ME 513 Session 3 8/23/19

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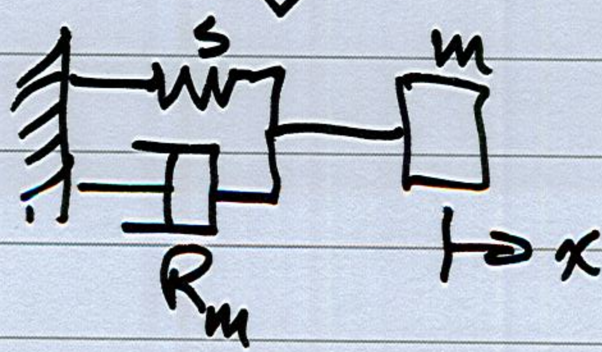
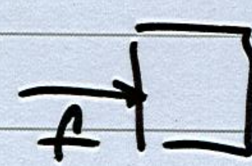
- governing eqns
- wave eqn
- possible solutions
- boundary condition



Restoring Force

$$f = -sx - R_m \frac{dx}{dt}$$



EOM

$$f = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \left(\frac{R_m}{m}\right)\frac{dx}{dt} + \omega_0^2 x = 0 \qquad \omega_0 = \sqrt{\frac{s}{m}}$$

$$\tilde{x} = \tilde{A}\underset{\sim}{e}^{\gamma t}$$

$$\gamma = \frac{-\left(\frac{R_m}{m}\right) \pm \sqrt{\left(\frac{R_m}{m}\right)^2 + 4\omega_0^2}}{2}$$

$$\beta = \frac{R_m}{2m}$$

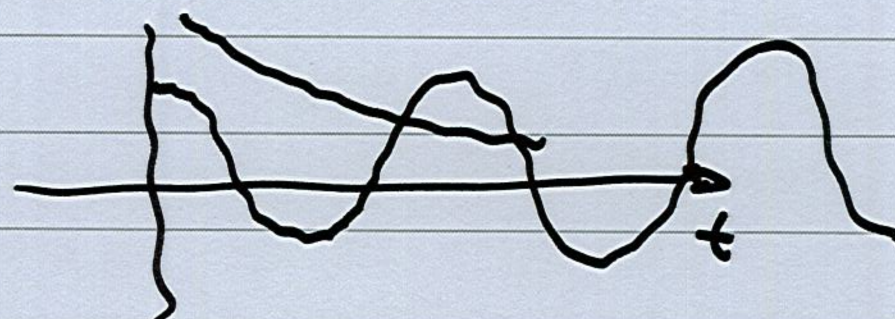
$$\gamma = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

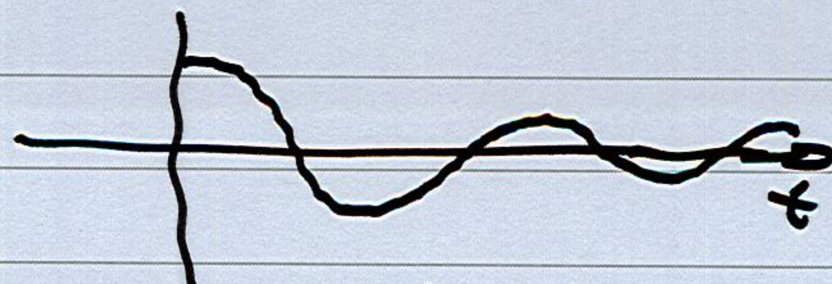
usually system is underdamped

$$\gamma = -\beta \pm j\sqrt{\underbrace{\omega_0^2 - \beta^2}_{\omega_d^2}}$$

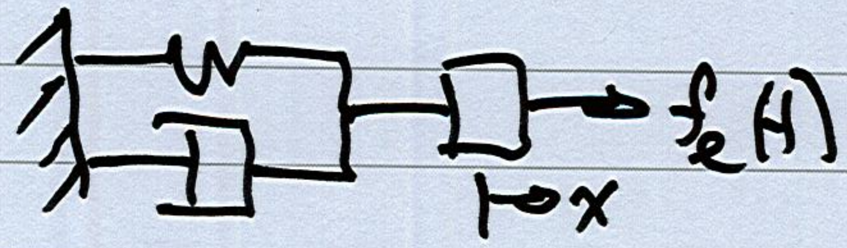
$$\dot{\gamma} = -\beta \pm j\omega_d$$

$$\tilde{x} = \tilde{A}e^{-\beta t}e^{\pm j\omega_d t}$$ ✓





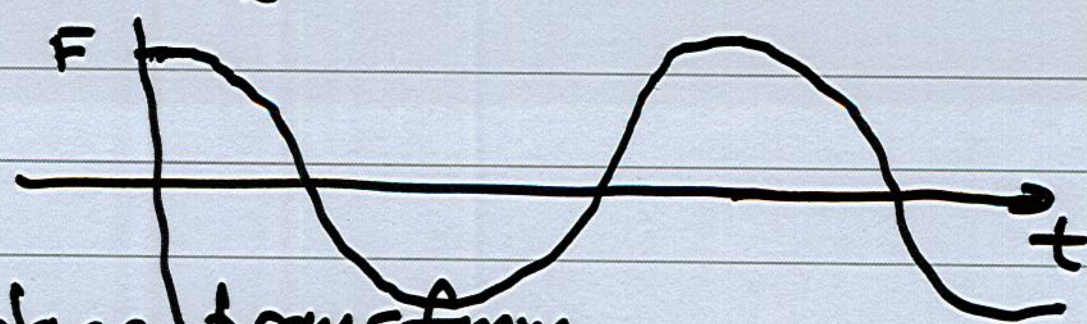
$$\omega_d^2 = \omega_0^2 - \beta^2 \quad < \omega_0$$



$$\frac{m\,d^2x}{dt} + R_m\frac{dx}{dt} + s\,x = f_e(t) \qquad (3)$$

inhomogeneous ODE

Example $\quad f_e(t) = F\cos\omega t \qquad t \geq 0$



Laplace transform

-algebraic solution

$$x = \underset{\text{homogeneous}}{x_{\text{transient}}} + \underset{\text{particular}}{\underline{x_{\text{steady state}}}}$$

For real systems

since $R_m > 0$ always

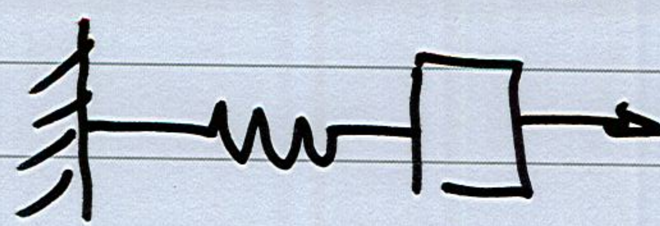
transient solution negligible
as t increases

$\Rightarrow$ so we concentrate on
the steady-state solution

1.3.2 ~~stead~~ Steady State solution

Assume complex form for driving force

$\hat{f}_e = F e^{j\omega t}$ driving frequency $\qquad \mathrm{Re}\{\hat{f}_e(t)\} = F\cos\omega t$



$\hat{f}_e(t) = F e^{j\omega t}$

Linear Systems
- At steady state - system responds at the driving freq.

ω - forcing freq

You control

ω_0 - natural freq

- property of the system

Assumed Solution

$$\underbrace{\hat{f}_e = F\underline{e^{j\omega t}} \qquad \tilde{x} = \hat{A}\underline{e^{j\omega t}}}_{\text{sub into Eq. (3)}}$$

$$-\omega^2 m \hat{A}\cancel{e^{j\omega t}} + j\omega R_m \hat{A}\cancel{e^{j\omega t}} + s\hat{A}\cancel{e^{j\omega t}} = F\cancel{e^{j\omega t}}$$

$$\hat{A} = \frac{F}{-\omega^2 m + j\omega R_m + s}$$

$$= \frac{F}{j\omega\left(\underline{R_m + j\left(\omega m - \frac{s}{\omega}\right)}\right)}$$

steady-state response of a damped SDOF

$$\tilde{x} = \tilde{A} e^{j\omega t}$$

$$= \frac{F e^{j\omega t}}{j\omega\left(R_m + j\left(\omega m - \frac{s}{\omega}\right)\right)}$$

physical displacement = $Re\{\tilde{x}\}$

and

$$\tilde{u} = j\omega \tilde{x} \quad \text{velocity}$$

$$\tilde{u} = \frac{F e^{j\omega t}}{R_m + j\left(\omega m - \frac{s}{\omega}\right)}$$

Acceleration

$$\hat{a} = \frac{d^2\hat{x}}{dt^2} = \frac{d\hat{u}}{dt} = -\omega^2\hat{x} = j\omega\hat{u}$$

acoustic displacements & velocities

progressive plane wave

$$\left|\frac{p}{u}\right| = \rho_0 c \qquad \rho_0 = 1.2\ \mathrm{kg/m^3}$$

$$c = 340\ \mathrm{m/s}$$

$p = 1$ Pa $\rightarrow$ 94 dB

$$|u| = \frac{|p|}{\rho_0 c} = \frac{1}{400} = 2.5\ \mathrm{mm/s}$$

$$\omega |x| = |u|$$

$$|x| = \frac{|u|}{\omega} = \frac{2.5 \times 10^{-3}}{2\pi \times 1 \times 10^{3}}$$

$$\simeq 0.5 \times 10^{-6}\ \mathrm{m}$$