ME 513 Session 7 9/4/19 1

TA office hours

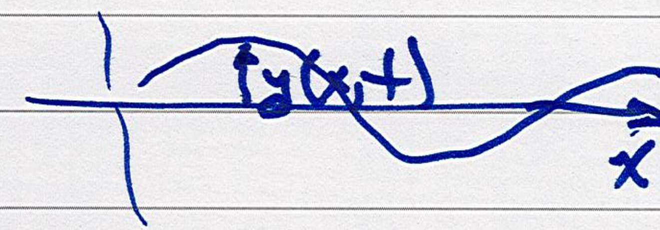
Wed 6-7 PM offsite folks

Friday 3-4 ME 6005

HLAB 2002 9-11 Weds

Vibration of strings

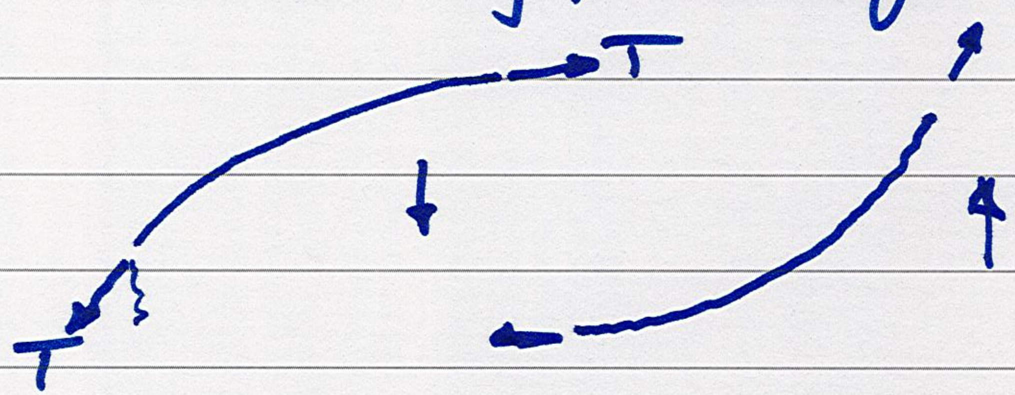
$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$



$$c = \sqrt{\frac{T}{\rho_L}}$$

Wave eqn governing <u>small amplitude</u> transverse displacement of a tensioned string

Restoring force eqn



2.2 Solutions of the Wave Eqn

Two independent ~~v~~ variables
x & t space & time

2.2.1 General Solution

$$y(x,t) = y_1(\underbrace{ct - x}_{w_1}) + y_2(\underbrace{ct + x}_{w_2}) \checkmark$$

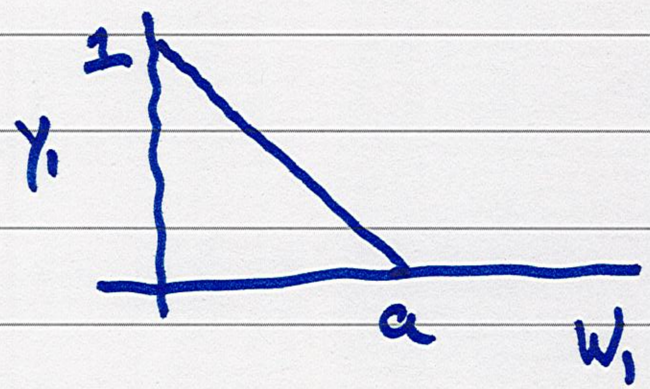
$$= y_1(w_1) + y_2(w_2)$$

$$w_1 = ct - x \qquad w_2 = ct + x$$

y_1 & y_2 are <u>any</u> functions of a single variable.

Prove by direct substitution that that this is the sol'n

Simple example



$y_1 = 1$ at $w_1 = 0$

$y_1 = 0$ at $w_1 = a$

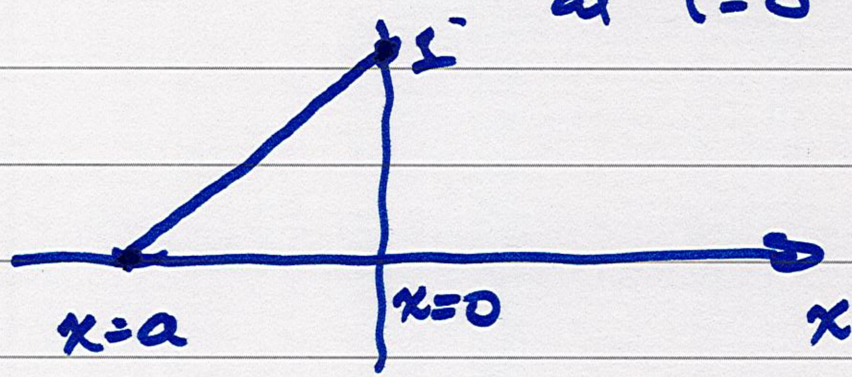
plot as a function of x at $t=0$

$w_1 = ct - x$ $\qquad$ $x = -w_1$ $\qquad$ $w_1 = 0$ $\quad$ $x = 0$

at $t=0$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $w_1 = a$ $\quad$ $x = -a$

$w_1 = -x$

at $t=0$

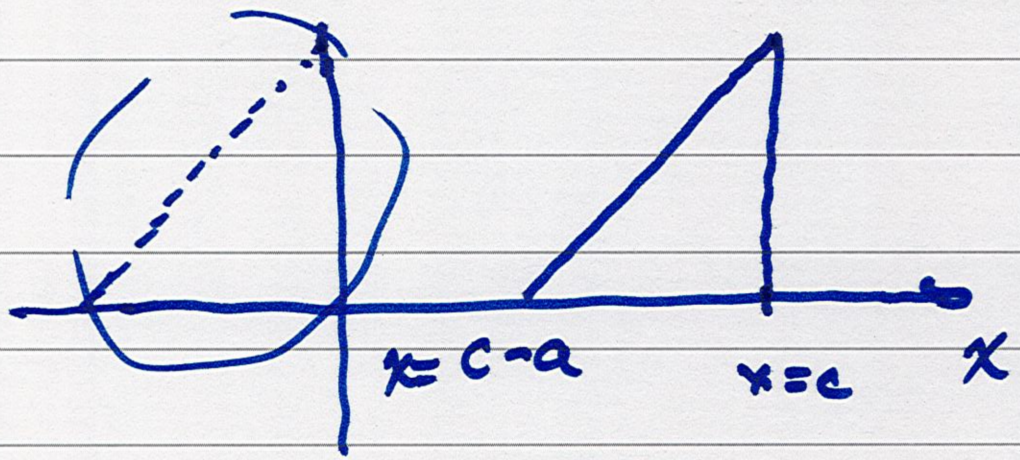


plot as a function of x at $t = 1s$

$w_1 = c - x$ at $t = 1s$ $\qquad x = c - w_1$

$w_1 = 0 \quad x = c$

$w_1 = a \quad x = c - a$



speed of propagation

$\frac{\text{distance}}{\text{time}} = \frac{c}{1}$

$c = \sqrt{\frac{T}{\rho_L}}$

* shape has remained the same - linear wave propagation
  - non-dispersive

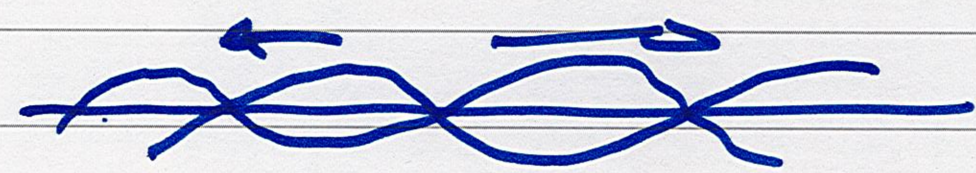
$y_1(\underbrace{ct - x}_{w_1})$ disturbance traveling in the +ve x-direction without changing shape

$y_2(\underbrace{ct + x}_{w_2})$ disturbance that travels in the ~~+~~ -ve x-direction

by holding the value of w_1 constant
- follow a particular point on the wave as time advances

$y(x,t) = y_1 + y_2$

General solution - superposition of waves traveling to the right + left



What is the transverse of the string

$$v_t = \frac{\partial y_1}{\partial t} = \frac{\partial y_1(ct-x)}{\partial(ct-x)} \frac{\partial(ct-x)}{\partial t} \sim c$$

$$= c\,\underbrace{\frac{\partial y_1(ct-x)}{\partial(ct-x)}} \neq c$$

v_t depends on y_1

speed of wave propagation
$\neq$ transverse velocity of
the string

complete sol'n

$$y(x,t) = \underbrace{y_1(ct - x)}_{+ve\text{-going}} + \underset{-ve\text{-going}}{y_2(ct + x)}$$

~~Prov~~ - functions propagate without changing shape

- non-dispersive wave prop
  - all frequency components travel at the same speed

2.2.2 Harmonic Single Frequency Solutions

Assume a separable form of solution

$$\underline{y(x,t) = Y(x)e^{j\omega t}}$$

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$

$$\frac{d^2 Y}{dx^2}\cancel{e^{j\omega t}} + \left(\frac{\omega^2}{c^2}\right) Y \cancel{e^{j\omega t}} = 0$$

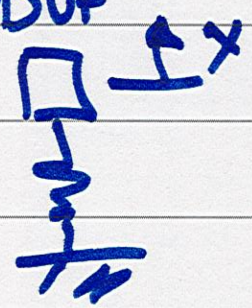
scalar Helmholtz Eqn.

define $k^2 = \frac{\omega^2}{c^2}$ $k = \frac{\omega}{c}$ wavenumber

$$\boxed{\frac{d^2y}{dx^2} + k^2 y = 0} \qquad (4) \qquad e^{j\omega t}$$

SHE governs the spatial dependence of the solution

SDOF



$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0 \qquad x = Ae^{\pm j\omega_0 t}$$

$$Y(x) = Ae^{\pm jkx}$$ two sol'ns

Solution for the spatial dependence of a single frequency transverse vib of a string

$$y(x,t) = A_1 e^{+ikx} e^{j\omega t} + A_2 e^{-ikx} e^{j\omega t}$$

$$= A_1 e^{+i(kx+\omega t)} + A_2 e^{+i(-kx+\omega t)}$$

$$= A_1 e^{+jk(x+ct)} + A_2 e^{+ik(-x+ct)}$$

$y_2(ct+x)$ $\qquad$ $y_1(ct-x]$

-ve going wave $\qquad$ +ve going wave

what is k?

e^{+ikx}

$e^{+i\theta}$