wavenumber $k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

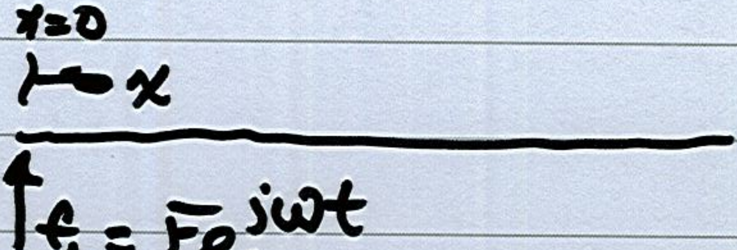
spatial frequency

$$y(x,t) = \underset{\approx}{A_1} e^{j(\omega t - \underset{\approx}{k} x)} + \underset{\approx}{A_2} e^{j(\omega t + \underset{\approx}{k} x)}$$

+ve -ve

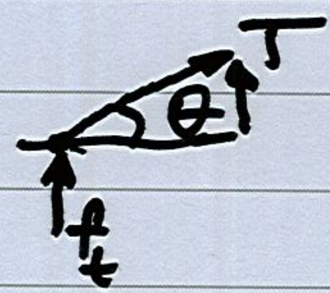
B.C.'s

2.3.2 Force B.C. at $x=0$

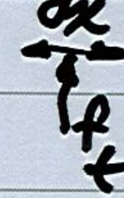


$f_t = \underline{Fe^{j\omega t}}$

FBD



$\sum f_y = 0$



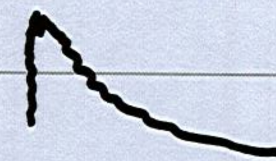
$f_t dx$

$$f_t + T\underline{\sin\theta}\Big|_{x=0} = 0$$

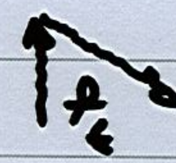
small θ's (linear small amplitude vibration)

$$\sin\theta \simeq \frac{dy}{dx}$$

$$f_t + T\frac{dy}{dx}\Big|_{x=0} = 0$$



$$\frac{\partial y}{\partial x}\Big|_{x=0} = -\frac{f_t}{T}$$



Is the special where $f_t = 0$

$\frac{\partial y}{\partial x} = 0$ at a free end

no <u>lateral</u> constraint

$$y(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$$

$$\frac{dy}{dx} = -jkAe^{j(\omega t - kx)} + jkBe^{j(\omega t + kx)}$$

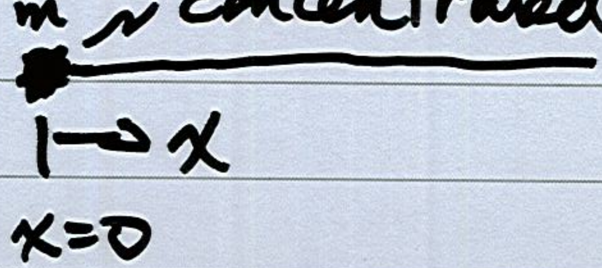
$$\left.\frac{dy}{dx}\right|_{x=0} = -jkA\cancel{e^{j\omega t}} + jkB\cancel{e^{j\omega t}} = -\frac{f_t}{T} = -\frac{F\cancel{e^{j\omega t}}}{T}$$

$$\boxed{-jkA + jkB = -\frac{F}{T}}$$

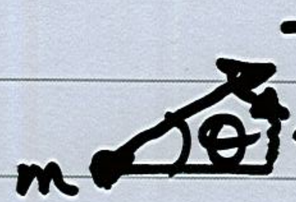
Force B.C.

2.3.3 Mass at $x = 0$

m ~ concentrated (point mass)



FBD

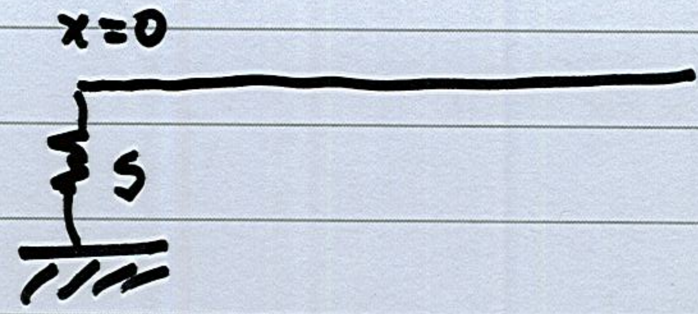


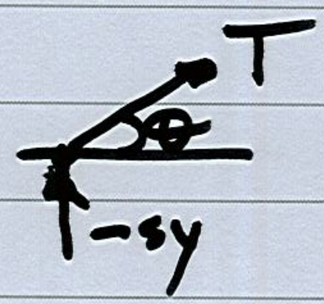
$\Sigma f_y = ma$

$$\underbrace{T\sin\theta}_{\frac{\partial y}{\partial x}}\Big|_{x=0} = m\frac{\partial^2 y}{\partial t^2}\Big|_{x=0}$$

$$\frac{\partial^2 y}{\partial t^2}\Big|_{x=0} = \frac{T}{m}\frac{\partial y}{\partial x}\Big)_{x=0}$$

2.3.4 Stiffness B.C.





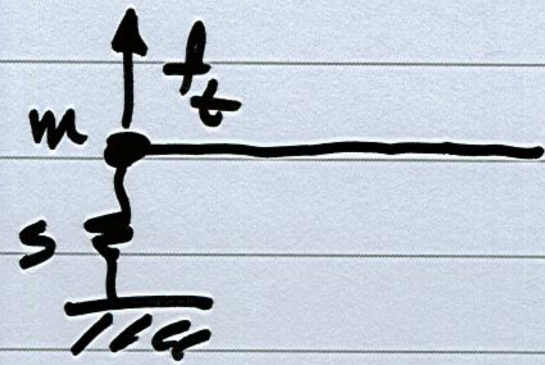
$$\sum f_y = 0$$

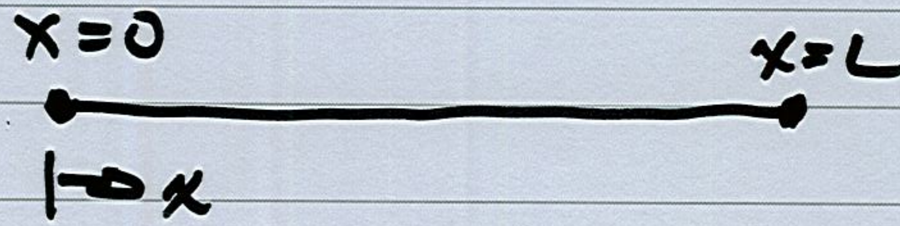
$$T\sin\theta\Big|_{x=0} - sy\Big|_{x=0} = 0$$

$$T\frac{dy}{dx}\Big|_{x=0} - sy\Big|_{x=0} = 0$$

$$\frac{dy}{dx}\Big|_{x=0} = \left(\frac{s}{T}\right) y\Big|_{x=0}$$

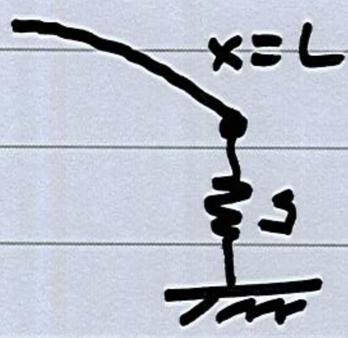
- fixed
- mass
- stiffness
- transverse

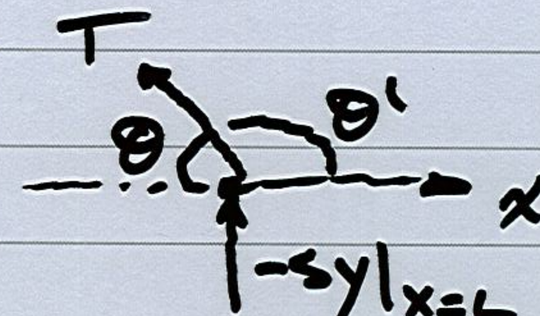




2.3.5 B.C applied at the positive x end of a string

e.g. stiffness





$$\Sigma f_y = 0$$

$$T\underline{\sin\theta'}\big|_{x=L} - sy\big|_{x=L} = 0$$

$$\sin\theta' \simeq -\frac{\partial y}{\partial x}$$

$$-T\frac{dy}{dx}\Big|_{x=L} = s\,y\Big|_{x=L}$$

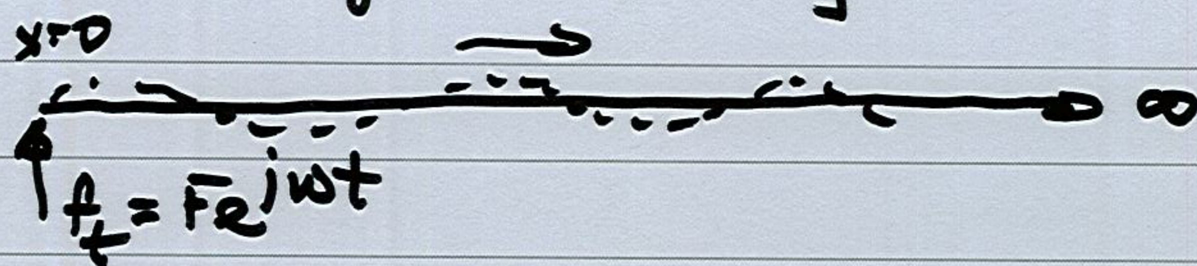
$$\frac{dy}{dx}\Big|_{x=L} = -\left(\frac{s}{T}\right)y\Big|_{x=L}$$

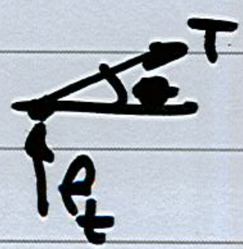
Sign of the b.c. depends on position

Exam

2.4 Forced Vibration

2.4.1 Semi-infinite string





$$\sum f_y = 0$$

$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = -\frac{f_t}{T}$$

$$y(x,t) = Ae^{j(\omega t - kx)} + \cancel{Be^{j(\omega t + kx)}}$$

nothing returns from infinity

$$y(x,t) = \underline{A}e^{j(\omega t - kx)}$$

$$\frac{\partial y}{\partial x} = -jkAe^{j(\omega t - kx)}$$

$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = -jkA\cancel{e^{j\omega t}} = -\frac{F}{T}\cancel{e^{j\omega t}}$$

$$A = \frac{F}{jkT}$$

$$\boxed{y(x,t) = \frac{F}{jkT}e^{j(\omega t - kx)}}$$

$\mathrm{Re}\{y(x,t)\}$ physical solution

Transverse Velocity

$$u(x,t) = \frac{\partial y}{\partial t} = j\omega \frac{F}{jkT} e^{j(\omega t - kx)}$$

$$k = \frac{\omega}{c}$$

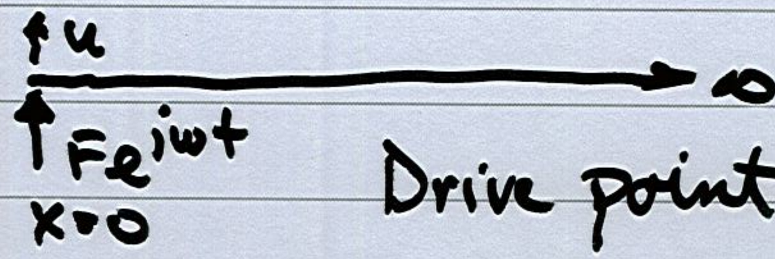
$$c = \sqrt{\frac{T}{\rho_L}} \Rightarrow T = \rho_L c^2$$

$$u(x,t) = \frac{F}{\rho_L c} e^{j(\omega t - kx)}$$

$\rho_L c$ ← characteristic impedance

"characteristic" of the medium & the wave type.

Input Mechanical Impedance



Drive point impedance

$$Z_{m0} = \frac{\text{Complex driving force at } x=0}{\text{velocity at the drive point}}$$

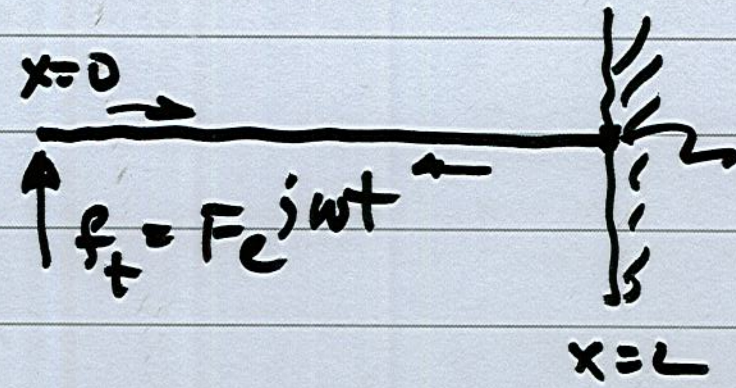
$$= \frac{\cancel{Fe^{j\omega t}}\,\rho_L c}{\cancel{Fe^{j\omega t}}}$$

$\boxed{Z_{m0} = \rho_L c}$ input mechanical impedance for a semi-infinite string

= characteristic impedance

- real - energy being carried to infinity

2.4.2 Finite Length String



fixed b.c. at $x = L$