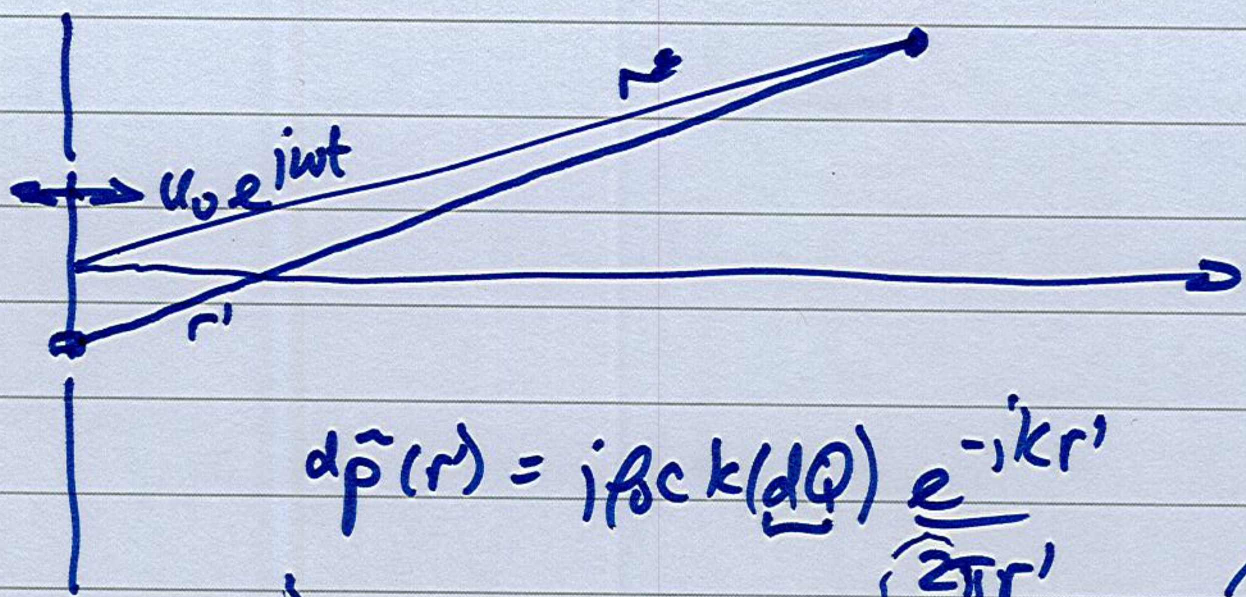
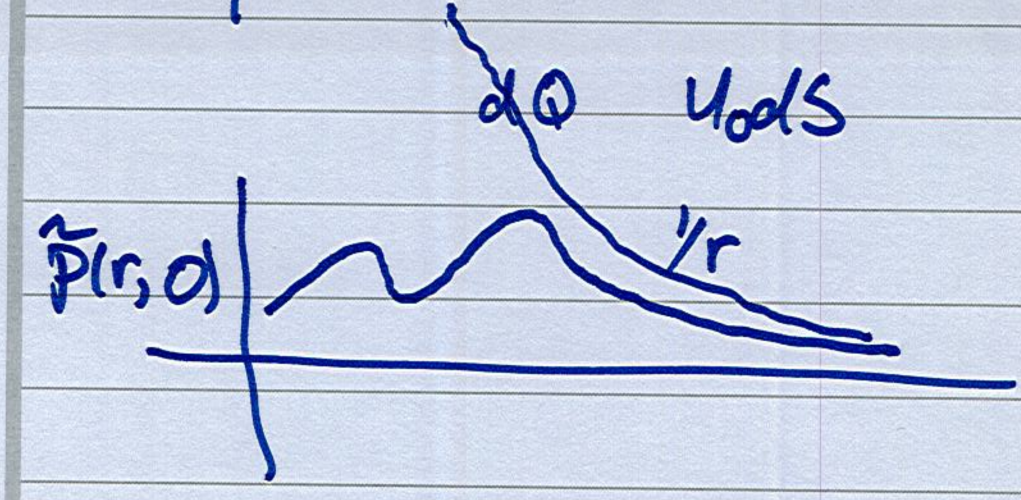
Piston in a baffle

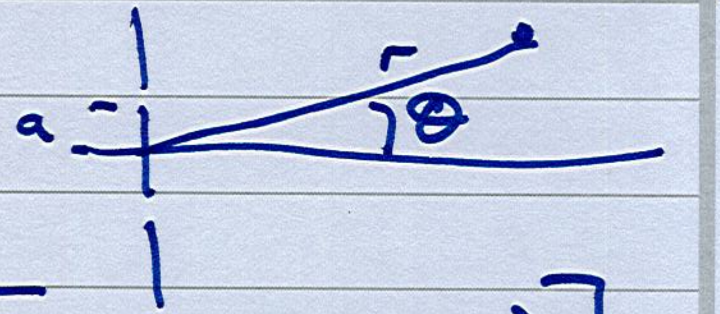


$$d\tilde{p}(r) = j\rho_0 c k (\underset{\sim}{dQ}) \frac{e^{-jkr'}}{2\pi r'}$$

$dQ \quad u_0 dS$

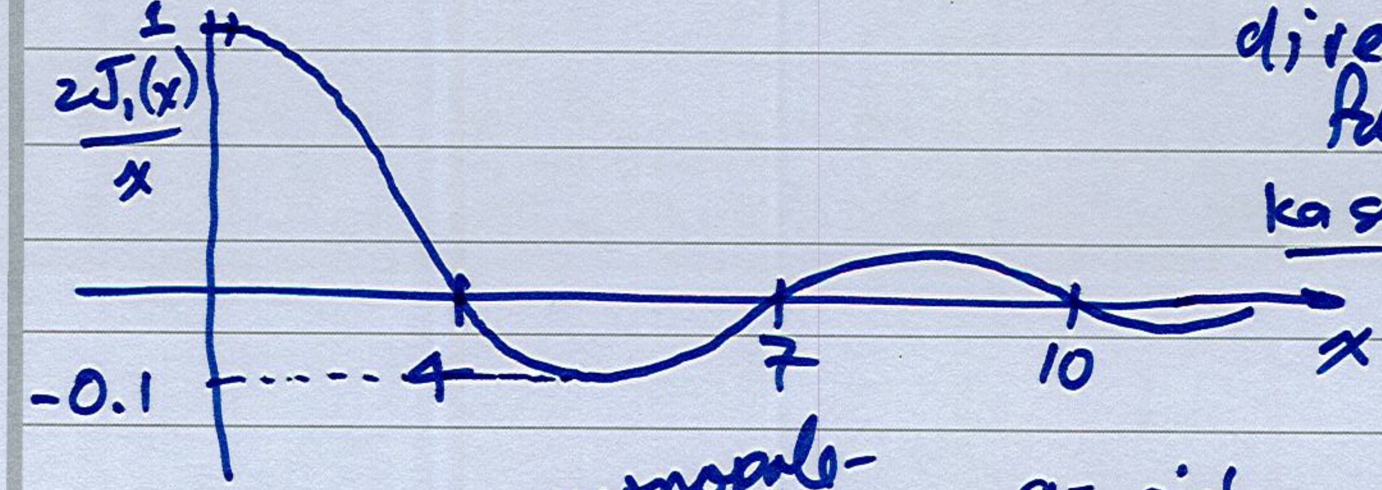


Farfield



$$\hat{p}(r,\theta) = j\frac{\rho_0 c}{2}U_0\underset{\uparrow}{\left(\frac{a}{r}\right)}e^{-jkr}\underbrace{\left[2\,\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]}_{\text{directivity factor}}$$

$ka\sin\theta = x$



if $ka \ll 1$ monopole-like case.

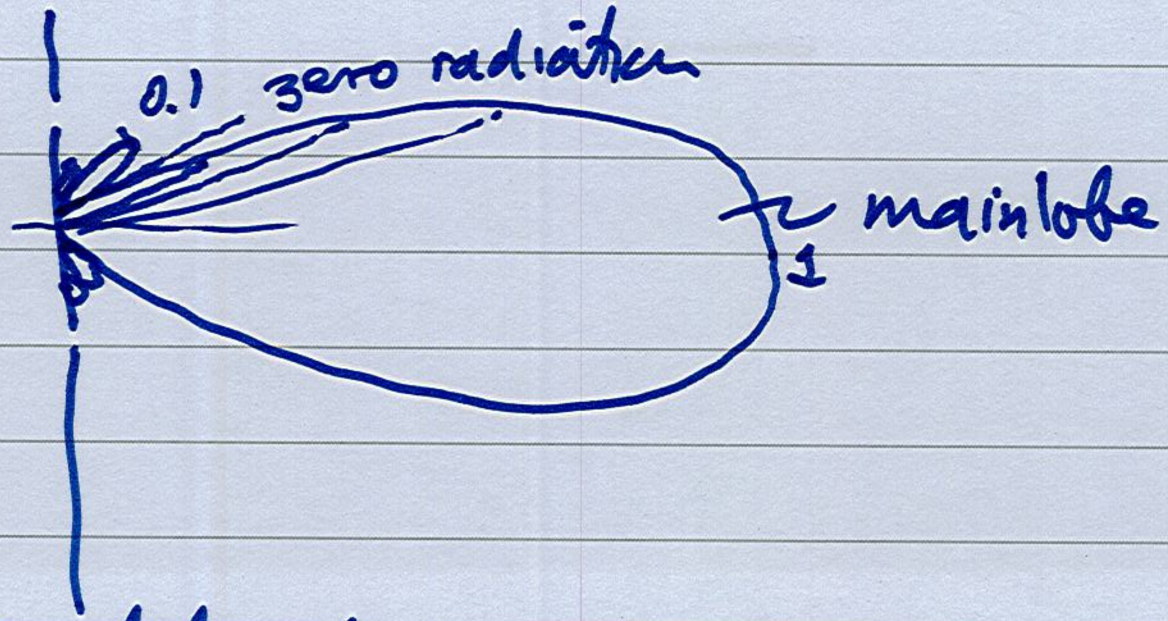
directivity factor $\simeq 1$ for all angles

a = piston radius

$k = \frac{\omega}{c}$

θ = polar angle

$ka > 1$ we have possibility of sidelobes & nulls

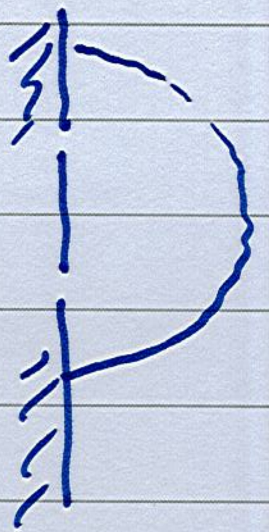


main lobe becomes narrower as the frequency increases

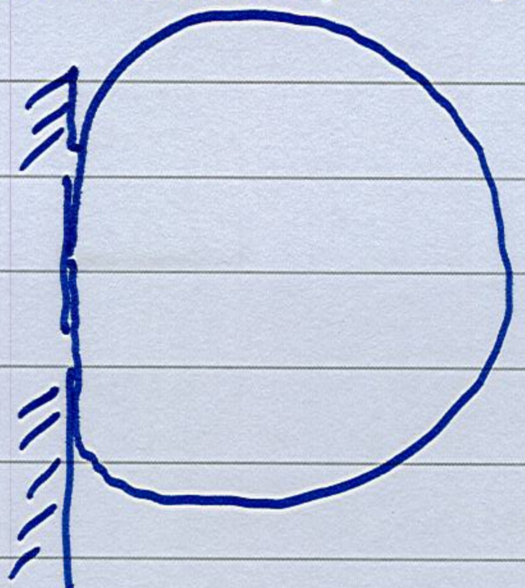
$ka \sin\theta = 4$

$ka \gg$ highly direction radiation

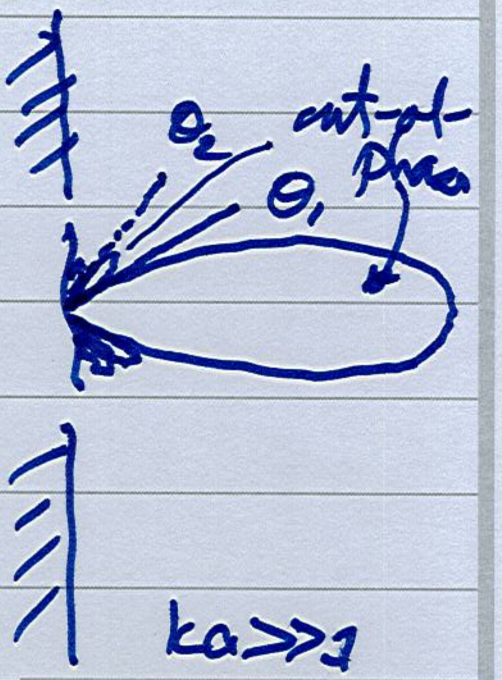
$ka << 1$ omnidirectional radiation



$ka << 1$



$ka \simeq 4$



$ka >> 1$

zeros of the Bessel function → AS

θ_1 = angle of first null

First minimum happens when $\underline{ka \sin\theta_1} = j_{11} \simeq$ first zero of the Bessel function $J_1(j_{11}) = 0$

$J_1(j_{11}) = 0$

$J_1(j_{12}) = 0$ $\quad ka \sin\theta_2 = j_{12}$

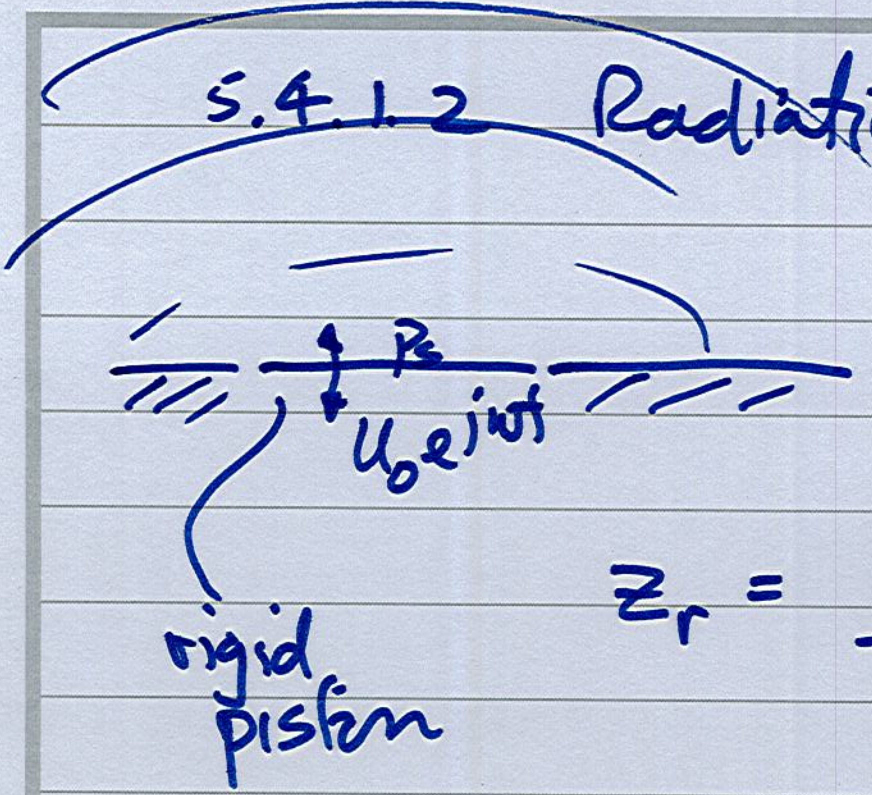
at high freqs $ka \gg 1$ j_{11} is
reached at progressively smaller
angles

"beam # width" becomes narrower

Public Address systems
- many h.f. drivers - pointing
in various direction to
ensure uniform coverage

- small number of large
low frequency drivers

5.4.1.2 Radiation Impedance



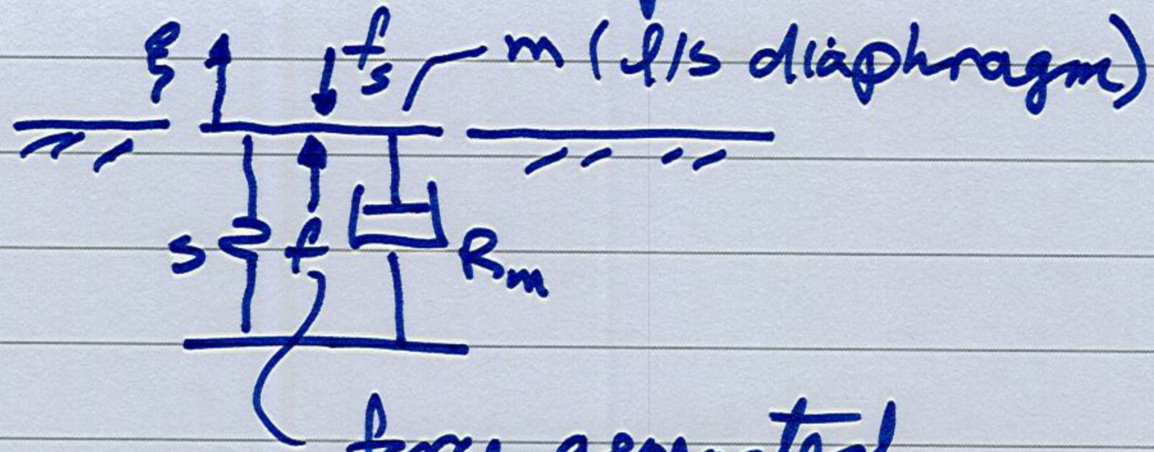
$$Z_r = \frac{\text{force exerted on the radiator by the sound field}}{\text{source velocity}}$$

$$Z_r = \frac{\int_S p_{surface}\, dS}{u_0}$$

total force acting on piston

u_0 2 rigid piston velocity

Loudspeaker - finite internal mechanical impedance



force generated by the voice coil.

Equation of Motion

$$f - f_s = m\frac{d^2\xi}{dt^2} + R_m\frac{d\xi}{dt} + s\xi$$

Assume harmonic motion $e^{j\omega t}$

$u = j\omega\xi$ diaphragm velocity

$$f - f_s = j\omega m u + R_m u + \frac{s}{j\omega} u$$

$$= \underbrace{\overbrace{\left[R_m + j\left(\omega m - \frac{s}{\omega}\right)\right]}^{z_m}}_{\text{in vacuo mechanical impedance of the l/s}} u$$

$$f = f_s + z_m u$$

$$= \left(\underset{\uparrow}{z_r} + z_m \right) u \qquad z_r = \frac{f_s}{u}$$

radiation impedance

in vacuo mechanical impedance

$$\underset{\sim}{u} = \frac{f}{\underset{\sim}{z_r} + \underset{=}{z_m}}$$

l/s response is determined by both mechanical & radiation impedances

$$z_r = R_r + jX_r$$

resistance reactance

Power delivered by the piston to the sound field

$$\Pi = \frac{1}{T}\int_0^T \mathrm{Re}\{f_s\}\,\mathrm{Re}\{u\}\,dt$$

harmonic case

$$\Pi = \frac{1}{2}\mathrm{Re}\{f_s u^*\}$$

$$z_r = \frac{f_s}{u}$$

$$f_s = z_r u$$

$$\overline{\Pi} = \frac{1}{2} Re\{ Z_r u u^* \}$$

$$= \frac{|u|^2}{2} \underbrace{Re\{ Z_r \}}_{R_r}$$

R_r radiation resistance

$$= \boxed{\frac{|u|^2}{2}} \underline{R_r}$$

mean square velocity
of the piston

Circular Piston

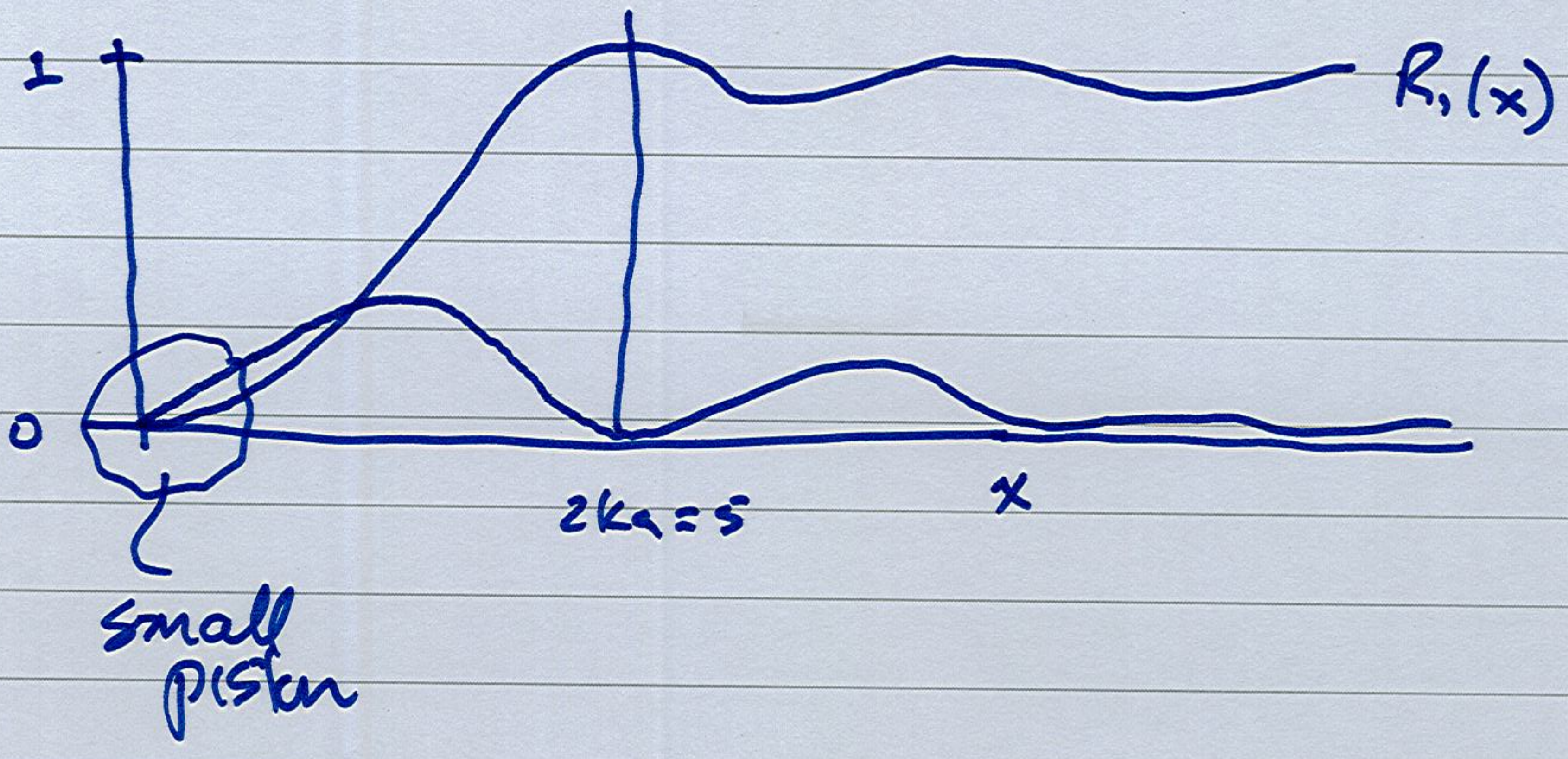
$$R_r = \pi a^2 \rho_0 c \; R_1(2ka)$$

$$X_r = \pi a^2 \rho_0 c \; X_1(2ka)$$

Circular rigid
piston in
an infinite
rigid baffle.

$$R_1(x) = 1 - 2\frac{J_1(x)}{x}$$

$$X_1(x) = \frac{4}{\pi}\left[\frac{x}{3} - \frac{x^3}{3^2 \cdot 5} + x^5 \ldots\right]$$


1
0
$R_1(x)$
$2ka=5$
x
small piston