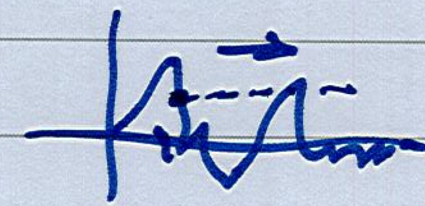
ME 513 session 8 9/6/19 1

Search for

dispersion of sound waves in ice sheets

$$\underbrace{\frac{\partial^2 y}{\partial x^2}} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$



$$y(x,t) = y_1(\underbrace{\overbrace{ct - x}}_{+ve}) + y_2(\underbrace{ct + x}_{-ve})$$

single frequency harmonic sol'ns

$$y(x,t) = Y(x)\, e^{i\omega t}$$

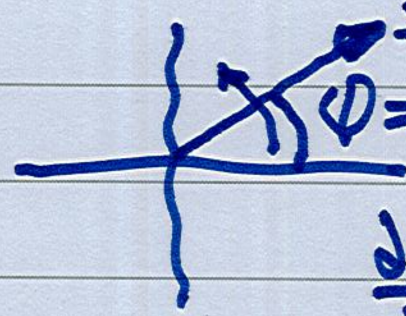
SHE $\quad \frac{d^2 Y}{dx^2} + k^2 Y = 0 \qquad k = \frac{\omega}{c}$

$y(x) = A e^{\pm jkx}$

$y(x,t) = A e^{\pm jkx} e^{j\omega t}$

what is k? wavenumber

time



$\Phi = \omega t$ $\quad e^{j\omega t}$ $\quad \omega t = \Phi$

$\frac{\partial \Phi}{\partial t} = \omega$

ω: rate of change of phase with time

T = period $\quad f = \frac{1}{T} \quad \omega = \frac{2\pi}{T} \; \frac{\text{rad}}{s}$

space

$\Phi = kx \quad e^{jkx}$

e^{-jkx}

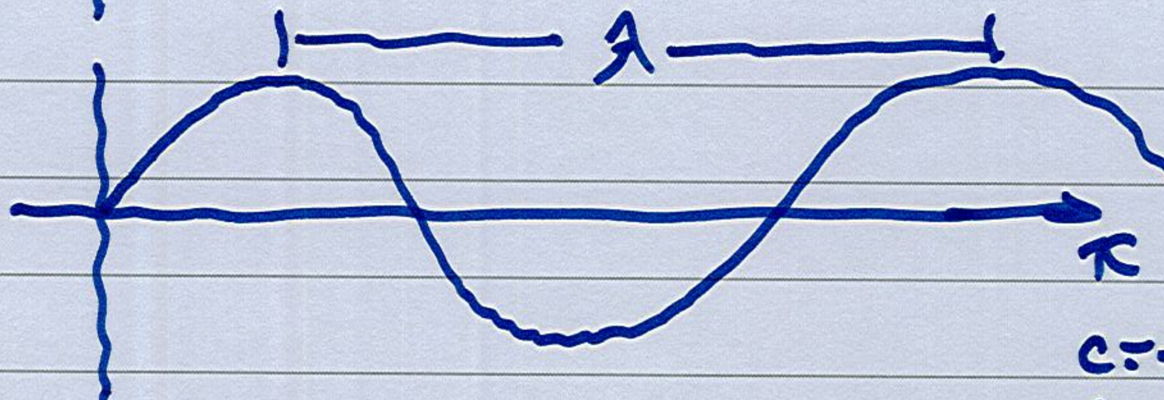
$\frac{\partial \Phi}{\partial x} = k$

k: rate of change of phase with position
- spatial frequency

time

$\omega = \frac{2\pi}{T}$

space



$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$

$c = f\lambda$

$\lambda = \frac{c}{f}$

$k_2 > k_1$

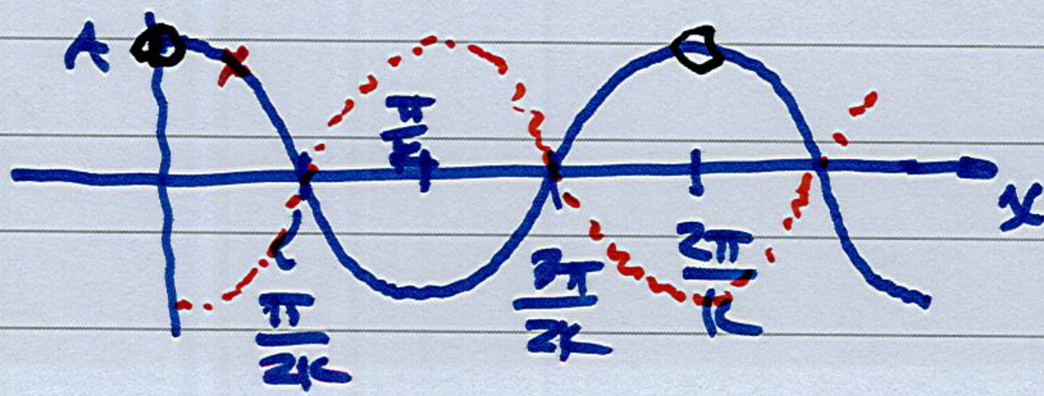
larger wavenumber, shorter wavelength

$$y = Ae^{j(\omega t - kx)}$$
$$= Ae^{j\omega t}\underbrace{e^{-jkx}}$$

$$\underline{\cos(-kx)} - j\sin(-kx)$$
$$\cos(-kx) = \cos(kx)$$

$Re\{y\}$ at $\underline{t=0}$ when A = real

$$Re\{y\} = A\cos \underbrace{kx}$$



$$kx = \frac{\pi}{2}$$
$$x = \frac{\pi}{2k}$$

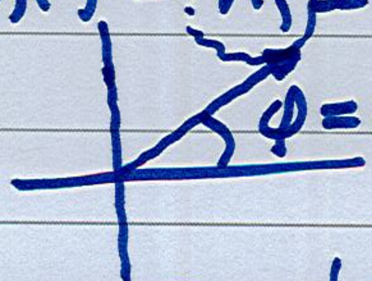
at $t = \frac{T}{2}$

when $t = T$
wave pattern
advances by
one wavelength

	Time	Space
frequency	ω	k
period	T	λ

General harmonic solution

$$y(x,t) = \hat{A}_1 e^{j(\omega t - kx)} + \hat{A}_2 e^{j(\omega t + kx)} \quad *$$



$\phi = \omega t - kx$ - hold constant

- follow a point on the wave

transverse velocity

$$\frac{\partial y}{\partial t}(x,t) = u(x,t) = j\omega\, y(x,t)$$

transverse acceleration

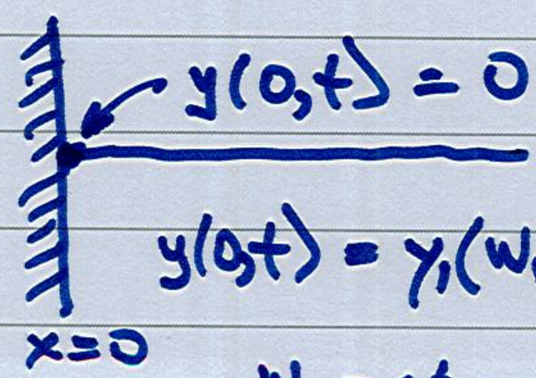
$$\frac{\partial^2 y}{\partial t^2}(x,t) = a(x,t) = \frac{\partial u}{\partial t} = (j\omega)(j\omega)\, y(x,t) = -\omega^2 y$$

2.3 ~~B~~ Boundary Conditions

General Soln: $y(x,t) = y_1(w_1) + y_2(w_2)$

$$w_1 = ct - x \qquad w_2 = ct + x$$

2.3.1 Fixed



$$y(0,t) = y_1\left(w_1\big|_{x=0}\right) + y_2\left(w_2\big|_{x=0}\right) = 0$$

$$w_1 = ct - x \qquad w_2 = ct + x$$

$$w_1\big|_{x=0} = ct \qquad w_2\big|_{x=0} = ct$$

$$y_1(ct) + y_2(ct) = 0$$

$$y_2(\underline{ct}) = -y_1(\underline{ct})$$

so generally true that

$y_2(w_2) = -y_1(w_1)$ when $w_1 = w_2$

at $x = 0$ y_1 & y_2 must be equal and opposite - cancelling with each other to create zero displacement

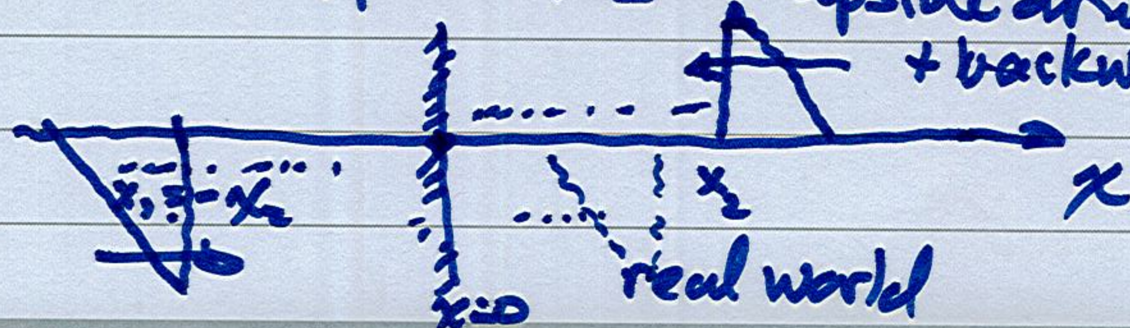
so at a particular time $t = t_1$

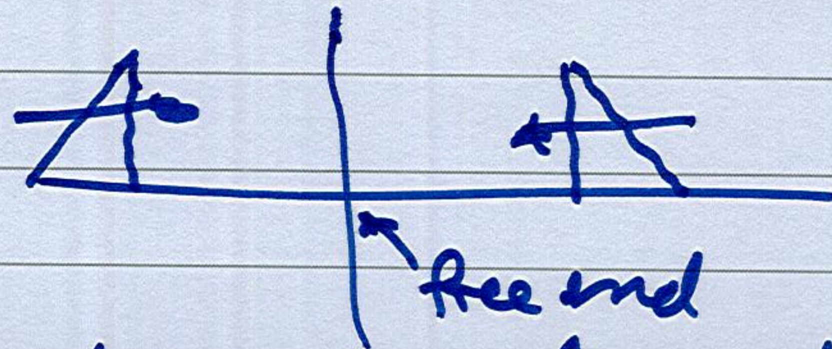
$w_1 = \cancel{ct_1} - x_1$ $w_2 = \cancel{ct_1} + x_2$

$w_1 = w_2$

$x_1 = -x_2$

- hard boundary causes a reflection upside down + backwards





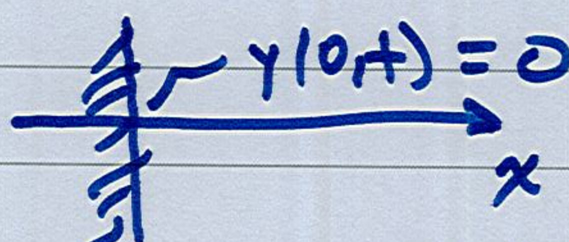
zero transverse force at the free end of a string

$$\frac{\partial y}{\partial x} = 0$$

frequency domain

$$y(x,t) = \underset{\sim}{A}\, e^{j(\omega t - kx)} + \underset{\sim}{B}\, e^{j(\omega t + kx)}$$

two bc's necessary to solve for A + B



$$\sin\theta = \frac{e^{+j\theta} - e^{-j\theta}}{2j}$$

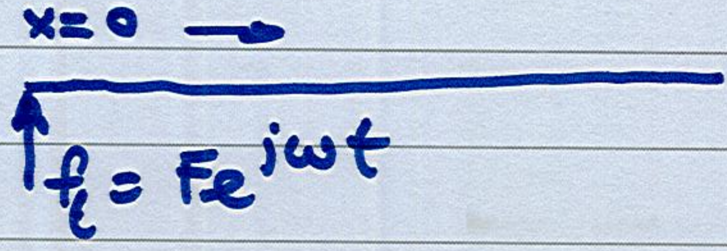
$$y(0,t) = A\cancel{e^{j\omega t}} + B\cancel{e^{j\omega t}} = 0$$

$$B = -A$$

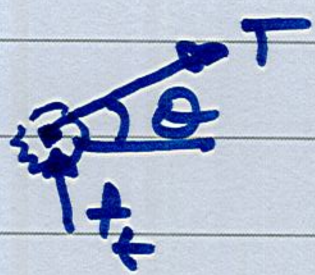
$$y(x,t) = A e^{j\omega t}\left(e^{-jkx} - e^{+jkx}\right)$$

$$= -2j A e^{j\omega t} \sin kx$$

2.3.2 Force b.c. at $x=0$



FBD



$$\Sigma f_y = 0$$