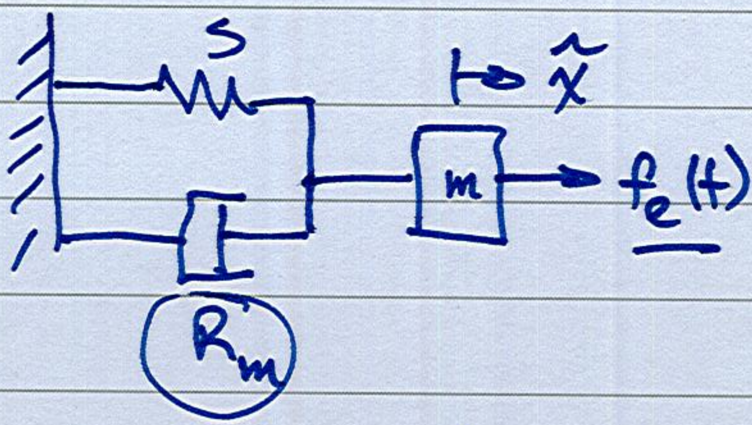
ME 513 session 4 8/26/12



$$f_e = F_e e^{j\omega t}$$

steady-state response
- system responds at the driving frequency

$$\tilde{x} = \frac{F_e e^{j\omega t}}{j\omega\left(R_m + j\left(\omega m - \frac{s}{\omega}\right)\right)}$$

zero at the natural freq
- large response
- resonance

1.3.3 Mechanical Impedance

Define $\tilde{Z}_m = \dfrac{\text{complex driving force}}{\text{complex steady-state velocity at the driven point}}$

$$= \frac{\tilde{f}}{\tilde{u}}$$

Damped SDOF

$$\tilde{Z}_m = \frac{F e^{j\omega t}}{\dfrac{F e^{j\omega t}}{R_m + j\left(\omega m - \frac{s}{\omega}\right)}}$$

$$= R_m + j\left(\omega m - \frac{s}{\omega}\right)$$

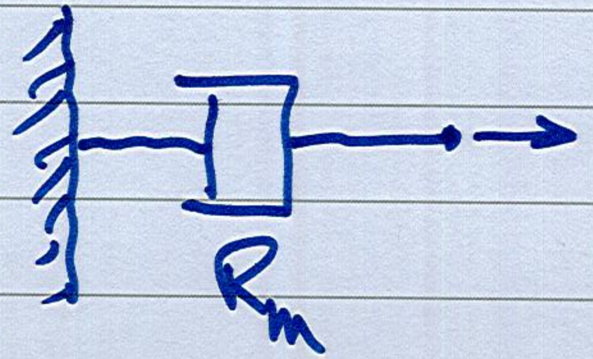
$$\tilde{z}_m = \underbrace{R}_{\text{mechanical resistance}} + j\underbrace{X}_{\text{mechanical reactance}}$$

$$R = Re\{\tilde{z}_m\}$$

$$X = Im\{\tilde{z}_m\}$$

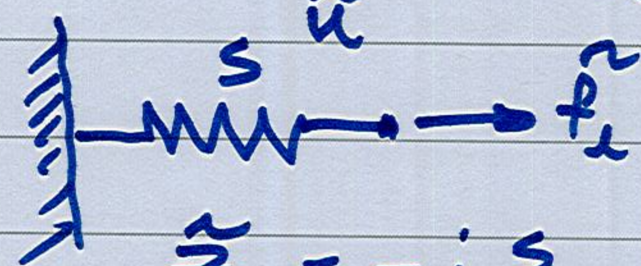
$$R = R_m$$

$$X = \left(\omega \underset{\sim}{m} - \frac{\tilde{s}}{\omega}\right)$$

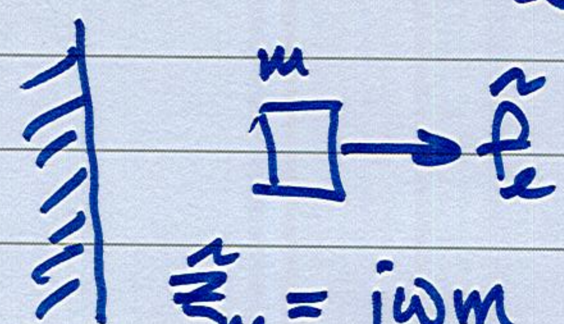


$\tilde{f}_e = F_e^{\,j\omega t}$

$\tilde{z}_m = \dfrac{\tilde{f}_e}{\tilde{u}} = R_m$ <u>real</u> and positive



$\tilde{z}_m = -j\dfrac{s}{\omega}$ imaginary, negative & inversely proportional to frequency



$\tilde{z}_m = j\omega m$ imaginary, positive proportional to freq

1.3.4 Mechanical ~~Response~~ Resonance

Defn: Occurs when the imaginary of the input mechanical impedance goes to zero

$$\mathrm{Im}\{\tilde{Z}_m\} = 0$$

$$\tilde{Z}_m = R_m + i\left(\omega m - \frac{s}{\omega}\right)$$

$$\mathrm{Im}\{\tilde{Z}_m\} = \omega m - \frac{s}{\omega}$$

$$\omega m - \frac{s}{\omega} = 0$$

$$\omega_0^2 = \frac{s}{m}$$

ω_0 - undamped natural freq of the system

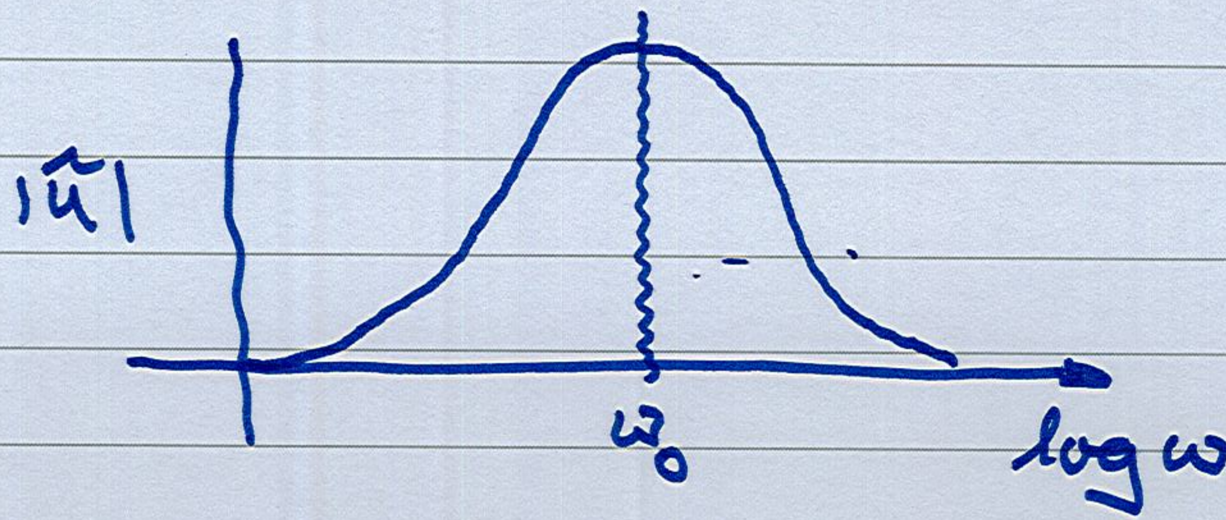
Resonance occurs when the system is driven at the undamped natural frequency

$$\hat{u} = \frac{F e^{j\omega t}}{R_m + j\underbrace{\left(\omega m - \frac{s}{\omega}\right)}_{=0 \text{ when } \omega = \omega_0}}$$

drive at the natural freq

velocity → maximum

& $\tilde{z}_m$ is purely real $= R_m$



3 frequency ranges of interest

(i) $\omega < \omega_0$

(ii) $\omega \simeq \omega_0$

(iii) $\omega > \omega_0$

$$\tilde{z}_m = R_m + i\left(\omega m - \frac{s}{\omega}\right)$$

(i) $\omega < \omega_0$

stiffness controlled regim

$\tilde{z}_m \simeq -i\frac{s}{\omega}$ $\qquad \tilde{x} \simeq \left(\frac{F}{s}\right) e^{i\omega t}$

$|\tilde{x}|$ is independent of frequency

(ii) near resonance

$\omega \simeq \omega_0$

$\tilde{z}_m \simeq R_m$ $\qquad \tilde{x} \simeq \frac{F e^{i\omega t}}{j\omega R_m}$

$$\tilde{Z}_m = R_m + j\left(\omega m - \frac{s}{\omega}\right)$$

$$\tilde{u} = j\omega\tilde{x} \approx \frac{F}{R_m}e^{j\omega t}$$

$|\tilde{u}| \approx \dfrac{F}{R_m}$ independent of frequency

"damping controlled region"

(iii) $\omega > \omega_0$ "mass controlled region"

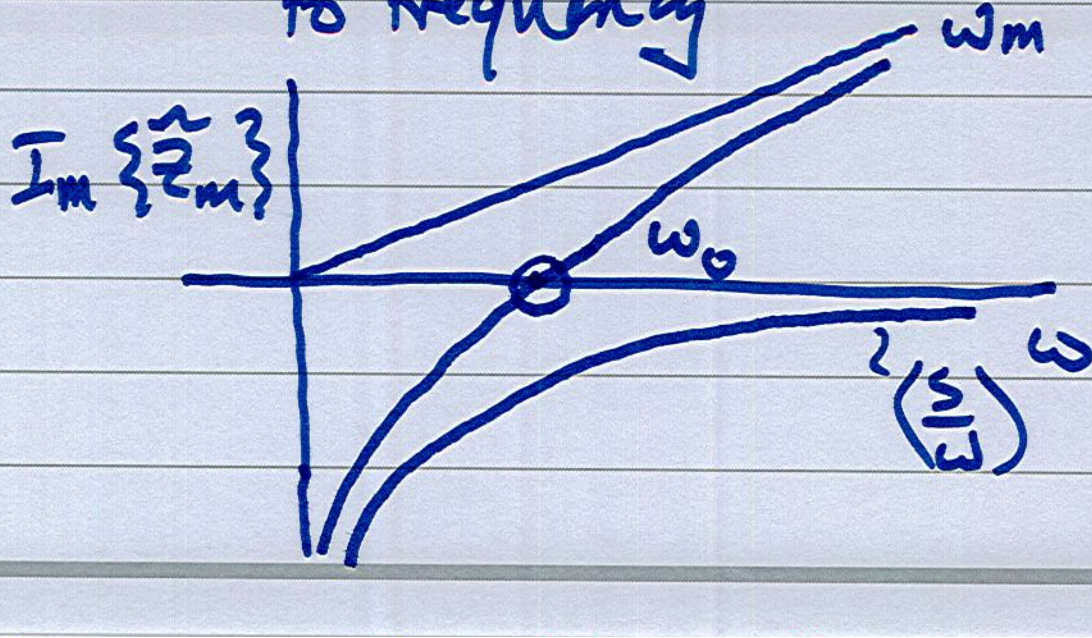
$$\tilde{Z}_m \approx j\omega m$$

$$\tilde{a} = j\omega\tilde{u} \approx \left(\frac{F}{m}\right)e^{j\omega t}$$

acceleration is independent of frequency

transducers are designed to operate in one of these regions

$\tilde{z}_m \simeq j\omega m$	$\tilde{z}_m \simeq -j\frac{s}{\omega}$
- imag	- imag
- positive	- negative
- proportional to frequency	- inversely prop. to freq.



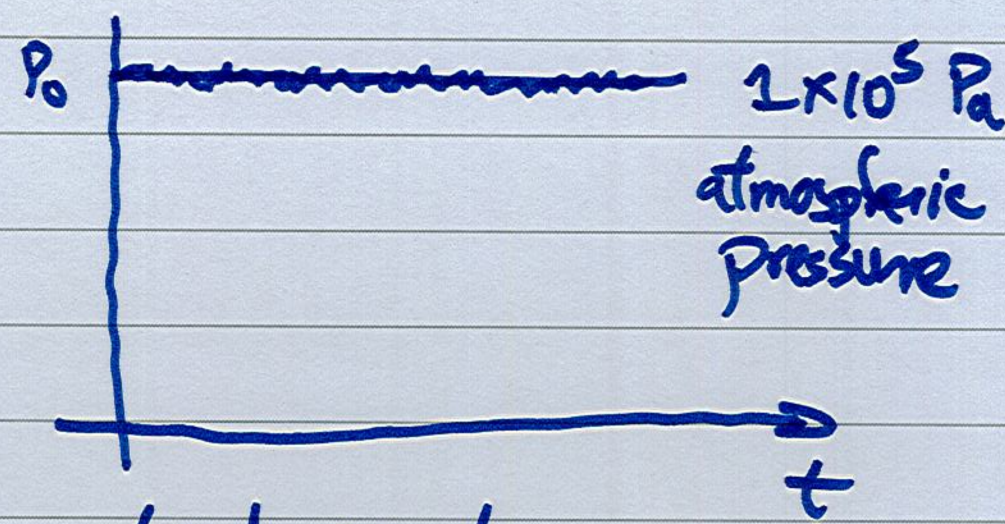
$$\tilde{z}_m = \frac{\tilde{f}_e}{\tilde{u}}$$

$$\tilde{u} = \frac{\tilde{f}_e}{\tilde{z}_m}$$

can predict the response to
any force if the impedance
is known (from measurement
or analysis) and if
the system is linear

1.9 Superposition

"linear" acoustics
"linear" vibration
- small amplitude fluctuations in sound pressure or velocity



loud sound
94 dB → 1 Pa

* linear system, small amplitudes

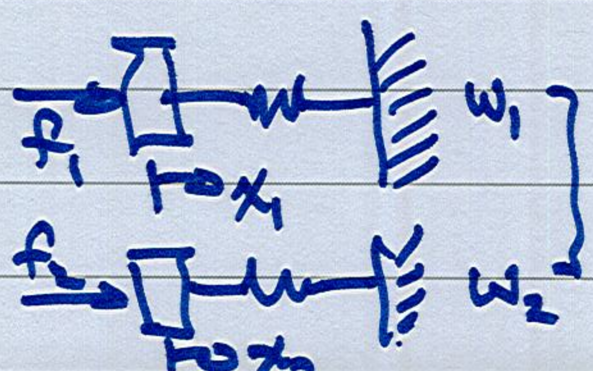
- output of a linear system is linearly proportional to the input

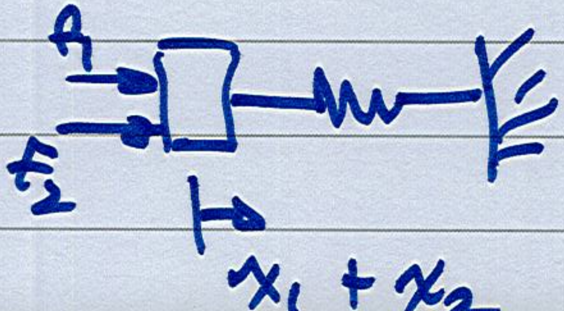
- double the force - response doubles

- linear system responds at the forcing frequency

$$Fe^{j\omega t} \longrightarrow \tilde{A}e^{j\omega t} = \tilde{x}$$

- response to twos or inputs is the linear sum of the individual responses





- convenient to break up input forces into individual freq components (freq analysis)
- find the response to each freq component
- sum up individual responses to give the total response