ME 513 Session 11 9/13/19

Homework Hints

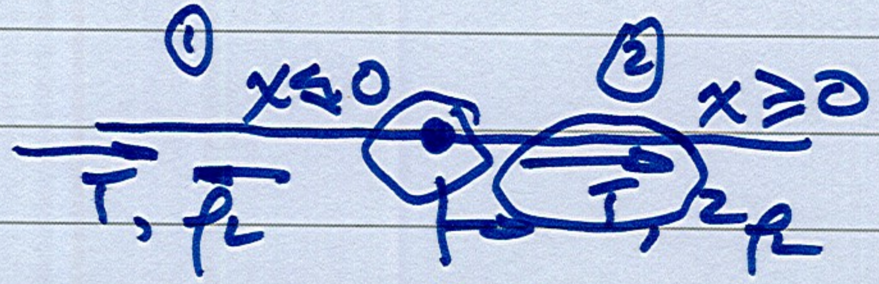
2.4.1 $$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t} = 0$$

(c) $y = a(ct - x)^2$

2.8.1 $y = 4\cos(3t - 2x)$

$y = A\cos(\omega t - kx)$

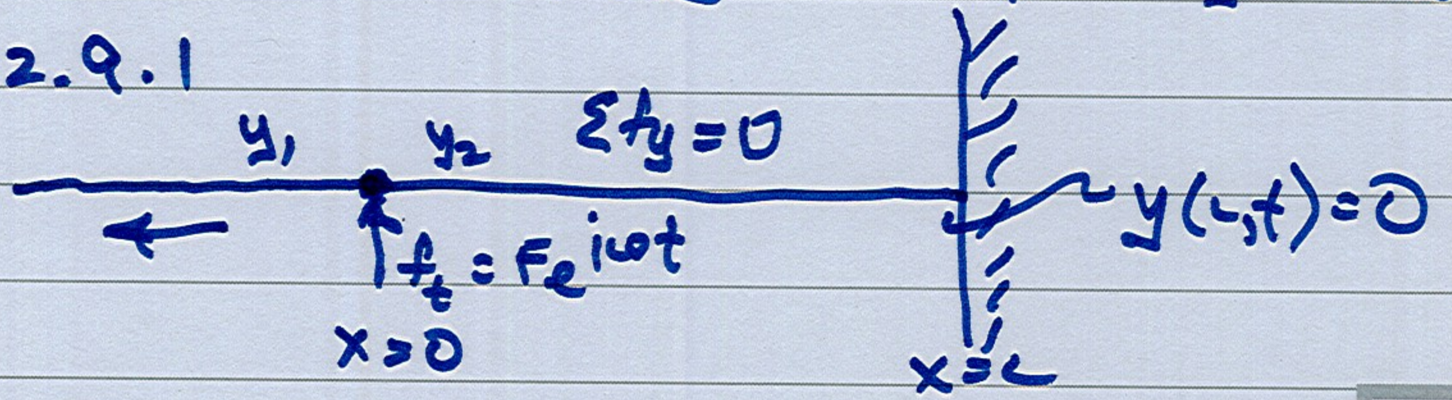
2.8.2



$\left(\frac{C}{A}\right)$

$y_1 = Ae^{j(\omega t - k_1 x)} + Be^{j(\omega t + k_1 x)}$ $\qquad y_2 = Ce^{j(\omega t - k_2 x)}$

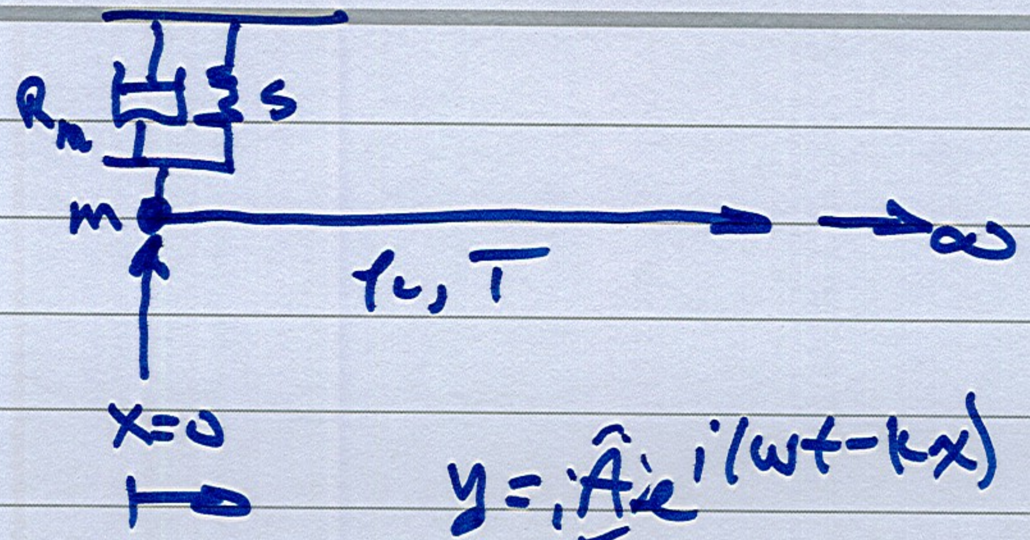
2.9.1



$Z_{m0} = \underline{-j\rho_L c \cot kL}$

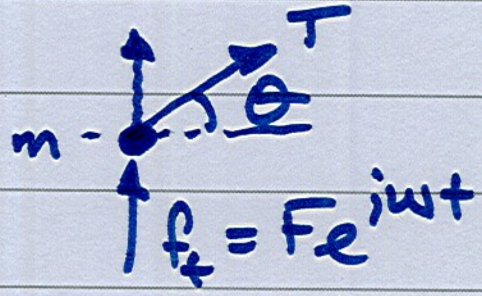
Review your notes

2.9.2



$$y = \underline{\hat{A}}\, e^{i(\omega t - kx)}$$

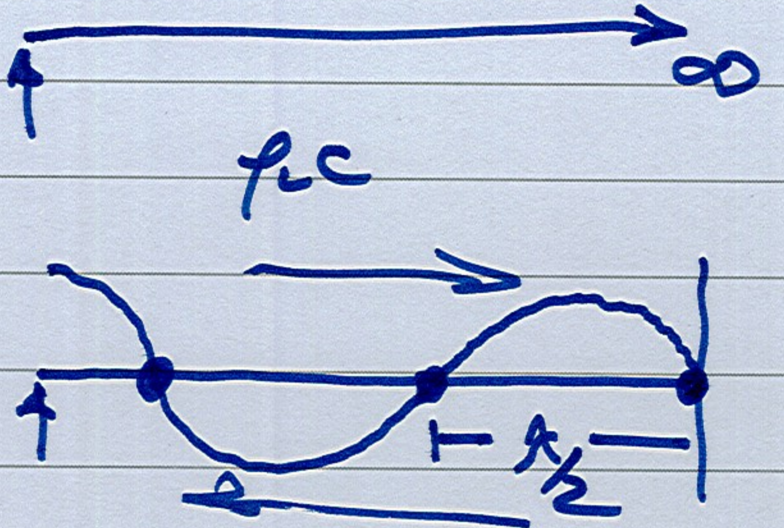
FBD



$$\underline{\sum f_y = m a} \qquad \left.\frac{\partial^2 y}{\partial t^2}\right|_{x=0}$$

A

$$Z_{m0} = \frac{Fe^{j\omega t}}{\left.\frac{\partial y}{\partial t}\right|_{x=0}}$$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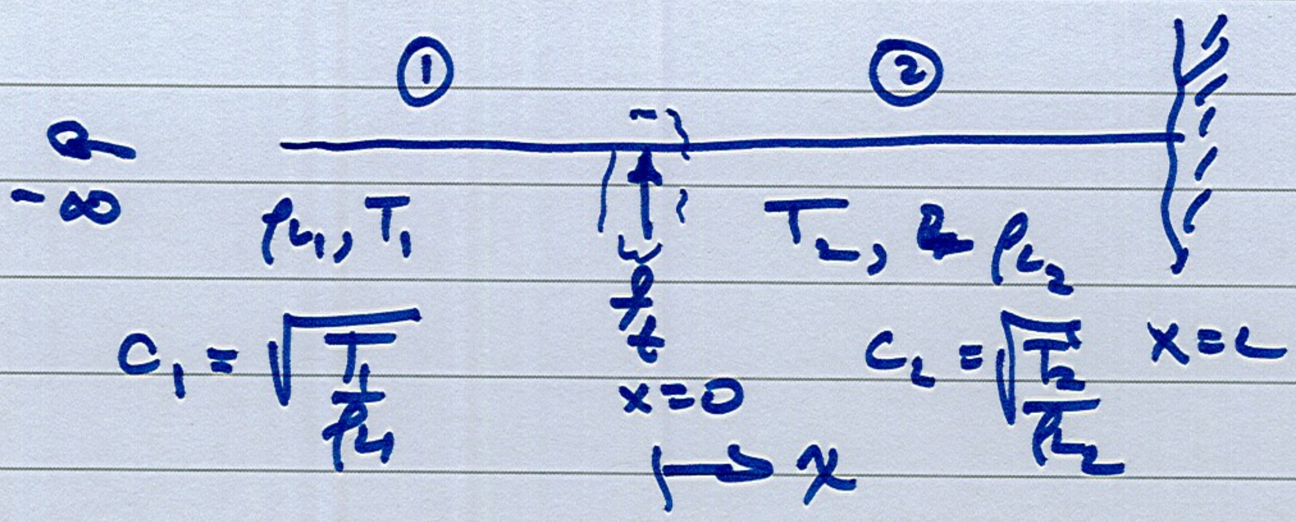
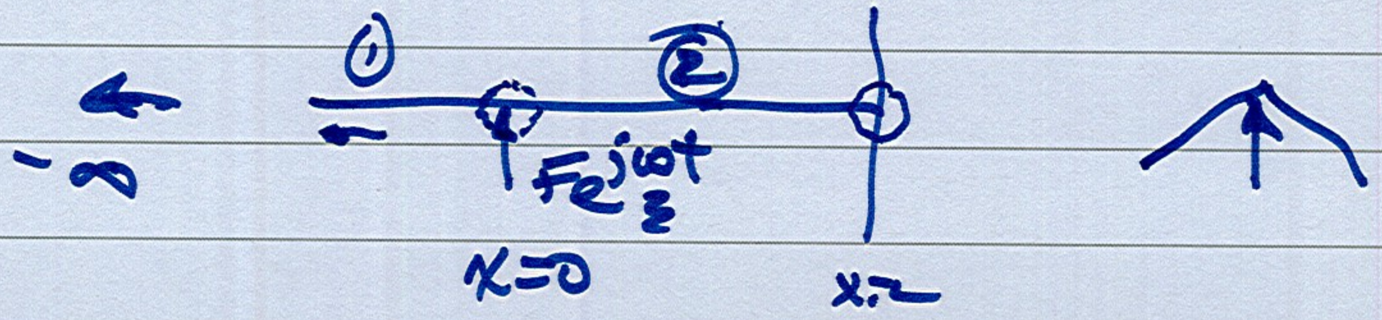
2.4.3 Strings with multiple segments



$c_1 = \sqrt{\frac{T_1}{\rho_{L1}}}$ $\qquad c_2 = \sqrt{\frac{T_2}{\rho_{L2}}}$

- wave eqn applies to homogeneous
- must treat the two strings as distinct - coupled through b.c's
- even if physically one string treat as two different strings

$$\frac{\partial^2 y_1}{\partial x^2} - \frac{1}{c_1^2}\frac{\partial^2 y_1}{\partial t^2} = 0 \qquad \frac{\partial^2 y_2}{\partial x^2} - \frac{1}{c_2^2}\frac{\partial^2 y_2}{\partial x^2} = 0$$



In region ① $\quad x \leq 0 \qquad k_1 = \frac{\omega}{c_1}$

$$y_1(x,t) = \cancel{A e^{j(\omega t - k_1 x)}} + B e^{j(\omega t + k_1 x)}$$

$\underline{-ve}$

In region ② $\quad x \geq 0$

$$y_2(x,t) = C e^{j(\omega t - k_2 x)} + D e^{j(\omega t + k_2 x)}$$

$$k_2 = \omega / c_2$$

3 unknowns → 3 b.c.'s

1 force b.c.

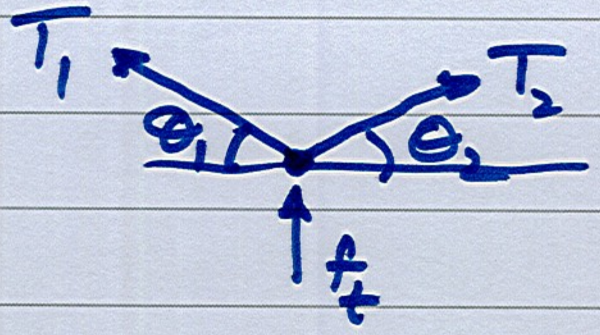
2 displacement conditions

(i) $y_2(L,t) = 0$

(ii) $y_1(0,t) = y_2(0,t)$

- displacement is continuous at the point the strings join
- but the slope can be discontinuous – no flexural stiffness

(iii) Force b.c. at $x = 0$

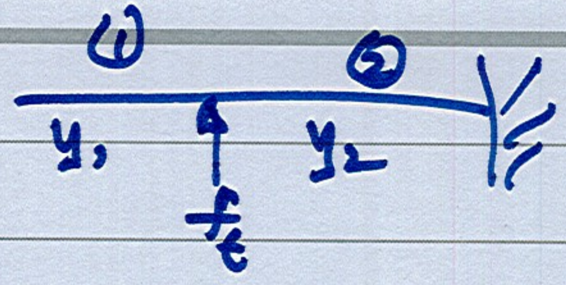


$$\sum f_y = 0 = f_t + T_2 \sin\theta_2\Big|_{x=0} + T_1 \sin\theta_1\Big|_{x=0}$$

$$\boxed{0 = f_t + T_2 \frac{dy_2}{dx}\Big|_{x=0} - T_1 \frac{dy_1}{dx}\Big|_{x=0}}$$

substitute the assumed sol'ns into (i), (ii) + (iii)

solve for B, C + D



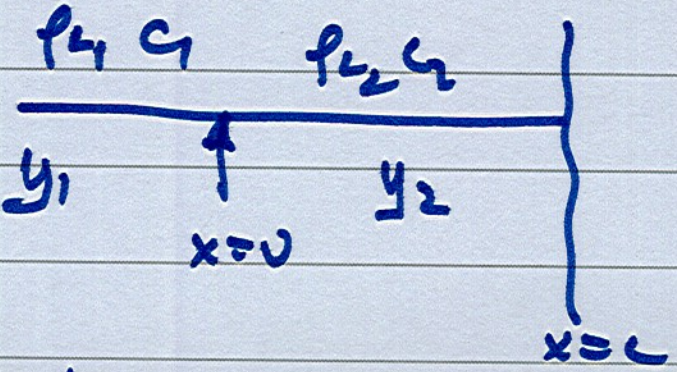
$$Z_{m_0} = \frac{f_t}{u_1|_{x=0}} = \frac{f_t}{u_2|_{x=0}}$$

$$\cancel{\frac{u_1+u_2}{2}}$$

$$= \rho_{L_1} c_1 \cancel{+} - j\, \rho_{L_2} c_2 \cot k_2 L \qquad k_2 = \frac{\omega}{c_2}$$

- impedances of the string segments add in <u>series</u>
- velocity is shared between the 2 segments at the drive point

Notes:

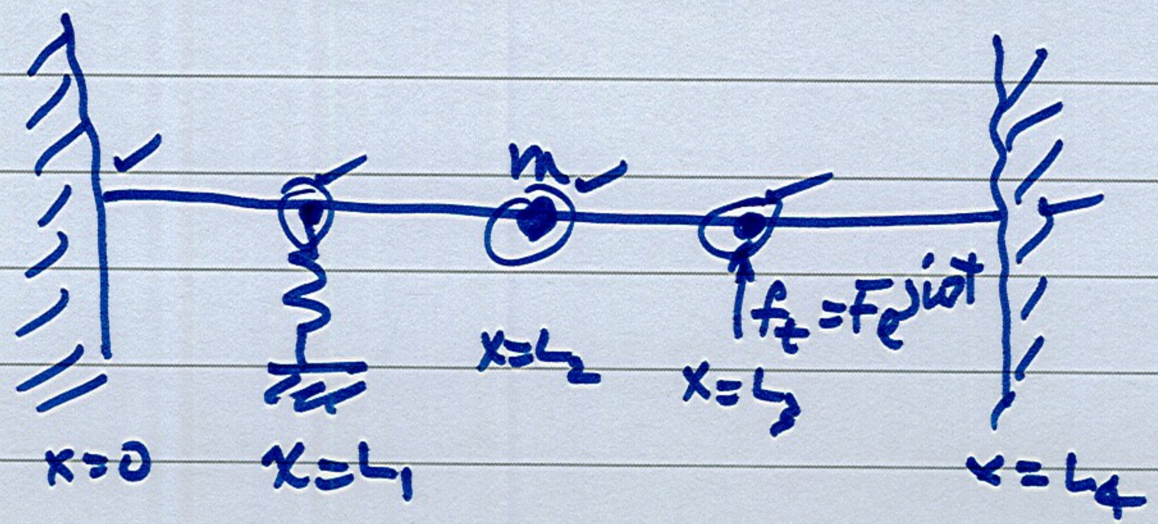


(i) solution for y_1 applies only in region $x \leq 0$

(ii) solution for y_2 applies only in region $0 \leq x \leq L$

(iii) same approach even if the two segments have the same properties

(iv) The same approach can be extended to any number of segments



$$y_1(x,t) = A e^{i(\omega t - k_1 x)} + B e^{i(\omega t + k_1 x)} \qquad k_1 = \frac{\omega}{c_1}$$

$$0 \le x \le L_1 \qquad c_1 = \sqrt{\frac{T_1}{\rho_1}}$$

$$y_2(x,t) = C \cdots + D \cdots \quad k_2$$

$$y_3(x,t) = E \cdots + F \cdots \quad k_3$$

$$y_4(x,t) = G \cdots + H \cdots \quad k_4$$

8 unknowns 5 displacement
3 Force

Homework hints

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2.8.1 $y = 4\cos(3t - 2x)$

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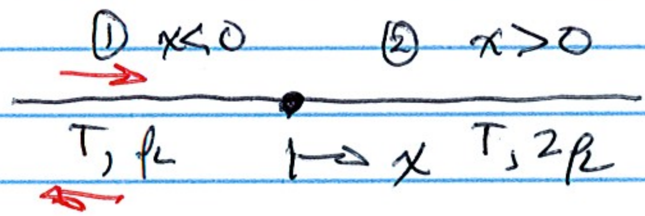
2.8.2



$c_1 = \sqrt{\dfrac{T}{\rho_L}}$ $\qquad$ $c_2 = \sqrt{\dfrac{T}{2\rho_L}}$

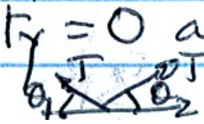
$k_1 = \dfrac{\omega}{c_1}$ $\qquad$ $k_2 = \dfrac{\omega}{c_2}$

$y_1 = Ae^{j(\omega t - k_1 x)} + Be^{j(\omega t + k_1 x)}$

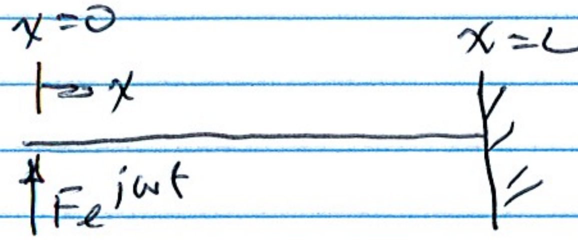
$y_2 = Ce^{j(\omega t - k_2 x)}$ $\qquad$ Two B.C.'s

(i) $y_1 = y_2$ at $x = 0$

Solve for $\left(\dfrac{C}{A}\right)$ $\qquad$ (ii) $\sum F_y = 0$ at $x = 0$

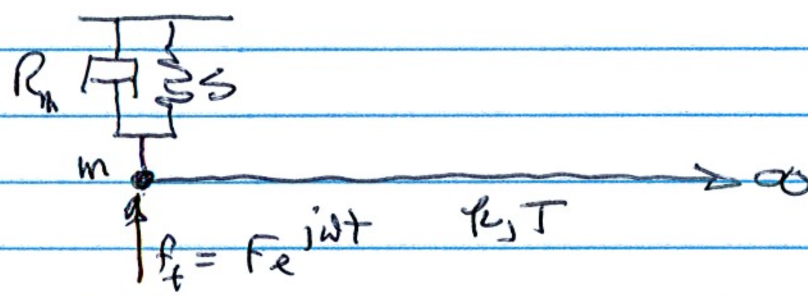


2.9.1



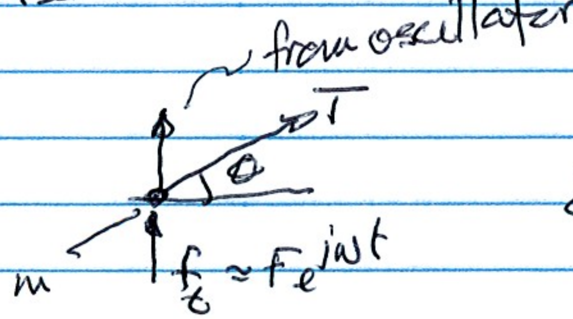
Review class notes

2.9.2



$$y = A e^{j(\omega t - kx)}$$

FBD



$$\sum f_y = ma$$

solve for A

Then $\quad Z_{m0} = \dfrac{F e^{j\omega t}}{\left.\dfrac{dy}{dt}\right|_{x=0}}$