Room Acoustics

$$T \propto \frac{V}{A} \,\Big]$$

Energy Acoustics
- kinetic
- potential

$$E = E_k + E_p$$

Total Energy in a volume V_0

$$\underline{E} = \frac{1}{2}\rho_0\left(\underset{\sim}{u}^2 + \frac{p^2}{\underset{\sim}{(\rho_0 c)^2}}\right)V_0$$

(iv) Energy Density

$$\varepsilon_i = \frac{E}{V_0}$$ instantaneous

Time-averaged energy density

$$\varepsilon = \langle \varepsilon_i \rangle = \frac{1}{T}\int_0^T \varepsilon_i \, dt$$ integral over 1 or a few cycles.

$\underline{\varepsilon(t)}$

time average is over a very short time interval so that $\mathcal{E}$ is itself a function of time

rate of change of $\mathcal{E}$ with time is slow compared to $\mathcal{E}_i$

$$\mathcal{E}_i = \frac{E}{V_0} = \frac{1}{2}\rho_0\left(u^2 + \frac{p^2}{(\rho_0 c)^2}\right)$$

for plane harmonic waves

$$p = \pm \rho_0 c\, u \qquad \mathcal{E}_i = \frac{p^2}{\rho_0 c^2}$$

when sound field is harmonic

$$p = \hat{P} e^{j\omega t} \qquad u = U e^{j\omega t}$$

time-averaged energy density

$$\mathcal{E} = \frac{1}{2} \frac{|P|^2}{\rho_0 c^2} = \frac{1}{2} \rho_0 |U|^2$$

mean-square pressure

- these relations are true for plane waves
- assuming here that sound fields consists of superposition of plane waves

- each of the plane wave components is <u>randomly phased</u>

- when mean square pressures are added

- $P^2_{total} = P_1^2 + P_2^2 + P_3^2 + \ldots$

because cross-terms are zero due to lack of correlation

$$\varepsilon_t = \frac{1}{2}\frac{P_t^2}{\rho_0 c^2}$$

total mean square pressure at a point in a diffuse field

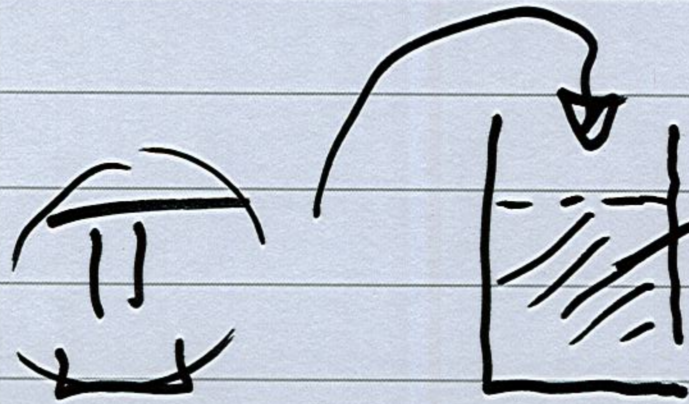
relation between total msp & total energy density

- when the sound field consists of randomly phased plane waves

6.3 Energy Model for Sound in a Room

sound source is turned on

- due to reflections energy density in the space increases until rate of energy absorption at the walls = source sound power



sound field

rate at which energy is absorbed at the walls.

rate of energy delivery - sound power

Rate at which energy is delivered

$\overline{\Pi}$ = Rate at which energy is stored in the space + Rate at which energy is lost at room surfaces

sound power

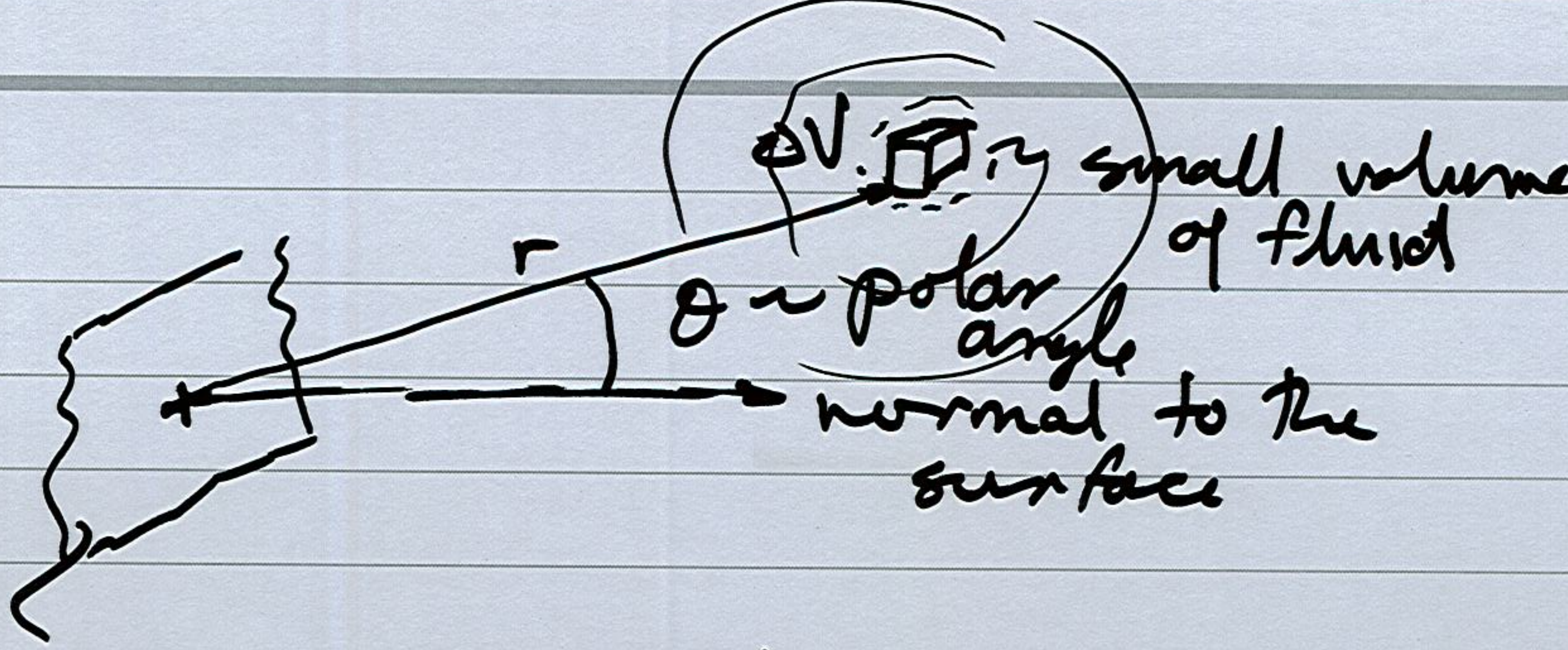
sound field

$$\overline{\Pi} = V\frac{d\mathcal{E}}{dt} + \text{rate at which energy is lost at walls.}$$

space & short-time averaged energy density in the room

last term = rate at which energy
arrives at surfaces
X the absorption at
The surfaces.

Relation between energy density
in a space &
The rate at which energy
arrives at The walls
when The sound field
is diffuse



ΔS
small surface segment

Acoustic energy associated with δV

$\mathcal{E}\delta V$

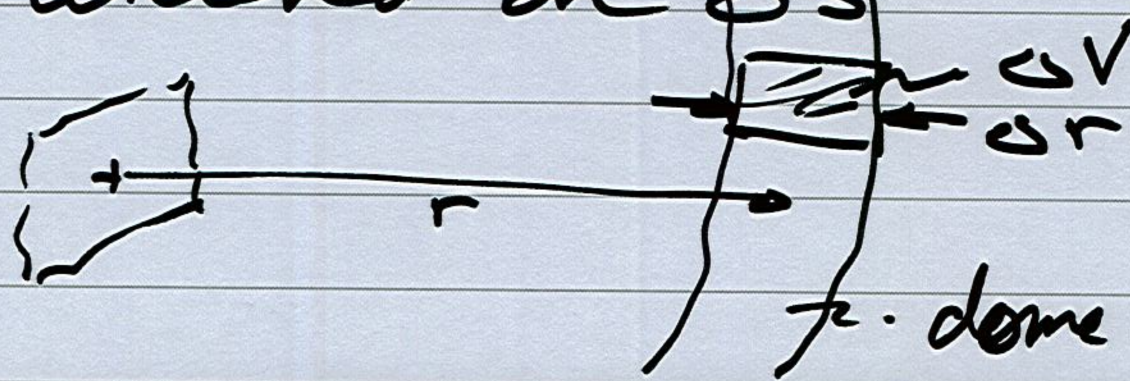
- assume energy radiates away uniformly in all directions

Amount of energy that strikes ΔS from ΔV

$$\frac{\mathcal{E}\,\Delta V}{4\pi r^2}\,\Delta S\,\underline{\cos\theta}$$

- no energy strikes the surface edge-on

let ΔV be part of a hemispherical shell of thickness Δr + radius r centered on ΔS



$$\frac{\Delta E}{\Delta t} = \frac{\mathcal{E} c \, \Delta S}{4}$$

rate at which energy falls on area ΔS

$\div S$

unit area

limit $\Delta t \to 0$

$$\frac{dE}{dt} = \frac{\mathcal{E} c}{4}$$

rate at which energy falls on the walls

A

rate at which energy is absorbed at the walls

$$\frac{\mathcal{E}c}{4}A$$

total room absorption

$\underbrace{[m^2]}_{\text{metric sabins}}$ "absorption area"

Energy Balance

$$\underset{\sim}{\Pi} = V\frac{d\mathcal{E}}{dt} + \frac{Ac}{4}\,\mathcal{E}$$

total absorption

ODE governing the space + short-time averaged energy density in a space.

steady-state $\quad \frac{d\mathcal{E}}{dt} = 0$

rate of energy delivery by source

= rate of energy absorption at the walls

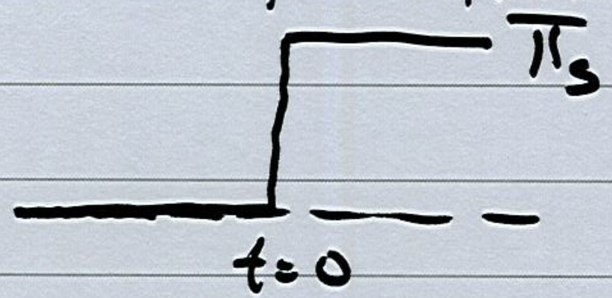
$$\boxed{\overline{\Pi} = \frac{Ac\,\mathcal{E}}{4}}$$

allows us to predict mean square pressure in a room

$$\mathcal{E} = \frac{4\,\overline{\Pi}}{Ac}$$

Recall $\quad \mathcal{E} = \frac{1}{2}\frac{p_+^2}{\rho_0 c^2}$ msp in the diffuse field.

Solve for time variation of the sound field by using Laplace transforms



source turns on

$$\hat{P_r^2} = \frac{4\Pi_s \rho_0 c}{A}\left(1 - e^{-t/\tau_e}\right)$$

$$\tau_e = \frac{4V}{Ac}$$