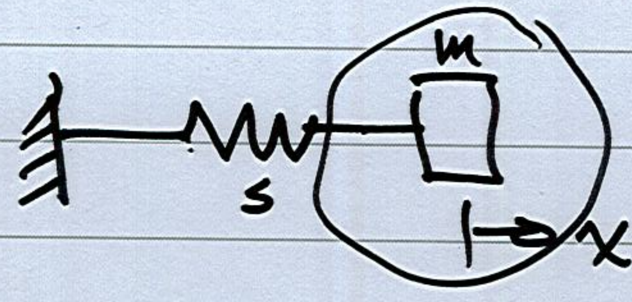
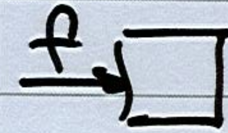
## 1. Fundamentals of Vibration




$$f = m\frac{d^2x}{dt^2}$$




$$f = -sx$$

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0 \qquad \omega_0 = \sqrt{\frac{s}{m}}$$

$$x = A_1 \cos \cancel{\omega_0}\,\gamma t \qquad x = A_2 \sin \gamma t$$

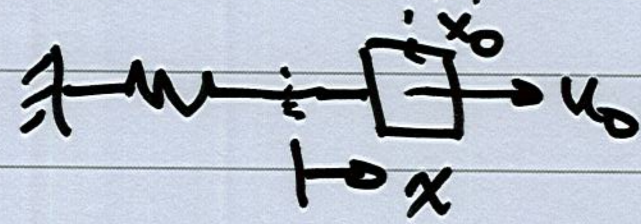
$$\gamma = \omega_0 \ [\text{rad/s}] \qquad f = \frac{\omega_0}{2\pi} \ [\text{Hz}]$$

complete sol'n

$$x = A_1 \cos \omega_0 t + A_2 \sin \omega_0 t$$

1.1.3 Initial Conditions (boundary conditions in time)

2 Constants $A_1$ & $A_2$
2 independent conditions




e.g. $t=0$ $\quad x = x_0$ displacement

$$\frac{dx}{dt} = \dot{x} = u_0 \text{ velocity}$$

since $x = A_1 \cos \omega_0 t + A_2 \sin \omega_0 t$

at $t=0$ $\quad x = A_1$

$$A_1 = x_0$$

Velocity

$$u = \dot{x} = \frac{dx}{dt} = -\omega_0 A_1 \sin\omega_0 t + \omega_0 A_2 \cos\omega_0 t$$

at $t=0$ $\quad u = \omega_0 A_2$

$$A_2 = \frac{u_0}{\omega_0}$$

complete soln

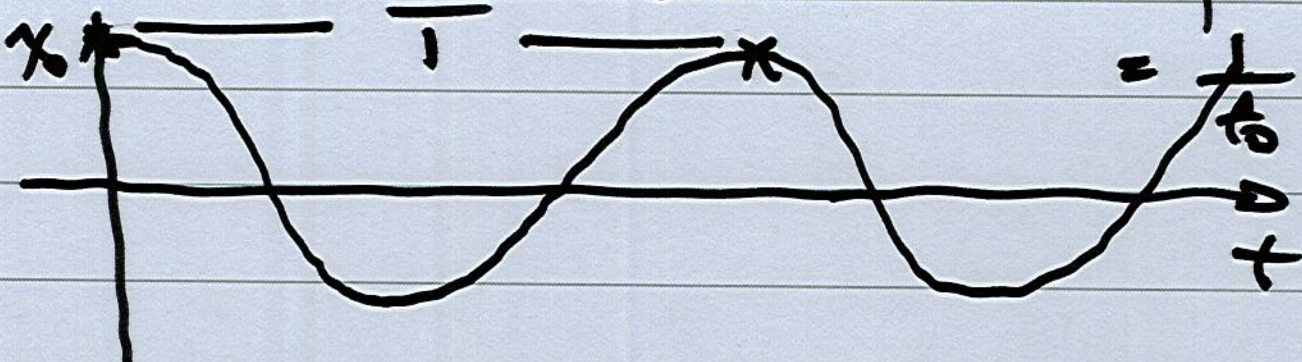
$$x = x_0 \cos\omega_0 t + \frac{u_0}{\omega_0}\sin\omega_0 t \quad (4)$$

$$\omega_0 = \sqrt{\frac{s}{m}} \quad ✓$$

if the initial velocity $= 0$ $\quad x_0 \neq 0$

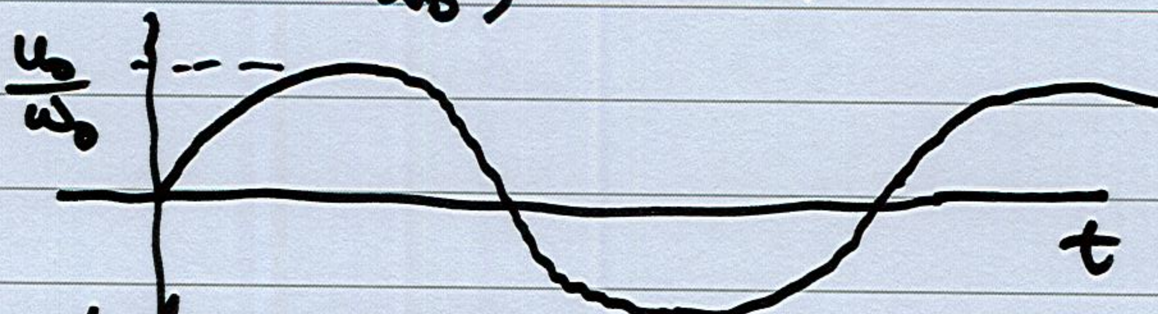
$x = x_0 \cos \omega_0 t$ $\qquad T = \text{period}$

$= \frac{1}{f_0} = \frac{2\pi}{\omega_0}$




if the initial displacement $= 0$ $\quad u_0 \neq 0$

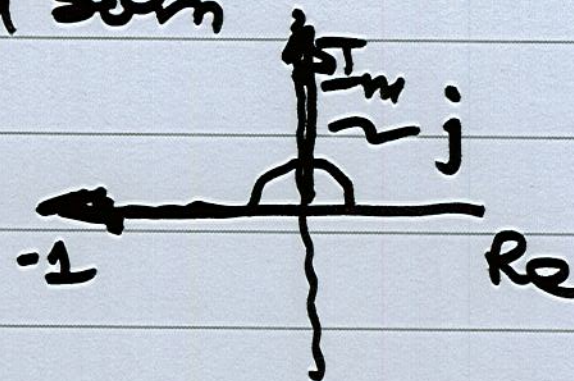
$x = \left(\frac{u_0}{\omega_0}\right) \sin \omega_0 t$




complete response
- sum of these two

1.1.4 ~~[illegible]~~ Complex Solution

$\boxed{\tilde{x} = \tilde{A} e^{\gamma t}}$ Assumed sol'n




$\tilde{x} = x_r + j x_i \quad j = \sqrt{-1}$

$\tilde{A} = a + jb$

$Re\{\tilde{x}\}$ = physical sol'n

(13) $\dfrac{d^2\tilde{x}}{dt^2} + \omega_0^2 \tilde{x} = 0$ $\qquad \underbrace{\tilde{x} = \tilde{A} e^{\gamma t}}$

$$\gamma^2 \cancel{\tilde{A} e^{\gamma t}} + \omega_0^2 \cancel{\tilde{A} e^{\gamma t}} = 0$$

$$\gamma^2 = -\omega_0^2$$ acceptable soln

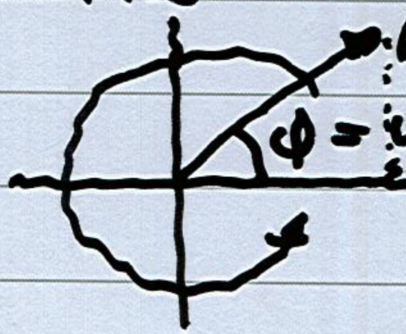
$$\gamma = \sqrt{-\omega_0^2} = \sqrt{-1}\sqrt{\omega_0^2}$$

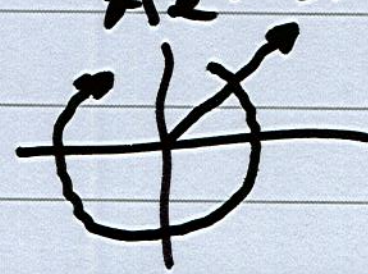
$$= \pm j\omega_0$$

$\tilde{A} e^{\underbrace{j\omega_0 t}}$ $\qquad \tilde{A} e^{-j\omega_0 t}$ $\qquad e^{j\omega_0 t}$

$\hat{A}$ $\qquad e^{i\phi}$

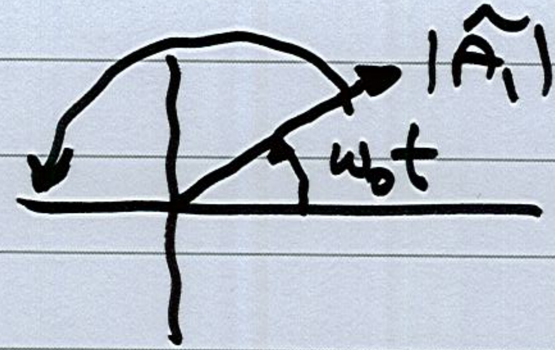
$\phi = \omega_0 t$

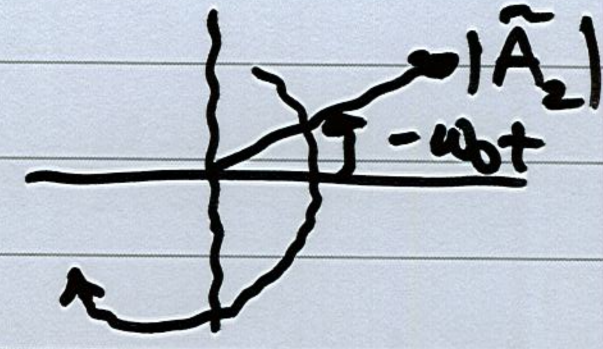




complete

$$\tilde{x} = \tilde{A}_1 e^{j\omega_0 t} + \tilde{A}_2 e^{-j\omega_0 t}$$




projections onto the real axis

Determine $\hat{A}_1$ & $\hat{A}_2$ by applying initial conditions

at $t = 0$

$$\boxed{\begin{aligned} \tilde{x} &= x_0 = \tilde{A}_1 + \tilde{A}_2 \\ \dot{\tilde{x}} &= u_0 = j\omega_0 \tilde{A}_1 - j\omega_0 \tilde{A}_2 \end{aligned}}$$

2 eqns in 2 unknowns

$$\tilde{A}_1 = \frac{1}{2}\left(x_0 - j\frac{u_0}{\omega_0}\right)$$

$$\tilde{A}_2 = \frac{1}{2}\left(x_0 + j\frac{u_0}{\omega_0}\right)$$

$$\tilde{A}_2 = \tilde{A}_1^*$$

so

$$\left.\begin{aligned}\tilde{A}_1 &= a + jb \\ \tilde{A}_2 &= a - jb\end{aligned}\right] \text{ just 2 real constants}$$

Complete soln

$$\tilde{x} = \frac{1}{2}\left(x_0 - i\frac{u_0}{\omega_0}\right)e^{\overbrace{j\omega_0 t}^{\cos\omega_0 t + j\sin\omega_0 t}} + \frac{1}{2}\left(x_0 + i\frac{u_0}{\omega_0}\right)e^{\overbrace{-j\omega_0 t}^{\cos\omega_0 t - j\sin\omega_0 t}}$$

$$\tilde{x} = x_0 \cos\omega_0 t + \frac{u_0}{\omega_0}\sin\omega_0 t$$

---

Exactly the same as before

Real part is the physical soln

$$x = \mathrm{Re}\{\tilde{x}\}$$

$$\tilde{x} = \tilde{A} e^{j\omega_0 t}$$ displacement

$$\frac{d\tilde{x}}{dt} = \dot{\tilde{x}} = \tilde{u} = j\omega_0 \tilde{A} e^{j\omega_0 t} = j\omega_0 \tilde{x}$$ velocity

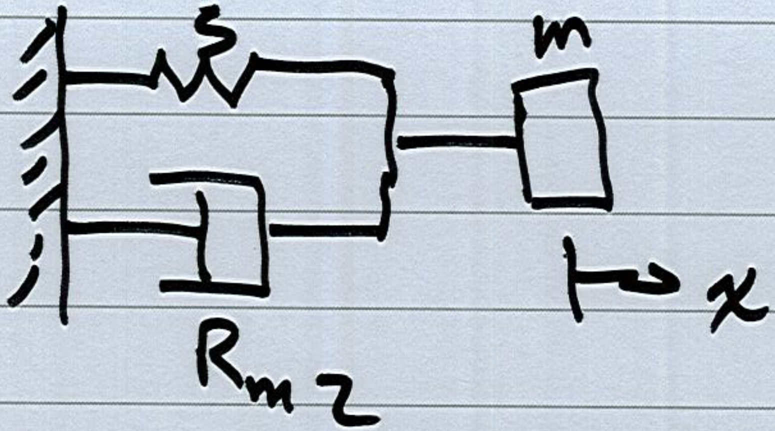
accel

$$\frac{d^2\tilde{x}}{dt^2} = \ddot{\tilde{x}} = \tilde{a} = -\omega_0^2 \tilde{A} e^{j\omega_0 t} = -\omega_0^2 \tilde{x} = j\omega_0 \tilde{u}$$

$$e^{\underline{j\omega_0 t}} \qquad e^{j\phi} \qquad \phi = \omega_0 t$$

$$\frac{d\phi}{dt} = \omega_0 =$$ time rate of change of phase

## 1.2 Damped Oscillations

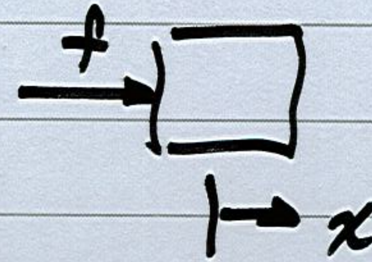



viscous damper

most realistic systems dissipate energy

1.2.1 Governing Eqn

(i) Restoring force




$$f = -sx - R_m \frac{dx}{dt} \qquad (1)$$

(ii) EOM

$$f = ma$$

$$= m\frac{d^2x}{dt^2} \qquad (2)$$

$(1) \rightarrow (2)$

$$m\frac{d^2x}{dt^2} + R_m \frac{dx}{dt} + sx = 0$$

$$\frac{d^2x}{dt^2} + \left(\frac{R_m}{m}\right)\frac{dx}{dt} + \omega_0^2 x = 0 \quad (3)$$

$$\omega_0 = \sqrt{\frac{s}{m}}$$

1.2.2 soln

$$\tilde{x} = \hat{A} e^{\gamma t}$$

Result

$$\cancel{\gamma^2 \tilde{A} e^{\gamma t}} + \gamma\left(\frac{R_m}{m}\right)\cancel{\tilde{A} e^{\gamma t}} + \omega_0^2 \cancel{\tilde{A} e^{\gamma t}} = 0$$

$$\gamma^2 + \left(\frac{R_m}{m}\right)\gamma + \omega_0^2 = 0$$

$$\gamma = \frac{-\left(\frac{R_m}{m}\right) \pm \sqrt{\left(\frac{R_m}{m}\right)^2 - 4\omega_0^2}}{2}$$

let $\frac{R_m}{2m} = \beta$