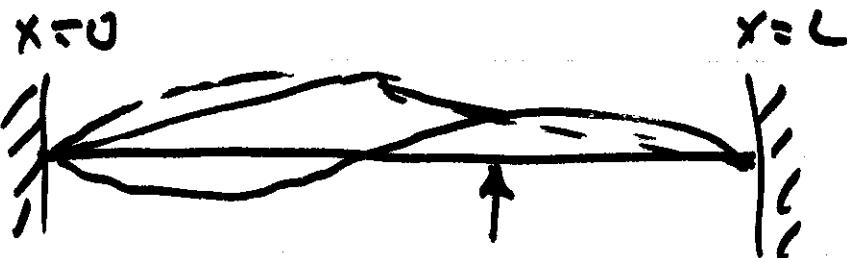


"forced" $\vec{F}_0 e^{i\omega t}$



b.c.'s

$$\sin(k_n) = 0$$

$$L = \frac{n}{2} \lambda_n \quad f_n's$$

- no external force
- motion results from initial conditions

responde only at the natural frequencies.

$$\text{nth mode } y_n(x, t) = \underbrace{\{-2; A_n\}}_{C_n} \sin k_n x e^{j\omega_n t}$$

$$= C_n \sin k_n x e^{j\omega_n t}$$

nth normal mode.

mode

- individual solution that satisfies the wave eqn and the boundary conditions
- k_n 's are the allowed wave numbers
- $\omega_n = 2\pi f_n$ natural freqs
- C_n = modal amplitude

Complete = sum of the possible soln's

2.5.3 Complete Solution

complete soln = superposition of possible solns

$$y(x,t) = \sum_{n=1}^{\infty} \tilde{a}_n \sin k_n x e^{j\omega_n t}$$

weighted
sum of
weight + jsinwt modes.

$$\tilde{a}_n = a_n + j b_n$$

Real displacement of the nth mode

$$\operatorname{Re}\{y_n(x,t)\} = (a_n \cos \omega_n t - b_n \sin \omega_n t) \sin k_n x$$

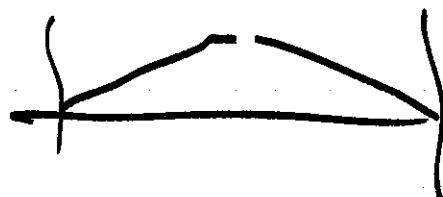
Say $\operatorname{Re}\{\gamma(x, 0)\}$ = known

at $t=0$

$$\operatorname{Re}\{\gamma(x, 0)\} = \sum_{n=1}^{\infty} a_n \sin nx$$

Fourier Series

$$\tilde{a}_n = \frac{2}{\pi} \int_0^{\pi} \operatorname{Re}\{\gamma(x, 0)\} \sin nx dx$$

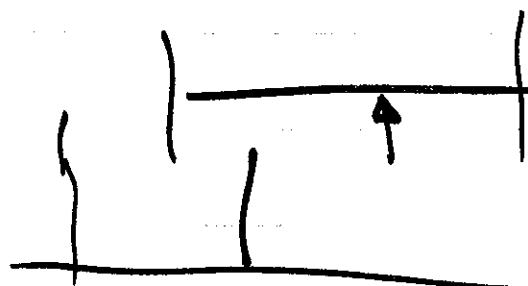


$$a_n + jb_n = \tilde{a}_n$$

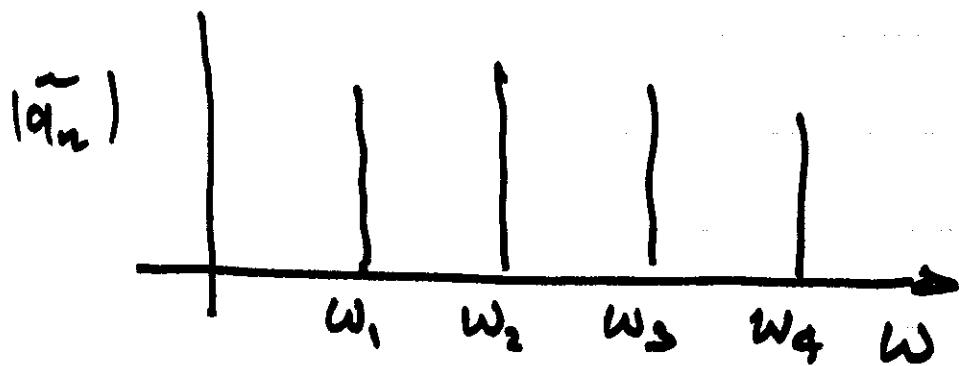
Use initial velocity of the string to solve
for the b.c.s b_n 's

$$u(x,t) = \frac{a}{2t} \sum_{n=1}^{\infty} \tilde{a}_n \sin k_n x e^{i\omega_n t}$$

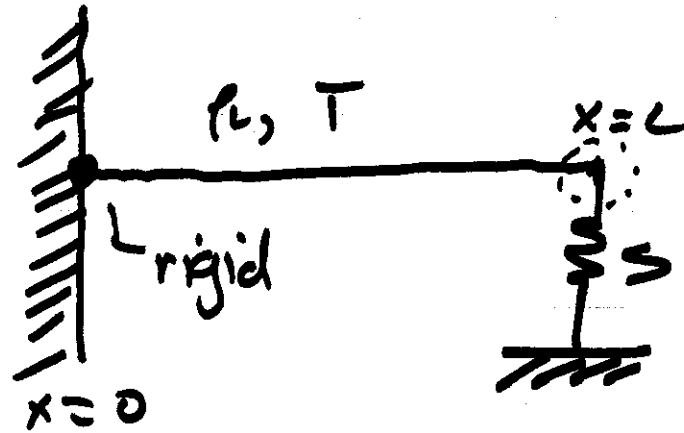
$$\text{Re}\{u(x,0)\} \rightarrow \underline{b_n}$$



$$\tilde{a}_n \Rightarrow = a_n + j b_n$$



2.5.4 Other b.c.'s



$$k = \frac{\omega}{c}$$

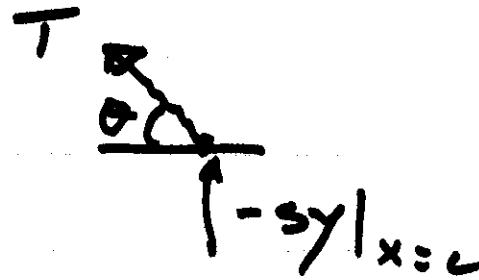
$$c = \sqrt{\frac{T}{\rho_L}}$$

$$y(x,t) = A e^{i(\omega t - kx)} + B e^{i(\omega t + kx)}$$

$$\text{at } x=0 \quad y(0,t) = 0 \quad \cancel{B = A} \quad B = -A$$

$$y(x,t) = -2j A \sin kx e^{i\omega t}$$

at $x=L$



$$\sum F_y = 0$$

$$T \underline{\sin \theta}|_{x=L} - sy|_{x=L} = 0$$

$$-T \frac{dy}{dx}|_{x=L} - sy|_{x=L} = 0$$

$$\frac{dy}{dx} = -2jkA \cos kx e^{j\omega t}$$

$$+ z_j k T \cancel{(\Delta x)^{\text{sat}}} \cos kL + z_j s(\Delta) e^{i\omega t} \sin kL = 0$$

$$k_n T \cos k_n L = -s \sin k_n L$$

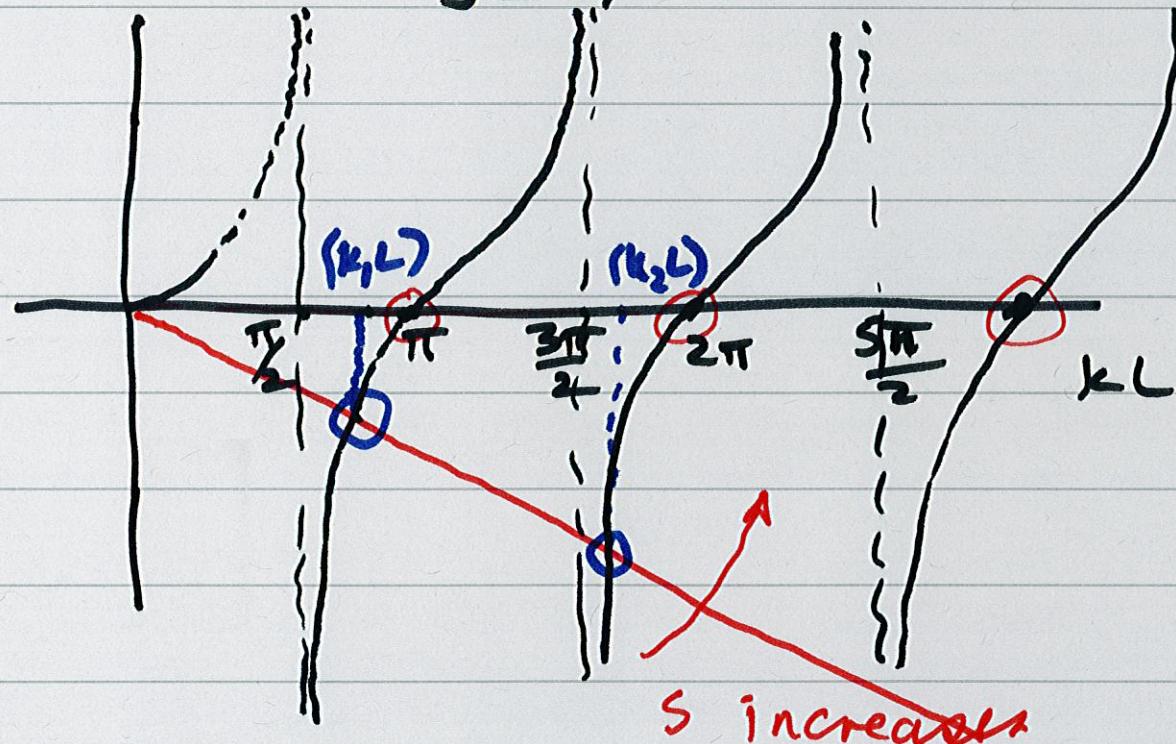
$$-\frac{k_n T}{s} = \tan(k_n L)$$

$$-(k_n L) \left(\frac{T}{sL} \right) = \tan(k_n L)$$

Characteristic Eqn.

$$-(k_n L) \left(\frac{I}{sL} \right) = \tan k_n L$$

9



$$k_1 L \rightarrow k_1$$

$$k_1 = \frac{\omega_1}{c} \rightarrow \omega_1$$

$$\frac{\omega_1}{2\pi} \rightarrow f_1$$

fixed-Fixed case

$$\text{slope} - \frac{T}{sL}$$

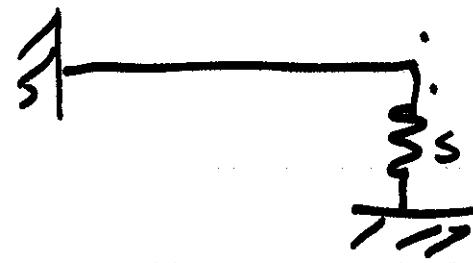
$$k_n L = n\pi$$

here at $s \rightarrow \infty$ approach
Fixed-Fixed case

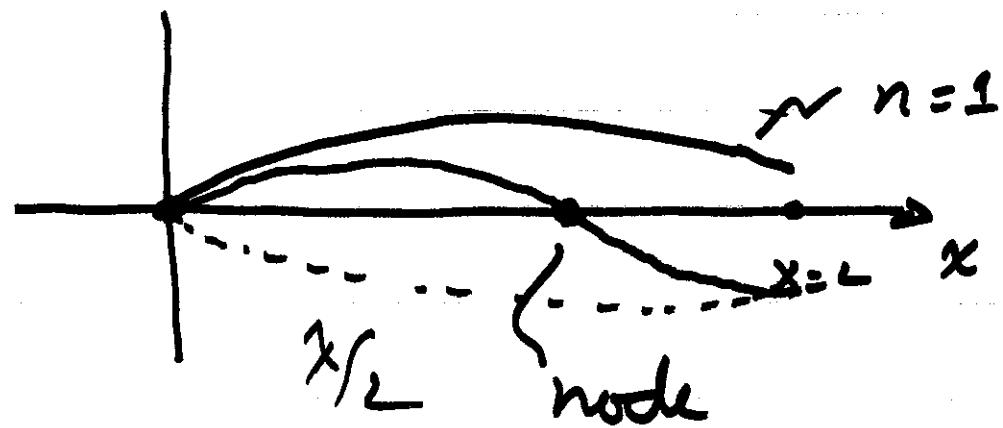
- otherwise natural freqs are lower

n

mode shape



$$y_n(x,t) = C_n \sin k_n x e^{j\omega_n t} \quad n^{\text{th}} \text{ mode.}$$



$$y(x,t) = \sum_{n=1}^{\infty} \text{modes } e_n$$

Summary

- Derivation of a wave Eqn]
(modeling) ✓
- restoring force] wave eqn
- equation of motion]
- inertia]
- stiffness]