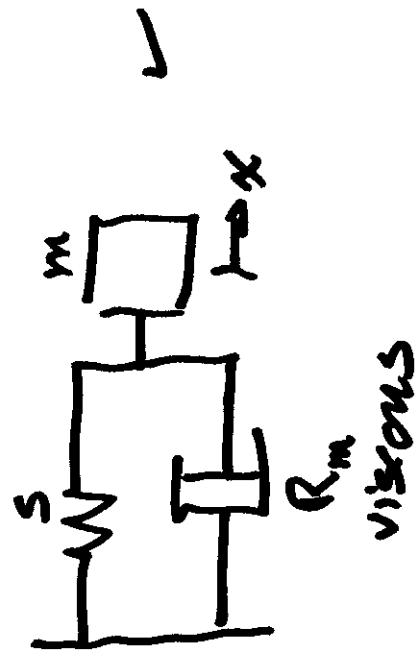


Session 3 8/25/17

1

- governing eqns
- wave equation
- possible solutions
- boundary conditions

SDOF



$$\ddot{x} = \tilde{A} e^{i\omega t}$$

Restring force



$$f = -s x - R_m \frac{dx}{dt}$$

EOM  $f = m \frac{d^2x}{dt^2}$

$$\frac{d^2\tilde{x}}{dt^2} + \left(\frac{R_m}{m}\right) \frac{dx}{dt} + \omega_0^2 x = 0 \quad \rightarrow \quad \tilde{x} = \tilde{A} e^{j\tilde{\gamma} t}$$

$$\omega_0 = \sqrt{\frac{s}{m}}$$

$$\alpha = -\left(\frac{R_m}{m}\right) \pm \sqrt{\left(\frac{R_m}{m}\right)^2 - 4\tilde{\omega}_0^2}$$

$$\beta = \frac{R_m}{2m} \quad \gamma = -\beta \pm \sqrt{\beta^2 - \tilde{\omega}_0^2}$$

usually system is underdamped  
 $\beta < \omega_0$

$$\gamma = -\beta \pm j\sqrt{\tilde{\omega}_0^2 - \beta^2}$$

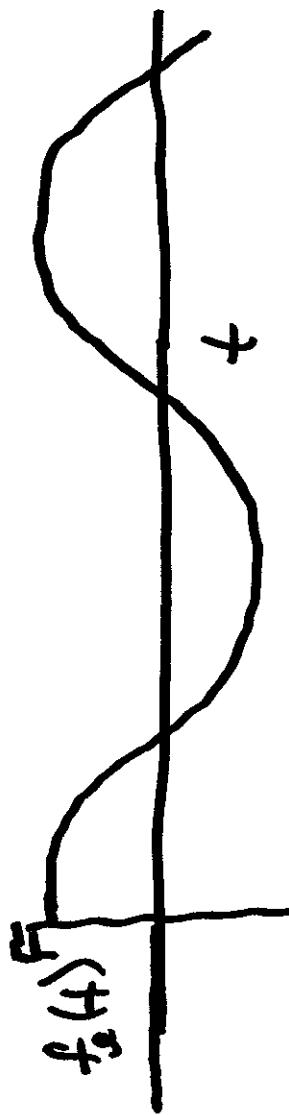
$\omega_d^2 = \tilde{\omega}_0^2 - \beta^2 < \omega_0^2$

$\omega_d$  natural freq  
 $\gamma$  damped

$$\boxed{m \frac{d^2x}{dt^2} + R_m \frac{dx}{dt} + Sx = f_e(t)} \quad (3)$$

inhomogeneous ODE

Example:  $f_e(t) = F \cos \omega t \quad t \geq 0$



Laplace transforms  
→ algebraic solution  
in transform

$$x = x_{\text{transient}} + x_{\text{steady state}}$$

for real systems

since  $R_m > 0$  always  
transient solution becomes  
negligible as  $t$  increases

### 1.3.2 Steady state Solution

Assume complex form for the driving force

$$\tilde{F}_2 = F_2 e^{j\omega t}$$

driving frequency

$$B \left\{ \tilde{F}_2(t) \right\} = \text{Fourier}$$

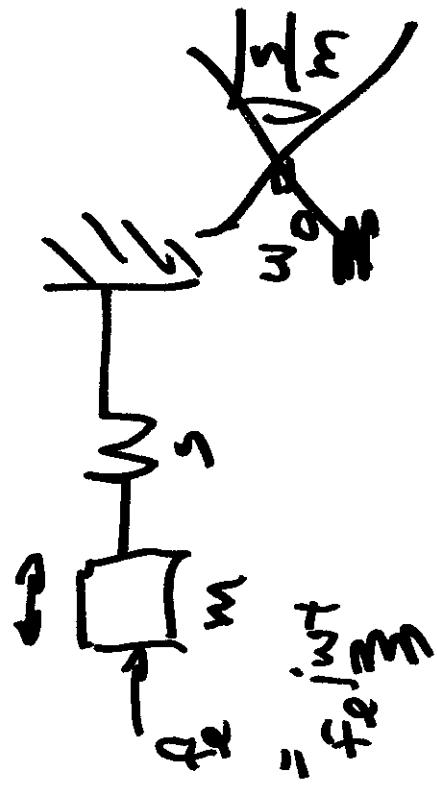
$$B \left[ \tilde{F}_2(t) \right] = \tilde{F}_2 e^{j\omega t}$$

### Linear Systems

- in a steady-state a linear system responds at driving frequency

$\omega$  - forcing frequency  
~~# your central~~

$\omega_0$  - natural frequency  
~~\* property of the system~~



## Assumed Solution

$$\begin{aligned}\tilde{f}_2 &= F_2 e^{j\omega t} & \tilde{\chi} &= \tilde{A} e^{j\omega t} \\ \boxed{\begin{array}{l} \tilde{f}_2 = F_2 e^{j\omega t} \\ \tilde{\chi} = \tilde{A} e^{j\omega t} \end{array}} & \text{Sub into (s)} & \end{aligned}$$

$$-\tilde{\omega_m^2} \tilde{A} e^{j\omega t} + j\omega R_m \tilde{A} e^{j\omega t} + s \tilde{A} e^{j\omega t} = \cancel{-F_2 e^{j\omega t}}$$

$$\tilde{A} = \frac{F}{-\tilde{\omega_m^2} + j\omega R_m + s}$$

$$= \frac{F}{j\omega (R_m + j(\omega_m - \frac{s}{\omega}))}$$

$$\ddot{x} = \tilde{A} e^{j\omega t}$$

$$= \frac{F_0 e^{j\omega t}}{j\omega (R_m + j(\omega_m - \frac{1}{m}))}$$

Steady state response of  
damped SDOF

# Physical Displacement

$$x(t) = R e^{\sum \tilde{x} (t)} \left\{ e^{j\omega t} \right\}$$

$$\text{Velocity: } \tilde{u} = \frac{d\tilde{x}}{dt} = j\omega \tilde{x}$$

$$\text{Acceleration: } \tilde{a} = \frac{d^2\tilde{x}}{dt^2} = \frac{d\tilde{u}}{dt} = -\omega^2 \tilde{x} = j\omega u$$

$$\tilde{u} = \frac{F e^{j\omega t}}{R_m + j(\omega_m - \frac{s}{m})}$$

acoustic displacements & velocities

progressive plane waves

$$|\vec{f}_u| = \rho c \quad \begin{matrix} \rho \\ c \end{matrix}$$

1.2 kg/m<sup>3</sup>  
340 m/s

$$P = 1 \text{ Pa} \rightarrow \frac{94 \text{ dB}}{\underline{}}$$

$$|u| = \frac{|P|}{\rho c} = \frac{0.0025 \text{ m/s}}{\underline{2.5 \text{ mm/s}}}$$

$$\omega(x) = |u|$$

$$|x| = \frac{|u|}{\omega} = \frac{2.5 \times 10^{-3}}{2\pi \times 1 \times 10^{-3}} \text{ m}$$
$$\approx 0.5 \times 10^{-6} \text{ m}$$

### 1.3.3 Mechanical Impedance

Define

$$\tilde{z}_m = \frac{\text{complex driving force}}{\text{complex velocity at the driven point}}$$

$$= \frac{f_e}{\dot{x}_e}$$

Damped SDOF

$$\tilde{z}_m = \frac{F_e^{\text{first}}}{R_m + j(\omega_m - \frac{s}{3})} = R_m + j(\omega_m - \frac{s}{3})$$

$$= R_m + j(\omega_m - \frac{s}{3})$$

$$\tilde{z}_m = R + iX$$

+ ↗

mechanical  
resistance      reactance

$$R = R_m \left\{ \begin{array}{l} \tilde{z}_m \\ \tilde{z}_m \end{array} \right\}$$

$$X = T_m \left\{ \begin{array}{l} \tilde{z}_m \\ \tilde{z}_m \end{array} \right\}$$

$$R = R_m$$

$$X = \frac{1}{2} (\omega_m - s) B_m$$