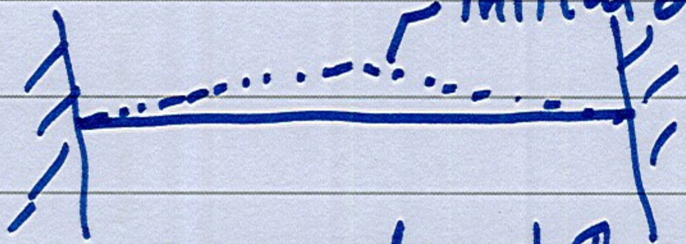
Forced

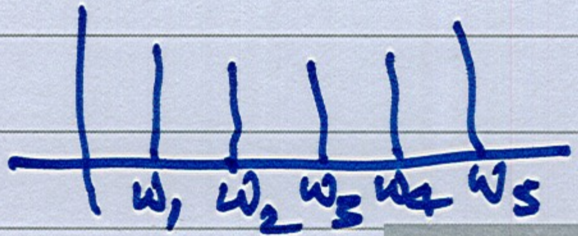




- responds only at the forcing freq

Free





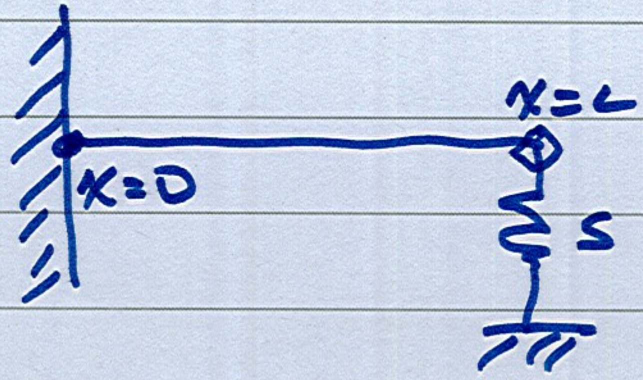
- responds at the natural freqs

$\sin kL = 0$

$k_n L = n\pi$

$$y(x,t) = \sum_{n=1}^{\infty} \hat{a}_n \sin k_n x \, e^{j\omega_n t}$$

2.5.4 Other b.c.'s



$$y(x,t) = -2jA\sin kx\, e^{j\omega t}$$

$$-T\frac{dy}{dx}\bigg|_{x=L} - sy\bigg|_{x=L} = 0$$

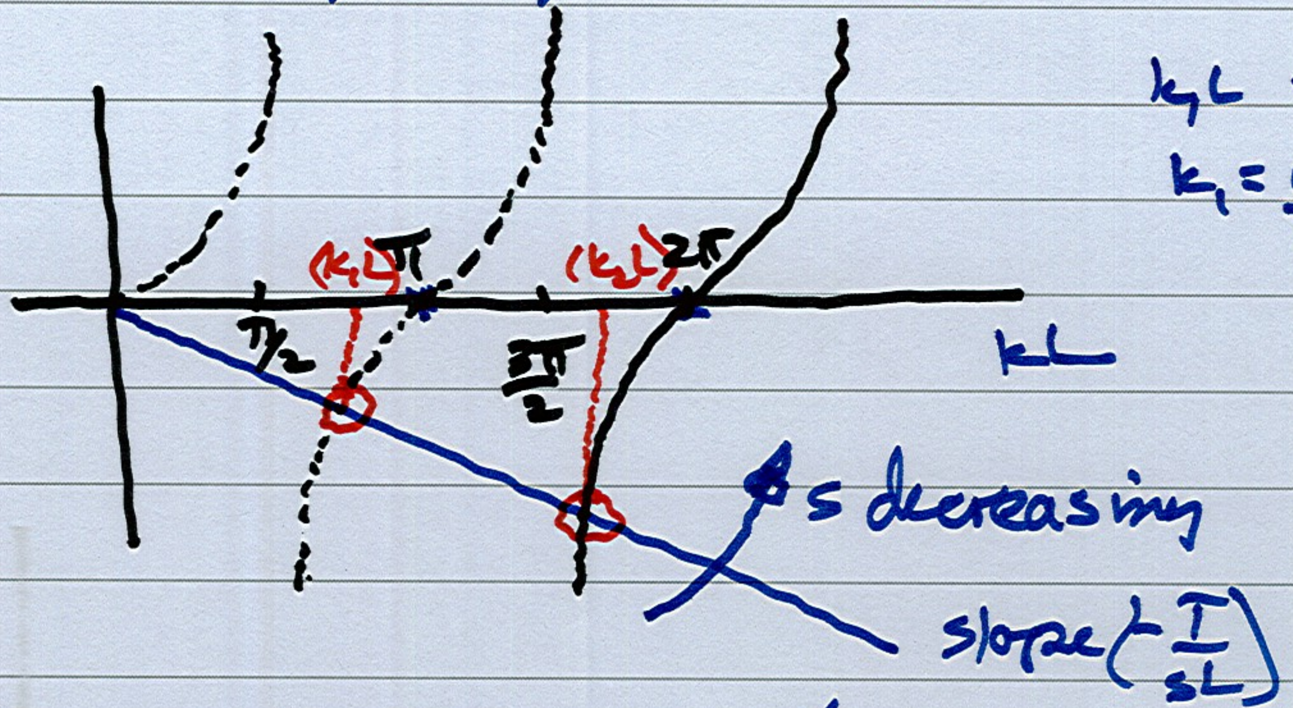
$$\cancel{2}jkT\cancel{A e^{j\omega t}}\cos kL + \cancel{2j}s\cancel{A e^{j\omega t}}\sin kL = 0$$

$$-\frac{k_n T}{s} = \tan(k_n L)$$

$$\boxed{-(k_n L)\left(\frac{T}{sL}\right) = \tan k_n L}$$

characteristic eqn

$$-(k_n L)\left(\frac{T}{sL}\right) = \tan k_n L$$



$k_1 L \to k_1$

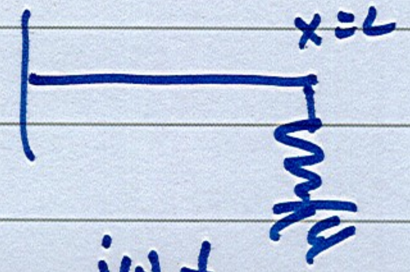
$k_1 = \frac{\omega_1}{c} \to \omega_1$

as $s \to \infty$ approach
The fixed-fixed case

$$k_n L = n\pi$$

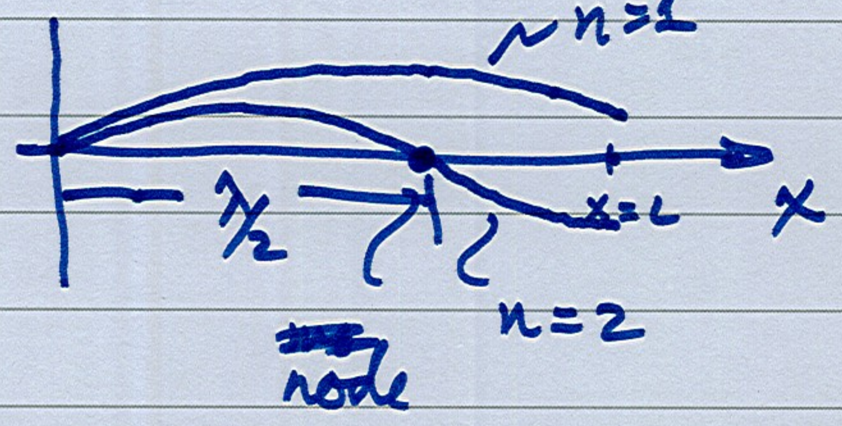
natural freqs are decreased
when s is finite compared
to fixed case

mode shape



$$y_n(x,t) = \hat{a}_n \sin k_n x \, e^{j\omega_n t}$$

nth mode



k_1 is smaller than in the fixed-fix case

$$y(x,t) = \sum_{n=1}^{\infty} \hat{a}_n \sin k_n x \, e^{j\omega_n t}$$

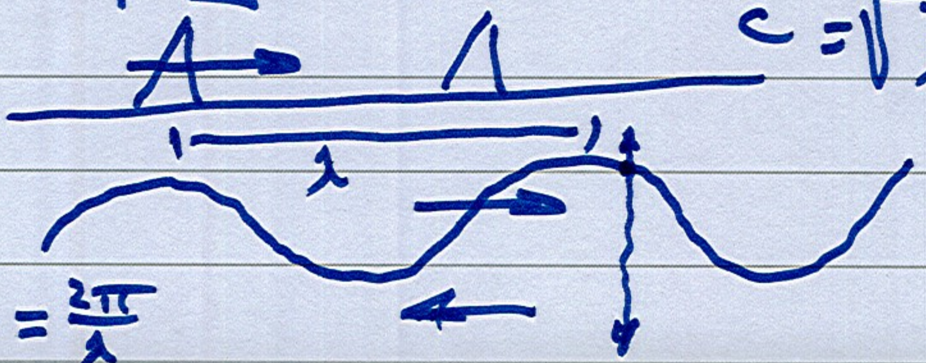
Summary

- Derivation of a wave eqn
  (modeling)
  - restoring force
  - eom
  - inertia
  - stiffness

wave propagation

$c = \sqrt{T/\rho}$

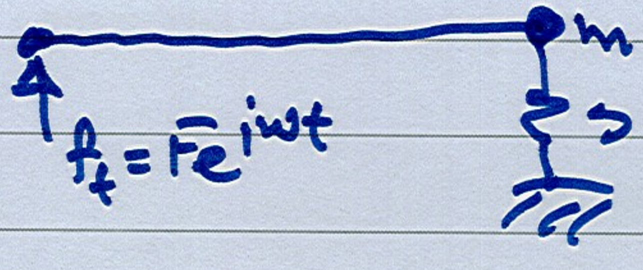
e^{-ikx}

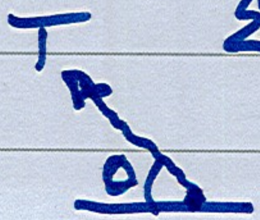


$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

Boundary Conditions

- fixed
- stiffness
- mass
- force





$$\sum f_y = ma$$

$$\sin\theta \simeq -\frac{dy}{dx}$$

Forced Response $Fe^{j\omega t}$

$$y(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$$

linear system

Impedance $\frac{\text{Force}}{\text{velocity}}$

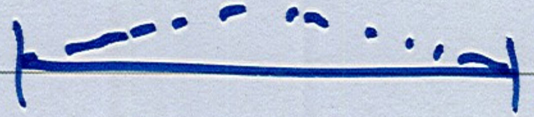
Z_{mo}

ρc characteristic impedance

- standing wave
- propagating

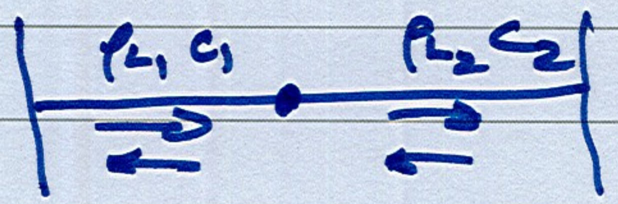
forced

free



- natural freqs
- mode shapes

characteristic eqn



3. The Acoustic Wave Eqn & Simple soln's

5.1 → 5.13

3.1 Introduction

- sound - propagating, small amplitude fluctuations in pressure ( ) in an elastic medium

- "ideal" acoustics
  - assume that fluid is ~~is~~ inviscid, lossless + adiabatic

- small amplitude fluctuations "linear" acoustics

- wave propagation
  - inertia 1.2 kg/m^3
  - stiffness

Derive a wave eqn

p + u - equation of state] - restoring force
           - continuity

p + u - momentum] - EOM

to eliminate u

single equation w/ p

- 1-D solutions
  - plane
  - cylindrical waves
  - spherical waves
- Acoustic Intensity
- specific Acoustic Impedance
- <u>Decibel</u>