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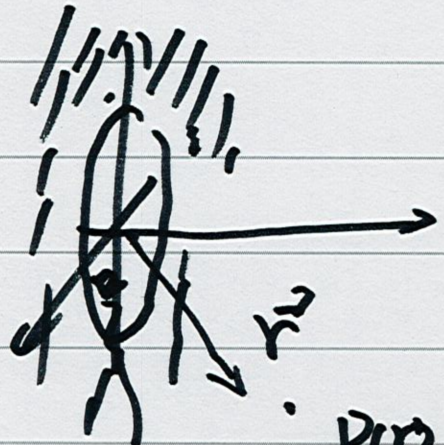
ME 513 Session 38

Recall. Piston in Baffle.

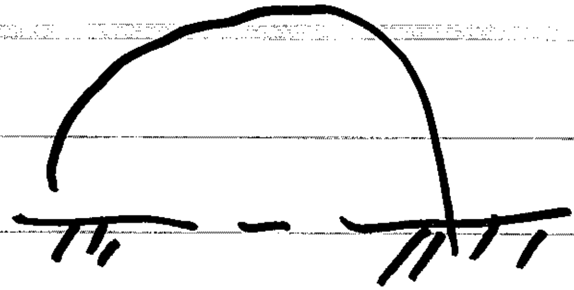


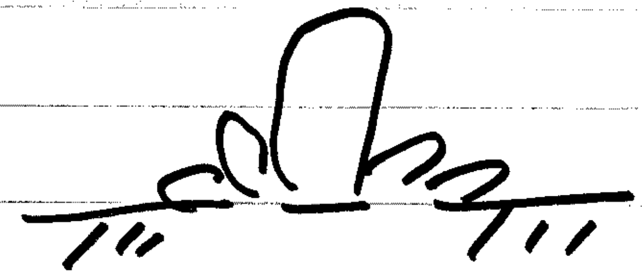
$dQ = u_0 \, ds$

$$p(r) = \int_S \underbrace{2}_{\text{Baffled}} \, j\rho_0 c k \underbrace{u_0 \, ds}_{\text{Volume Strength}} \frac{e^{-jkr}}{4\pi r}$$

Far field : ($\cancel{\quad}$ $r \gg a$)

$$p(r) = \frac{j\rho_0 c}{2} u_0 \left(\frac{a}{r}\right) e^{-jkr} \left[\frac{2 J_1(ka\sin\theta)}{ka\sin\theta} \right]$$

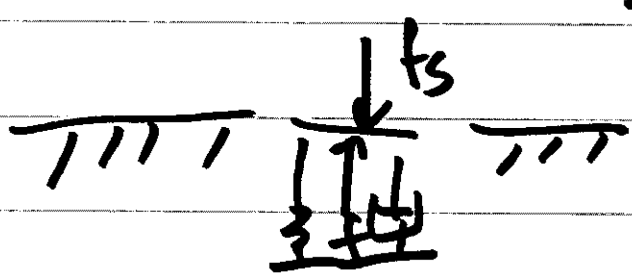




$ka \ll 1$ (monopole)

$ka \gg 1$

Radiation Impedance.



$$z_r = \frac{f_s}{u} = R_r + jX_r$$

Radiation Power

$$\Pi = \frac{1}{2} Re\{f_s u^*\} = \frac{|u|^2}{2} \underline{\underline{R_r}}$$

$$f_s = \int_S p(r)\,ds \quad , \quad p(r) = \int_S 2j\rho_0 c k\, u \frac{e^{-jkr}}{4\pi r}\,ds$$

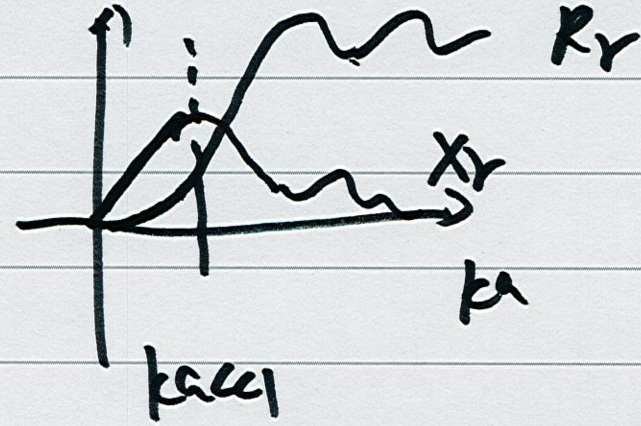
See Text for doing this integral.

$$\begin{cases} R_r = \pi a^2 \rho_0 c \left(1 - \dfrac{2J_1(2ka)}{2ka}\right) & J_1 - \text{Bessel function} \\ X_r = 2\pi a^2 \rho_0 c \left(\dfrac{H_1(2ka)}{2ka}\right) & H_1 - \text{Struve function} \end{cases}$$

when $ka \ll 1$

$$\begin{cases} R_r \approx \dfrac{\pi a^2}{2} \rho_0 c (ka)^2 \\ X_r \approx \pi a^2 \rho_0 c (ka) \dfrac{8}{3\pi} \end{cases}$$

X_r – dominates – mass-like

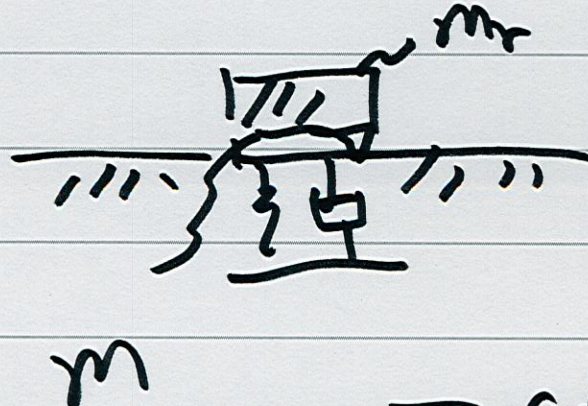


$X_r = w \underline{m_r}$

$$m_r = \pi a^2 \rho_0 \left(\frac{8a}{3\pi}\right)$$

↳ effective mass

for air : $m_r = 0.85\ a$



$$w_0 = \sqrt{\frac{s}{m + \underline{\underline{m_r}}}}$$

— added mass usually lower the natural freq.

High freq. $ka \gg 1$

$R_r \to \pi a^2 \rho_0 c$ $\quad (J_1(2ka) \to 0)$

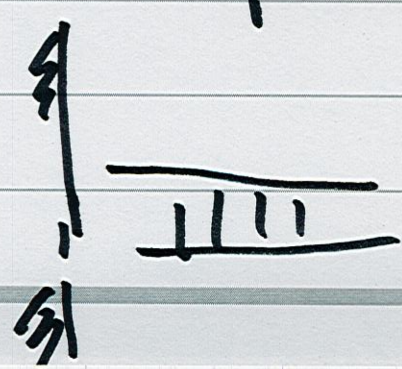
$X_r \to 0$ $\quad (H_1(2ka) \to 0)$

→ ~~Rad~~ Radiation power is higher than

Π_{accel}

$\Pi = \frac{1}{2} \rho_0 c \pi a^2 |u|^2$

→ plane wave.



Summary:

Sections: 7.1, 7.2, 7.4, 7.5, 7.10.

① Compact Sources $kD \ll 1$

- monopoles — Volume
- dipoles — point Force
- quadrupole — Moment
- ⋮

② Extended Source

- Piston in Baffle
  - $ka \ll 1$ — omnidirectional
  - $ka \gg 1$ — directivity

6. Room Acoustics

6.1 Introduction

Wallace Sabine (1868 - 1919)

- ~~Di~~ Discovered the empirical relation between Rev. Time (T), Volume of Room (V)

Total absorption (A)

$$T \propto \frac{V}{A}$$

Two way derive $T \propto \frac{V}{A}$

① Based on plane wave solution

(Theoretical Acoustics. P. Morse)

② Energy Acoustics ✓

① space Averaged energy

② rate of energy ~~[illegible]~~ added by the ~~[illegible]~~ source

③ rate of energy absorbed by the room