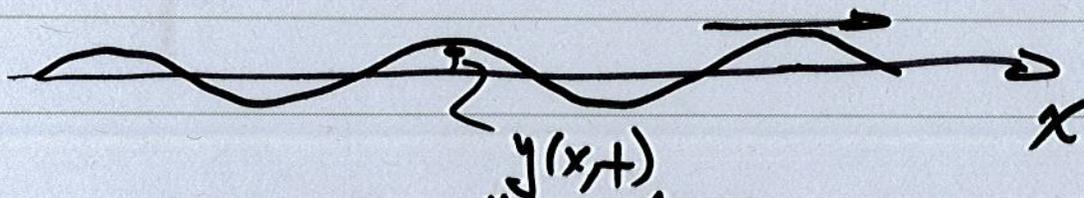


office homework

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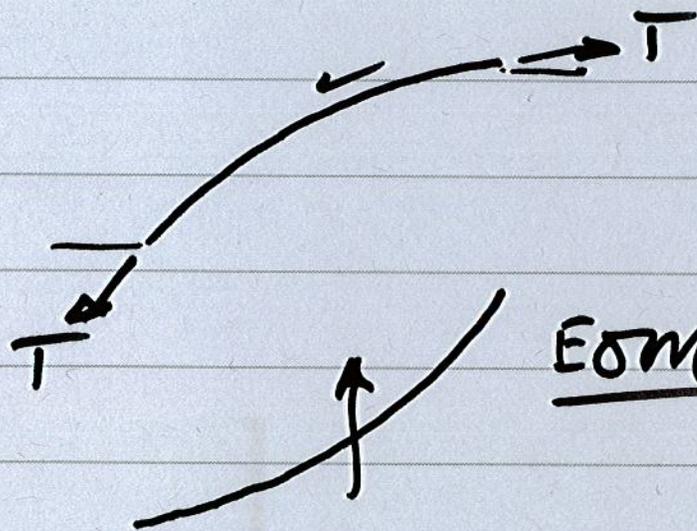
Vibration of Strings



transverse vibration

- assumptions - small amplitudes
- linear vibration

combine - restoring force eqn



$$df_y = T \frac{d^2 y}{dx^2} dx$$

$$df_y = f_c dx \frac{d^2 y}{dt^2}$$

$$\frac{d^2 y}{dx^2} - \frac{1}{c^2} \frac{d^2 y}{dt^2} = 0 \quad c = \sqrt{\frac{EI}{\rho}}$$

2.2 Solutions of The Wave Eqn

Two independent variables

x & t space & time

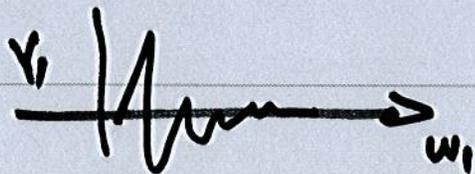
2.2.1 General Solution

$$y(x, t) = y_1(\underbrace{ct - x}_{w_1}) + y_2(\underbrace{ct + x}_{w_2})$$

$$= y_1(w_1) + y_2(w_2)$$

$$w_1 = ct - x \quad w_2 = ct + x$$

y_1 & y_2 are any functions of a single variable



$$y_1(ct - x)$$

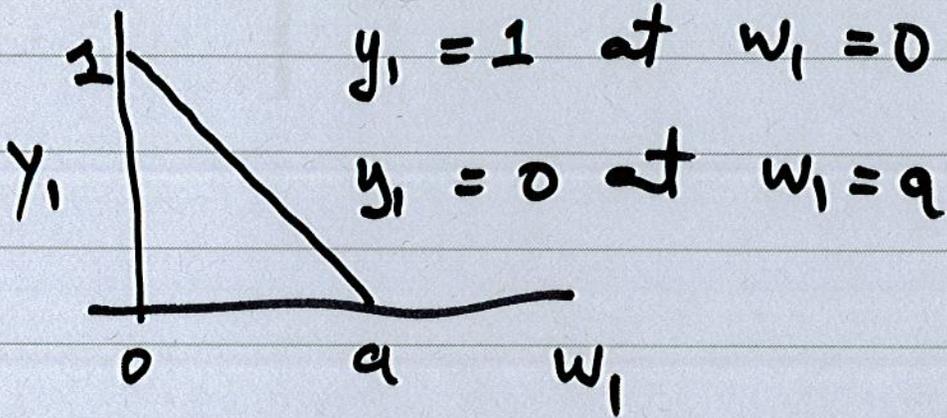
$\underbrace{\hspace{2cm}}_{w_1}$

$$y_2(ct + x)$$

$\underbrace{\hspace{2cm}}_{w_2}$

prove by direct substitution that these are solutions

Simple example



plot as a function of x at $t=0$

$$w_1 = ct - x$$

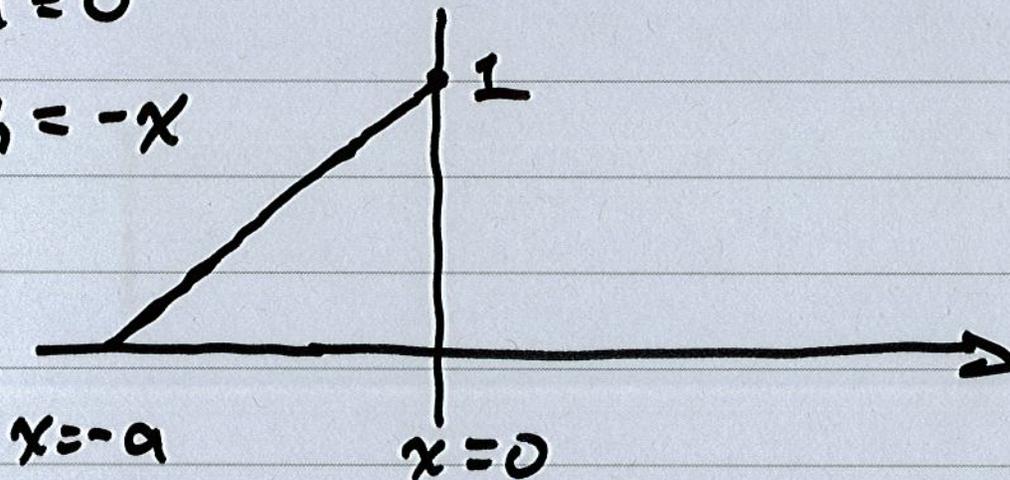
$$x = -w_1$$

$$w_1 = 0 \rightarrow x = 0$$

at $t=0$

$$w_1 = a \rightarrow x = -a$$

$$w_1 = -x$$



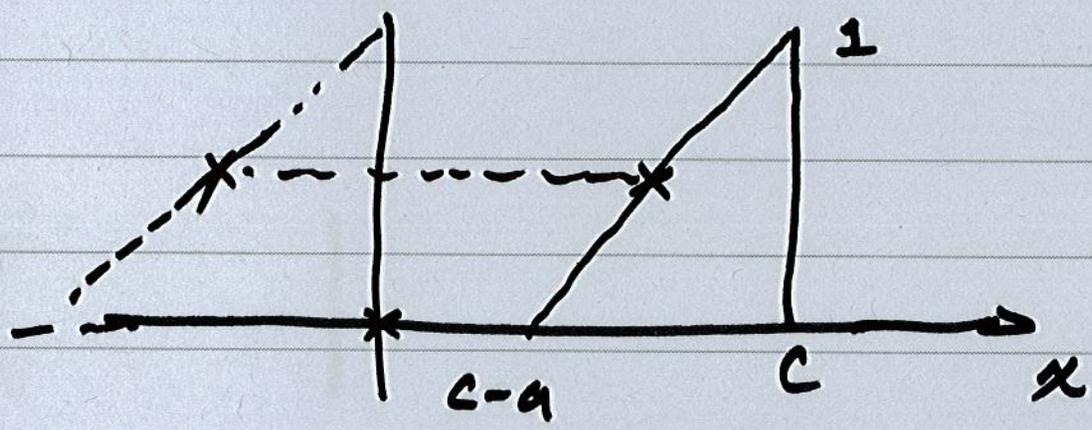
plot a function of x at $t=1s$

$w_1 = c - x$ at $t=1s$

$x = c - w_1$

$w_1 = 0 \rightarrow x = c$

$w_1 = a \rightarrow x = c - a$



$w_1 = ct - x$

$c = \frac{\sqrt{F}}{\rho L}$

$\frac{\text{distance}}{\text{time}} = \left(\frac{c}{1}\right) \rightarrow$ speed of wave propagation

* shape is unchanged
linear wave propagation

$$y_1(ct - x)$$

$\underbrace{\hspace{2cm}}$
 w_1

disturbance traveling in
The +ve x-direction
without changing shape at
the speed c

$$y_2(ct + x)$$

$\underbrace{\hspace{2cm}}$
 w_2

disturbance that travels
in the -ve x-direction

by holding the value of $w_1 = \text{constant}$ - following
a pt on the wave as time advances

$$y(x,t) = y_1 + y_2$$



General solution is a superposition
of waves travelling ~~to~~ to the right
& left

8

What is the transverse velocity of the string

$$v_{\perp} = \frac{\partial y_1}{\partial t} = \frac{\partial y_1(ct-x)}{\partial(ct-x)} \frac{\partial(ct-x)}{\partial t}$$
$$= c \frac{\partial y_1(ct-x)}{\partial(ct-x)} \neq c$$

value of v_{\perp} depends on y_1
speed of wave propagation
 \neq transverse velocity of
the string

Complete solution

$$y(x,t) = \underbrace{y_1(ct - x)}_{\text{+ve going}} + \underbrace{y_2(ct + x)}_{\text{-ve going}}$$

- functions propagate without changing shape
- non-dispersive
 - all frequency components travel at the same speed.

2.2.2 Harmonic Single Frequency Solution

Assume a separable form of solution

$$\underline{y(x,t) = y(x) e^{j\omega t}}$$

$$\frac{d^2 y}{dx^2} - \frac{1}{c^2} \frac{d^2 y}{dt^2} = 0$$

sub

$$\frac{d^2 y}{dx^2} e^{j\omega t} + \frac{\omega^2}{c^2} y e^{j\omega t} = 0$$

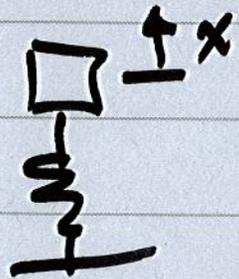
scalar Helmholtz Equ governs the spatial variation

define $k^2 = \frac{\omega^2}{c^2}$ & $k = \frac{\omega}{c}$ wave number

$$\frac{d^2 y}{dx^2} + k^2 y = 0 \quad (4)$$

$$e^{j\omega t}$$

SDOF



$$\frac{d^2 x}{dt^2} + \omega_0^2 x = 0$$

$$x = A e^{\pm j\omega t}$$

$$Y(x) = A e^{\pm jkx} \quad \text{two solutions}$$

Solution for the spatial dependence of single frequency transverse vibs of a string

$$y(x,t) = A_1 e^{+jkx} e^{j\omega t} + A_2 e^{-jkx} e^{j\omega t}$$

$$= A_1 e^{+j(kx + \omega t)} + A_2 e^{+j(-kx + \omega t)}$$

$$k = \frac{\omega}{c}$$

$$= A_1 e^{+jk(x+ct)} + A_2 e^{+jk(-x+ct)}$$

$y_L(ct+x)$

$y_1(ct-x)$

-ve going wave

+ve, going wave

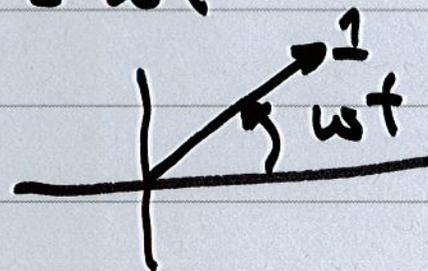
what is k ?



Recall $e^{i\omega t} = e^{i\phi}$

$$\phi = \omega t$$

$$\frac{d\phi}{dt} = \omega$$



ω : rate of change of phase with time

k :