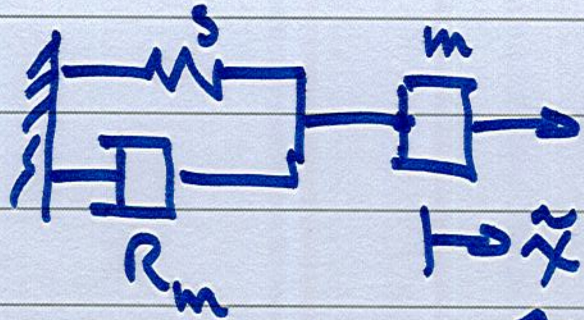
ME 513 Session 5 8/28/19

1

Mechanical Impedance



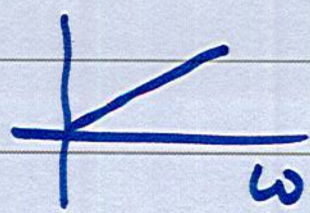
$\hat{f}_e(t) = Fe^{j\omega t}$

steady-state response

$\tilde{z}_m = \dfrac{\tilde{f}_e}{\tilde{u}}$

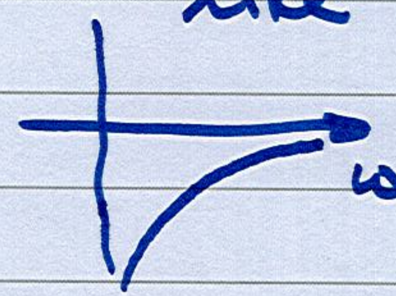
$j\omega m$

masslike



$-i\dfrac{s}{\omega}$

stiffness-like



if $\tilde{z}_m$ is known

$\tilde{u} = \dfrac{\tilde{f}_e}{\tilde{z}_m}$

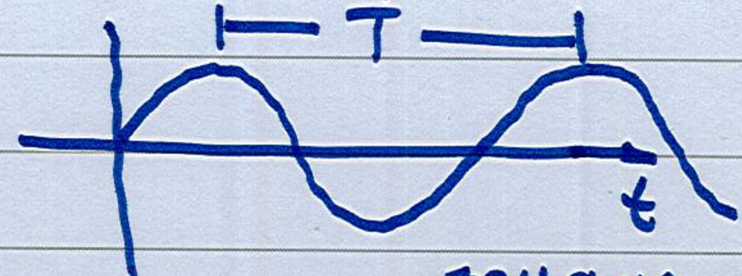
Resonance

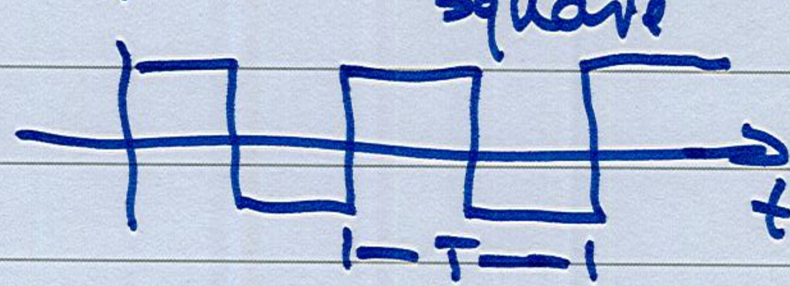
$\mathrm{Im}\{\tilde{z}_m\} \to 0$

$\omega \to \omega_0 = \sqrt{\dfrac{s}{m}}$

1.5 Fourier Analysis - ME 579

Periodic signals - repeats itself in T



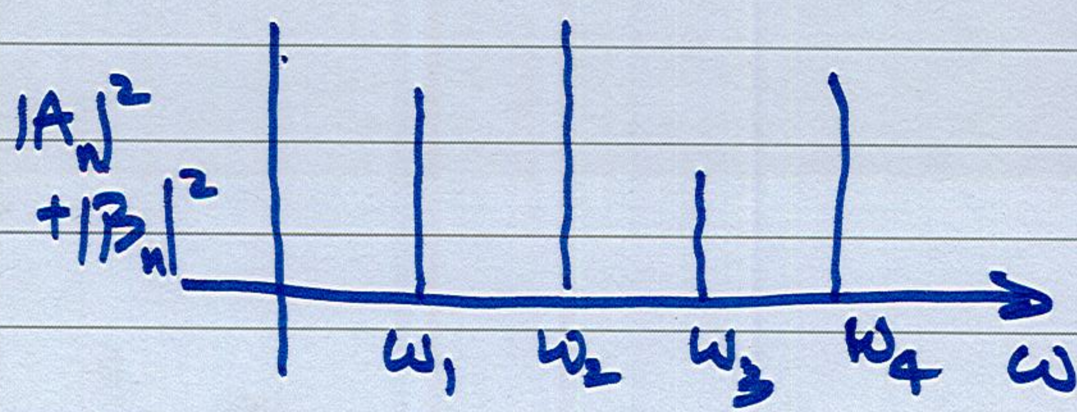


~~An~~ Any <u>single-valued</u> periodic function with period T can be represented <u>exactly</u> as a sum of sinusoids periodic in T

$f(t)$ periodic sV

$$f(t) = \frac{1}{2}A_0 + A_1 \cos\omega_1 t + A_2 \cos\omega_2 t + \ldots\ldots$$
$$+ B_1 \sin\omega_1 t + B_2 \sin\omega_2 t + \ldots\ldots$$

$\omega_1 = \frac{2\pi}{T}$ $\quad \omega_2 = 2\omega_1$ $\quad \omega_3 = 3\omega_1 \ldots\ldots$

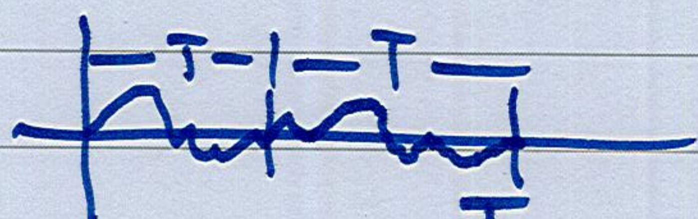


signal is represented by contributions at discrete frequencies

$\omega_1 = \frac{2\pi}{T}$ fundamental – first harmonic

$\omega_2 = 2\omega_1$ 2nd harmonic

$\omega_3 = 3\omega_1$ 3rd harmonic

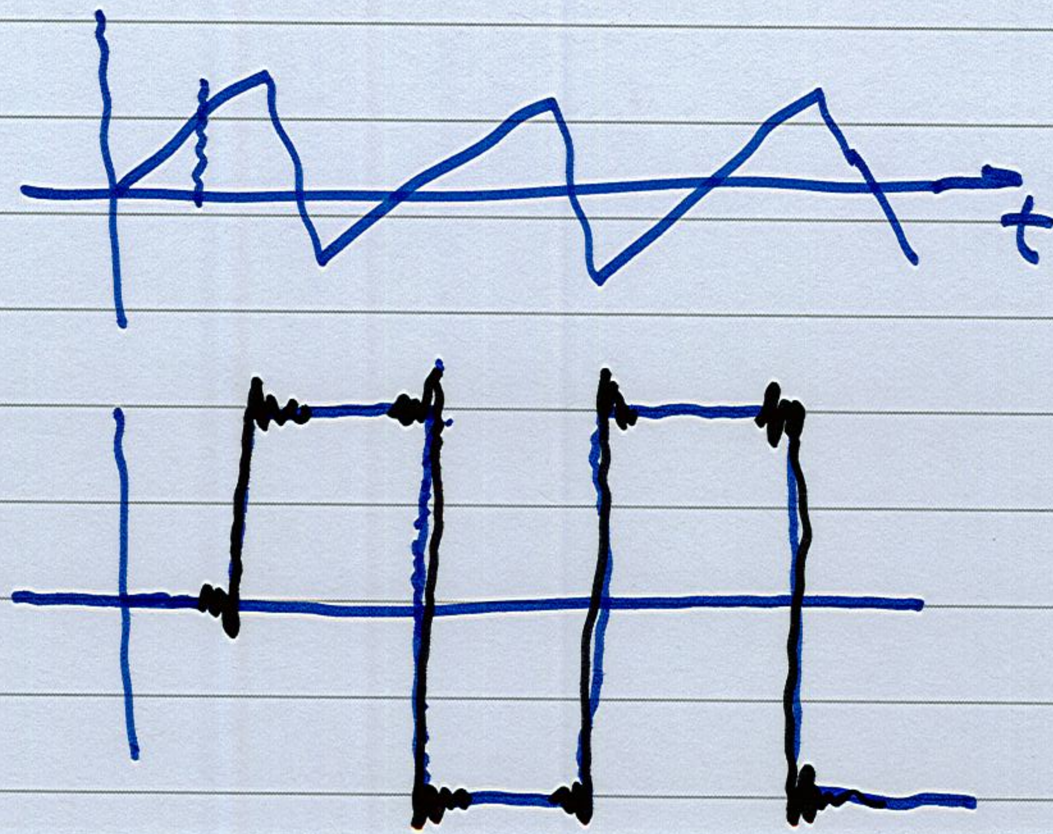


$$A_n = \frac{2}{T}\int_0^T f(t)\cos(\omega_n t)\,dt$$

3

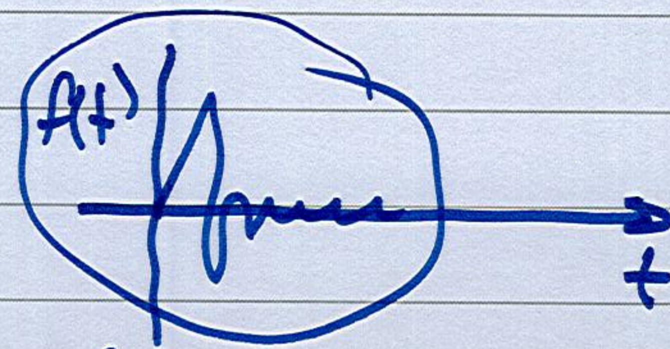
$$B_n = \frac{2}{T}\int_0^T f(t)\sin(\omega_n t)\,dt$$

Periodic signals → Fourier series



Gibbs phenomenon

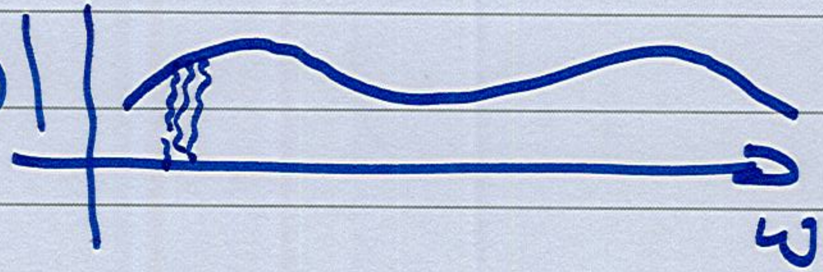
transient



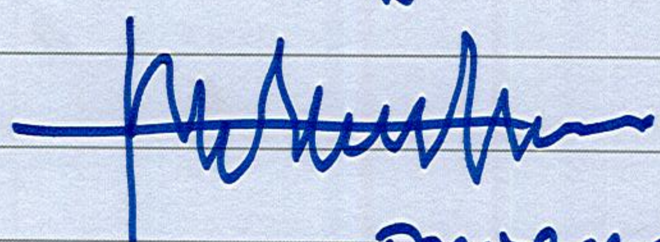
Fourier Transforms

$e^{j\omega t}$

$|F(\omega)|$



continuous - non-periodic

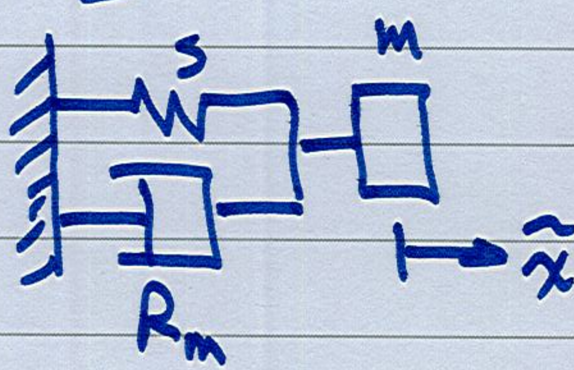


- power spectrum
- power spectral density

Summary

Physical System

SDOF



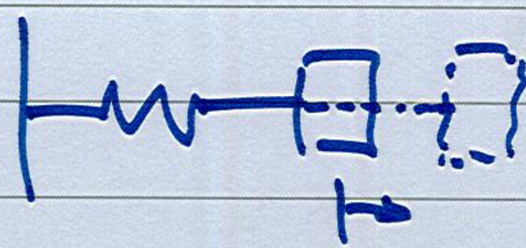
mathematical models

* To oscillate:
  mass + stiffness

Approach to problem solving

1. Identification of Governing Eqn's
   - restoring force
   - EOM ($F = ma$)
2. combine → 2nd order ODE
3. Identify possible solutions
   - expressed in a convenient form
4. Find the solutions that satisfy the boundary conditions

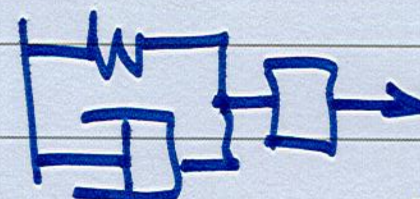
<u>Free response</u> – driven through initial conditions



x_0 , u_0

ω_0 system responds at the natural frequency

<u>Forced Response</u>



$f_e(t) = Fe^{j\omega t}$

- linear system in steady state response
- responds at the driving frequency

$Fe^{j\omega t} \rightarrow \hat{x}(t) = \underline{\hat{A}}\, e^{j\omega t}$

Resonance

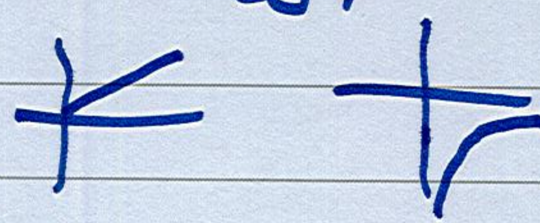
- system is driven at the natural frequency

$Im\{\tilde{Z}_m\} \to 0$ response of the system is a maximum

Mechanical Impedance

$$\tilde{Z}_m = R_m + i\left(\omega m - \frac{s}{\omega}\right)$$

$=$



$$\tilde{u} = \frac{\tilde{f}_e}{\tilde{Z}_m}$$