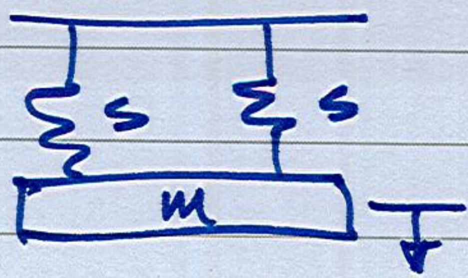
ME 513 Session 6 8/30/19

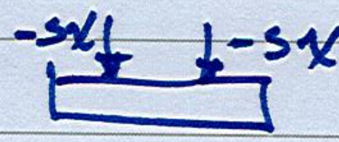
Homework Hints

1.2.1 $\frac{d^2x}{dt} + \omega_0^2 x = 0$

restoring force eqn]
EOM

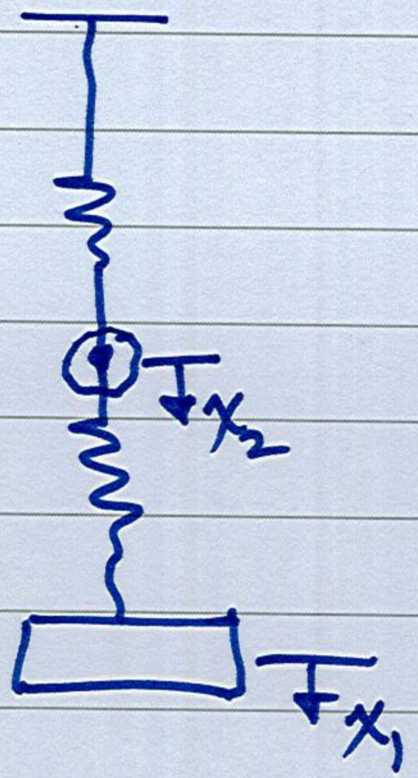
$\frac{2s}{m}$



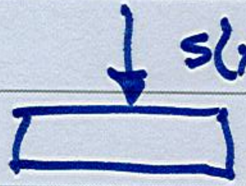


$f = -2sx$

$f = ma = m\frac{d^2x}{dt}$



$f = \frac{\cancel{ma}}{0}$



$s(x_1 - x_2) =$.

$f = -s(x_1 - x_2)$

$f = m\frac{d^2 x_1}{dt^2}$ $\approx$

1.3.3C

Eqn 1.3.1 $\quad x = x_0 \cos \omega_0 t + \frac{u_0}{\omega_0} \sin \omega_0 t$

Plot $\left(\frac{x}{A}\right)$ vs $\frac{t}{T}$ $\quad \omega_0 = \frac{2\pi}{T}$

three different sets of IC's
3 plots

1.5.4 $\quad A = x + jy \quad B = X + jY$

$|A|$

Additional problem #1

1.6.12 $\tilde{x} = \underline{e^{-\check{\beta} t}} \left(\check{A}_1 e^{j \check{\omega}_0 t} + \check{A}_2 e^{-j \check{\omega}_0 t} \right)$

find the real part of the solution

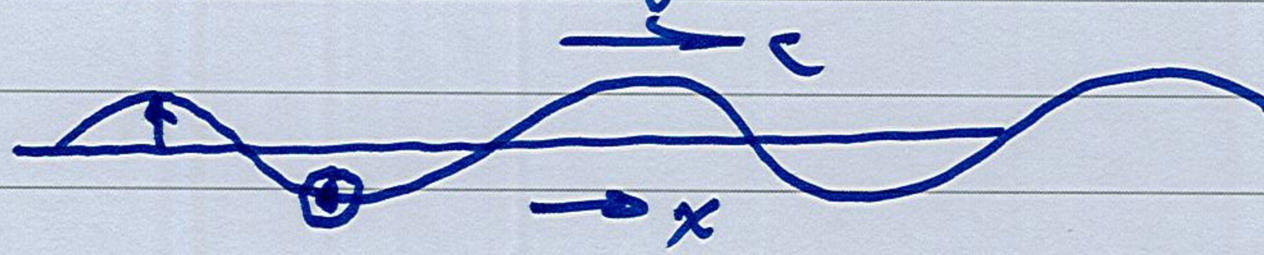
Additional problem #2

$Z_m = R_m + j\left(\underline{\omega m - \frac{s}{m}}\right)$

Imag - $\underline{\log \omega}$

2. Vibrating String

- vibration of extended systems
- derive a wave eqn

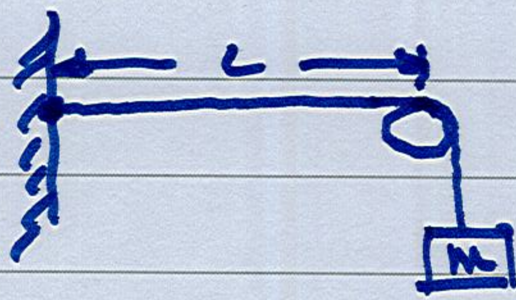


- transverse wave motion
  - particle motion is $\perp$ to direction of prop.
- phase speed
  - particle velocity
- wave number - spatial frequency

- application of b.c.'s
- characteristic impedance
- standing waves
  vs propagating waves

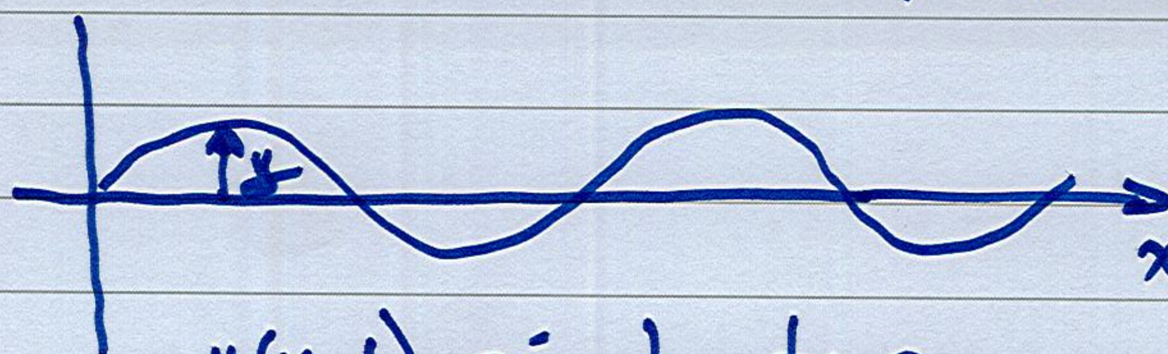
2.1 Derivation of a wave eqn

- governs transverse vibration of a tensioned string



μ = mass/unit length (linear density)

T = tension force



$y(x,t)$ - instantaneous transverse displacement vs position

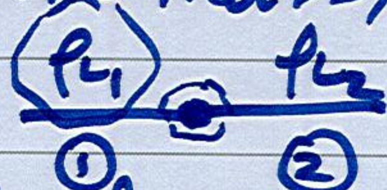
Assumptions:

1. Uniform static tension - independent of transverse displacement

2. Small amplitude responses - linear approximation

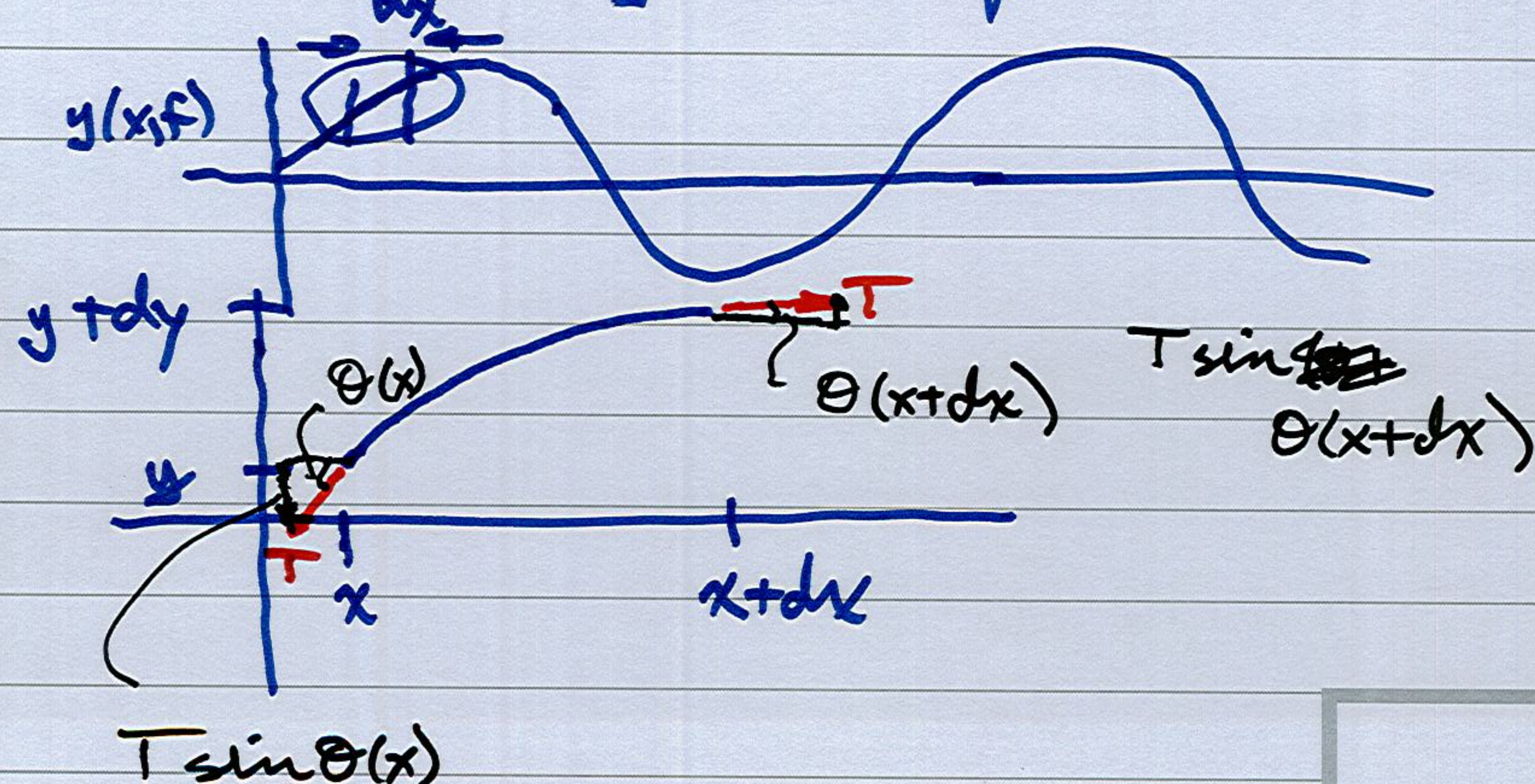
3. No losses

4. Uniform mass/unit length



5. Gravity forces are negligible - orientation is not significant

2.1.1 Restoring Force Eqn



net vertical force in
the y-direction

$$df_y = T\sin\theta(x+dx) - T\sin\theta(x)$$

Taylors Series Expansion

$$f(x+dx) = f(x) + \frac{df}{dx}dx + \underbrace{\cancel{\frac{1}{2}\frac{d^2f}{dx^2}(dx)^2}}_{\to 0} + \ldots$$

Small amplitude assumption
- series can be truncated

$$\sin\theta(x+dx) \simeq \sin\theta(x) + \frac{d[\sin\theta(x)]}{dx}dx + \ldots$$

$$df_y = \cancel{T\sin\theta(x)} + T\frac{d[\sin\theta(x)]}{dx}dx$$

$$- \cancel{T\sin\theta(x)}$$

$$df_y = T \frac{d[\sin\theta(x)]}{dx} dx$$

small displacements

θ is always small

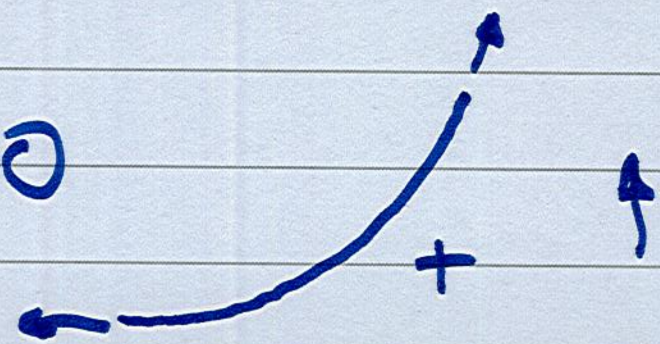
when θ is small

$$\sin\theta \simeq \tan\theta = \underline{\underline{\frac{dy}{dx}}} \quad \text{slope}$$

$$df_y \approx T \underline{\underline{\frac{\partial\left[\frac{dy}{dx}\right]}{\partial x}}} dx$$

$$\boxed{df_y \approx T\left(\frac{\partial^2 y}{\partial x^2}\right)dx} \quad (1)$$

$\frac{d^2y}{dx^2} > 0$



$\frac{d^2y}{dx^2} < 0$

no curvature

- no net force

2.1.2 EOM

$f = ma$

ρ_L = mass/unit length

$m = \rho_L \, dx$

$f = df_y$

$a = \frac{\partial^2 y}{\partial t^2}$

$$\boxed{df_y = \rho_L \, dx \frac{\partial^2 y}{\partial t^2}} \quad (2)$$

2.1.3 Wave Eqn

(1) = (2)

$$T \frac{\partial^2 y}{\partial x^2} \cancel{dx} = \rho_L \cancel{dx} \frac{\partial^2 y}{\partial x^2}$$

$$\frac{\partial^2 y}{\partial x^2} - \left(\frac{\rho_L}{T}\right) \frac{\partial^2 y}{\partial t^2} = 0$$

2nd order PDE

$$\boxed{\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2} = 0} \quad 3$$

$$c = \sqrt{\frac{T}{\rho_L}}$$