

## Student Chapters

ASA



AES

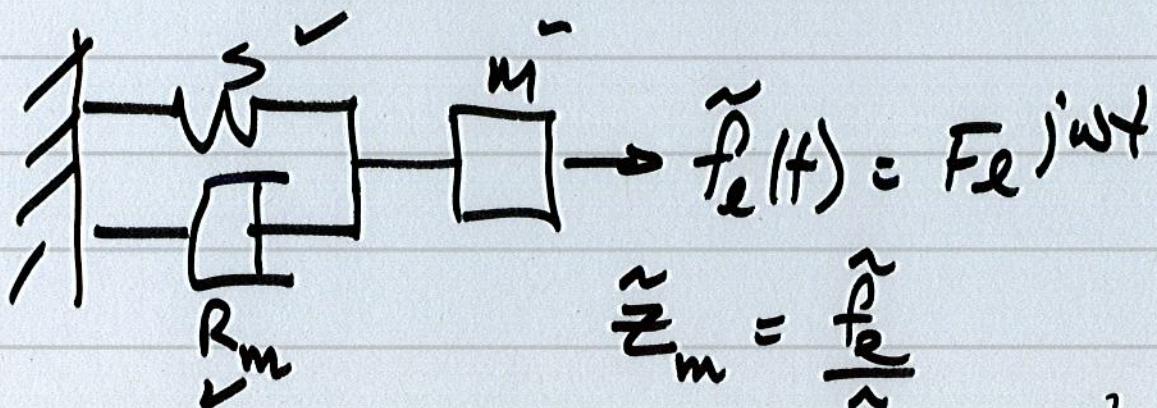


Midea

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- mechanical impedance



$$\underline{j\omega m} \quad \begin{array}{c} \nearrow \\ \dashrightarrow \\ \searrow \end{array} \quad \omega$$

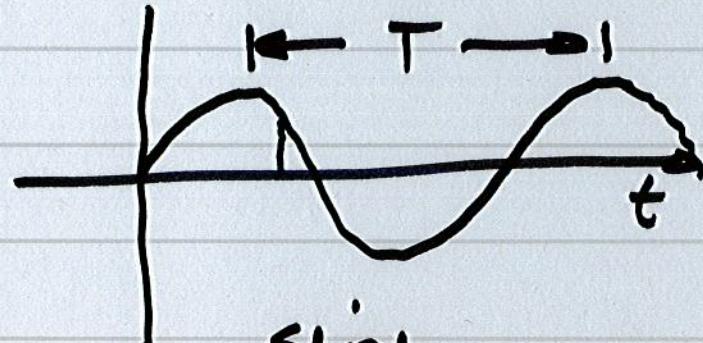
masslike

stiffness  
like

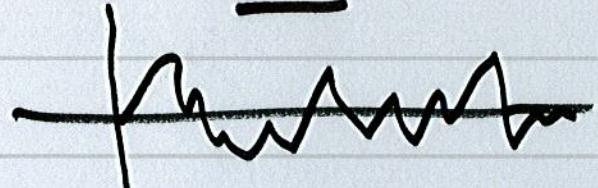
if  $\tilde{\Sigma}_m$  is known

$$\underline{\tilde{u}} = \frac{\tilde{f}_e}{\tilde{\Sigma}_m} \quad \text{Resonance} \quad \left. \begin{aligned} \text{Im} \{ \tilde{\Sigma}_m \} &\rightarrow 0 \\ \omega &\rightarrow \omega_0 \end{aligned} \right\} \quad \sqrt{\frac{\Sigma}{m}}$$

## 1.5 Fourier Analysis - ME 579

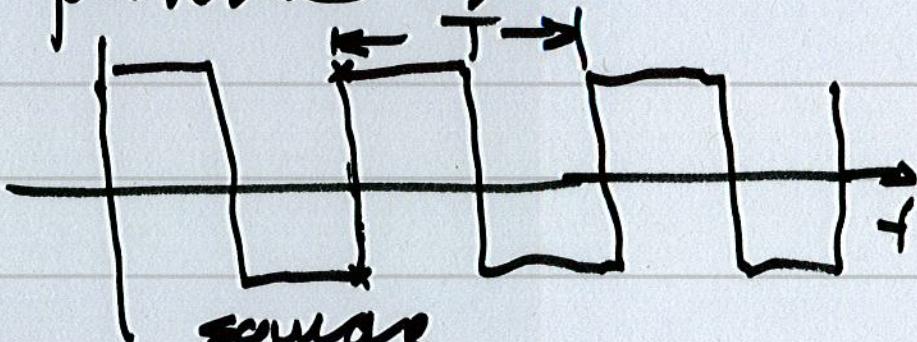


sine



signal repeats itself  
in  $\frac{1}{T}$

periodic signal



square

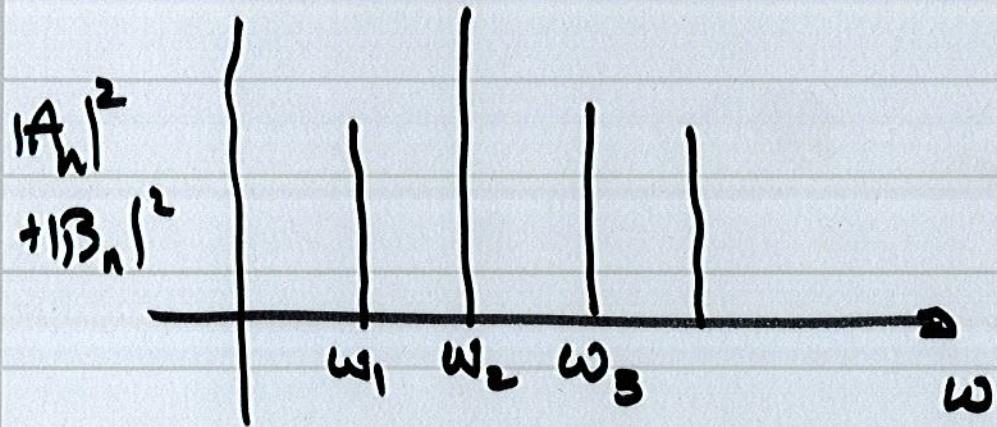
Any single-valued  
periodic function <sup>with period</sup>  $T$   
can be represented exactly as  
a sum of sinusoidal periodic  
in  $\frac{1}{T}$

$f(t)$  periodic  $\rightarrow$  SV

$$\begin{aligned} \underline{f(t)} = \frac{1}{2}A_0 + A_1 \cos \omega_1 t + A_2 \cos \omega_2 t + \dots \\ + B_1 \sin \omega_1 t + B_2 \sin \omega_2 t + \dots \end{aligned}$$

$$\omega_1 = \frac{2\pi}{T} \quad \omega_2 = 2\omega_1 \quad \omega_3 = 3\omega_1$$

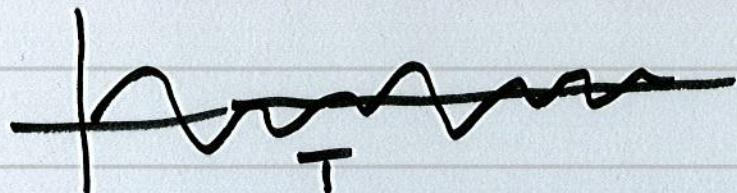
represented by contribution at discrete frequencies



$$\omega_1 = \frac{2\pi}{T} \quad \text{fundamental - first harmonic}$$

$$\omega_2 = 2\omega_1 \quad \text{2nd harmonic}$$

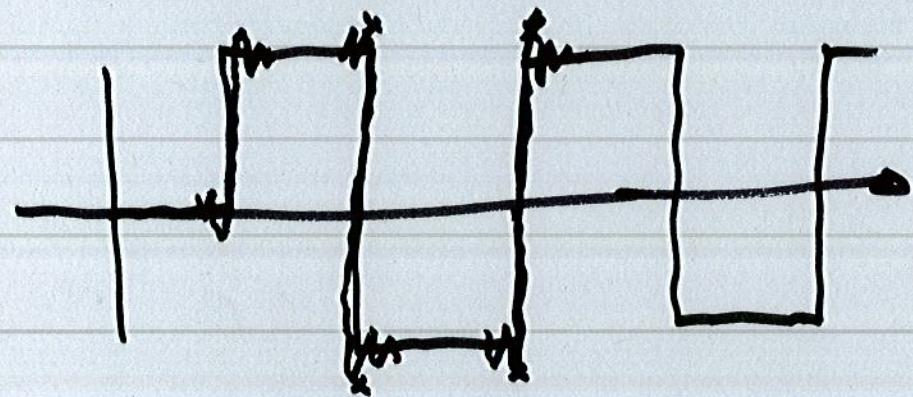
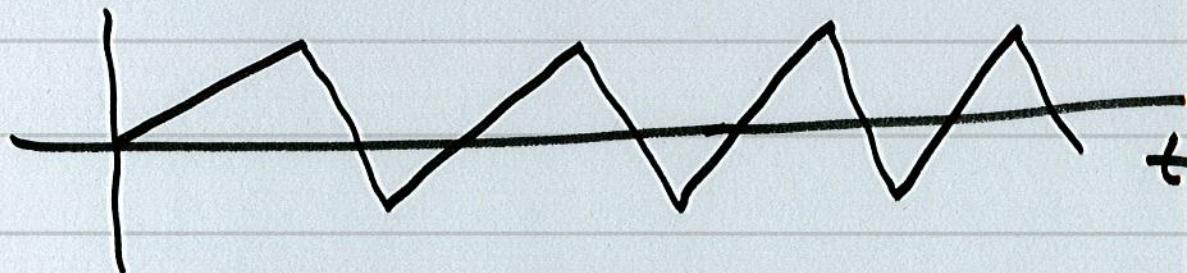
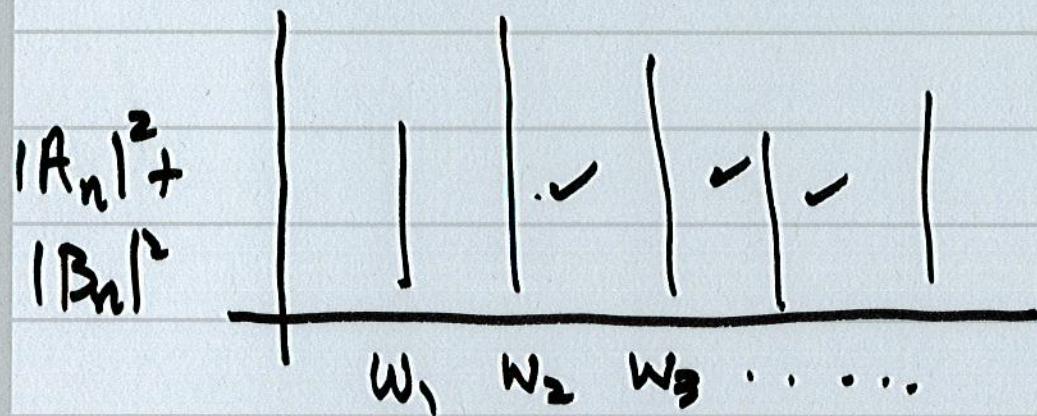
$$\omega_3 = 3\omega_1 \quad \text{3rd harmonic}$$



$$A_n = \frac{c}{T} \int_0^T f(t) \cos(n\omega_1 t) dt$$

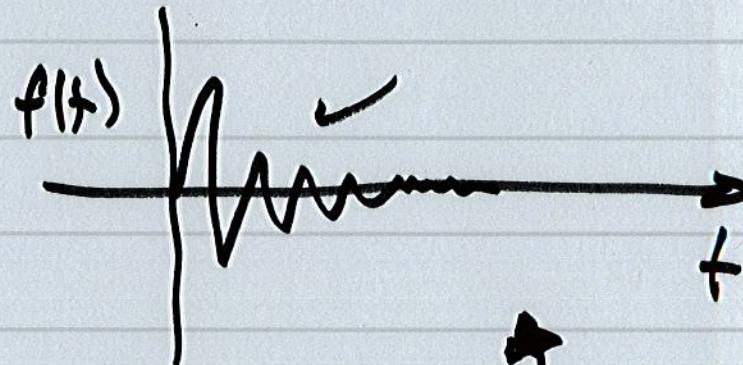
$$n = 1, 2, 3, \dots$$

$$B_n = \frac{c}{T} \int_0^T f(t) \sin(n\omega_1 t) dt$$

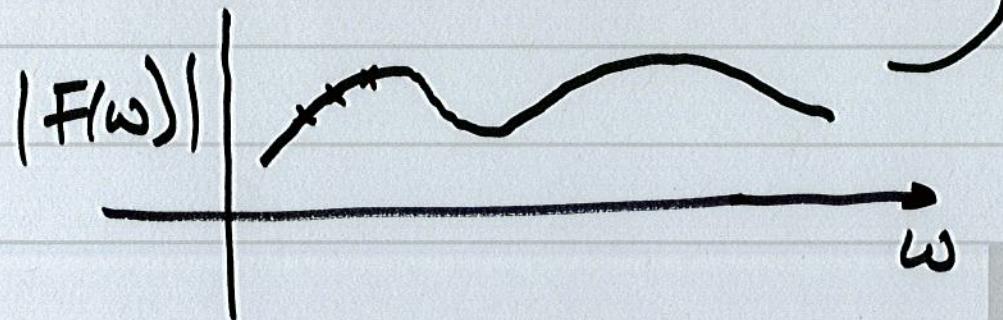


Gibb's phenomenon

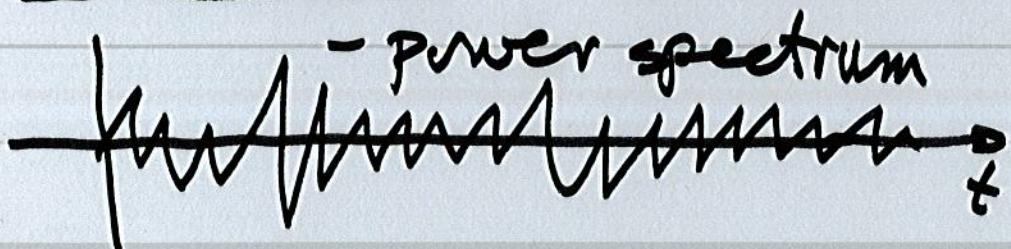
transient



Fourier transforms

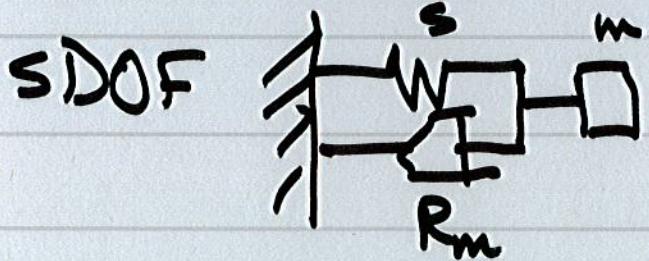


Continuous - non-periodic



## Summary

Physical System



mathematical  
model

To oscillate: mass + stiffness

# Approach to problem solving,

## ① Governing Equations

- restoring force

- Eom ( $f=ma$ )

## ② Combine $\rightarrow$ 2nd order ODE

## ③ Identify possible solutions

expressed in a  
convenient form

## ④ Find The solutions

That satisfy The boundary  
conditions

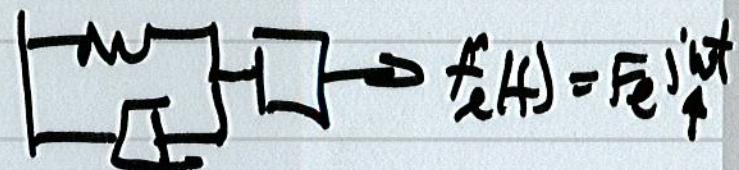
Free Response - driven through initial conditions

$$\omega_0$$

$$x_0, u_0$$

responds at  $\omega_0$ . The natural frequency

Forced Response



- responds at the driving  $\omega$

linear system

$$\hat{x}(t) = \hat{A} e^{j\omega t}$$

## Resonance

- system is driven at the natural frequency

$$\text{Im} \left\{ \tilde{\zeta}_m \right\} = 0 \quad \tilde{\zeta}_m = \frac{\tilde{f}_e}{\tilde{u}}$$

## Mechanical Impedance

$$\tilde{\zeta}_m = R_m + j \left( \underbrace{\omega_m}_{\frac{1}{R_m}} - \underbrace{\frac{s}{\omega}}_{\frac{1}{C_m}} \right)$$

$$\tilde{u} = \frac{\tilde{f}_e}{\tilde{\zeta}_m}$$