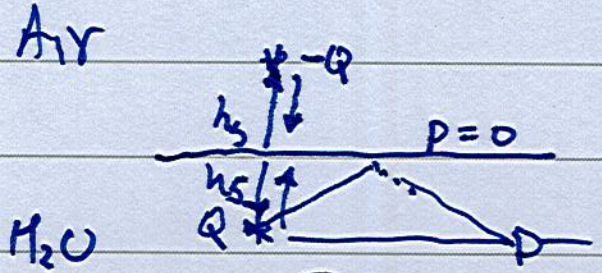
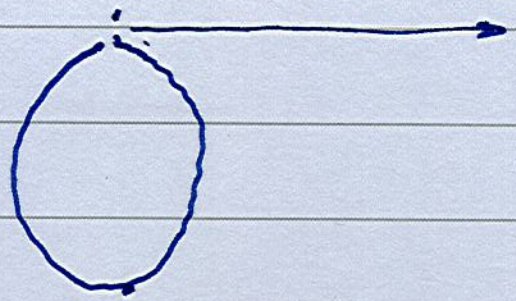
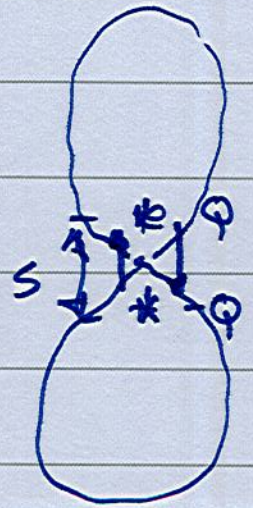
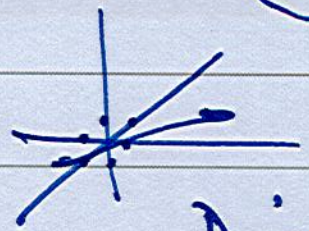




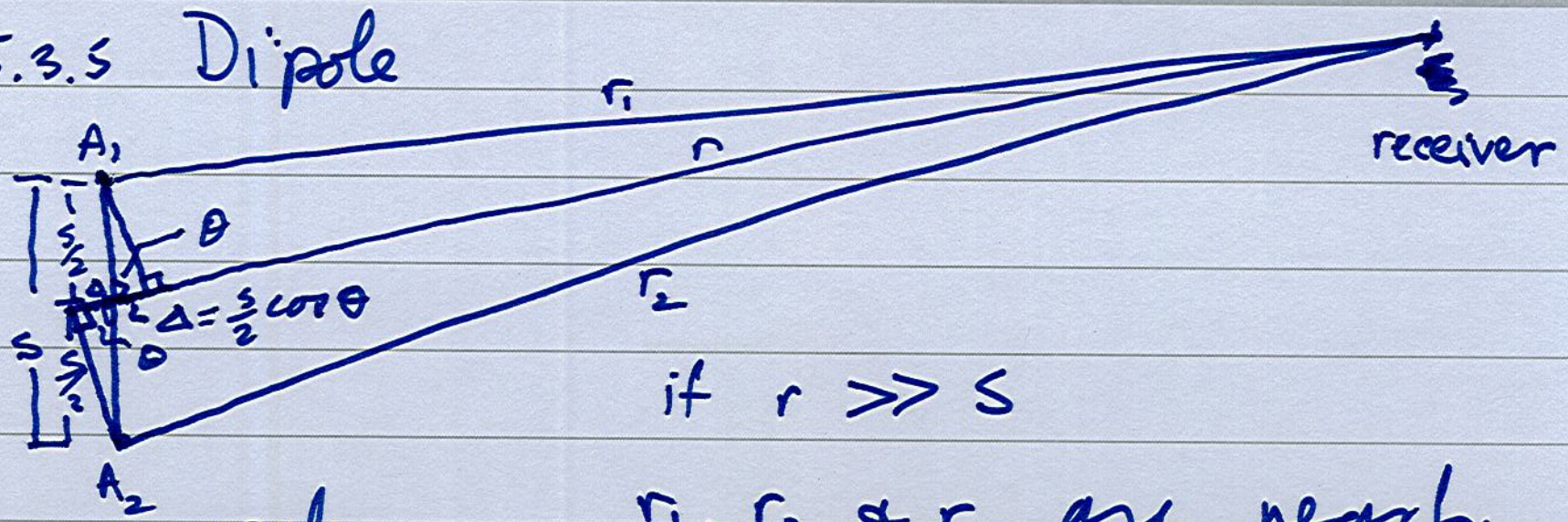


Dipole - represents a pt. force applied at a point in the fluid
- accelerating fluid



Directionality - 3 dipoles are required to represent an arbitrarily oriented source

5.3.5 Dipole



two monopoles strength A_1 & A_2

if $r \gg s$

r_1, r_2 & r are nearly parallel

$$r_1 \simeq r - \Delta = r - \frac{s}{2}\cos\theta$$

$$r_2 \simeq r + \Delta = r + \frac{s}{2}\cos\theta$$

θ is the angle from the dipole axis directed towards the positive monopole

$$\tilde{p}(r) = A_1 \frac{e^{-jkr_1}}{r_1} + A_2 \frac{e^{-jkr_2}}{r_2}$$

Assumed $r >> s$

Next $A_2 = -A_1$

$A_1 = A$

$$\tilde{p}(r) = A\left\{\frac{e^{-jk(r-a)}}{r-a} - \frac{e^{-jk(r+a)}}{r+a}\right\}$$

$$= A e^{-jkr}\left\{\frac{r(e^{+jka} - e^{-jka}) + a(e^{+jka} + e^{-jka})}{r^2 - a^2}\right\}$$

far field $r \gg s$, $a \ll r$

$$\tilde{p}(r) \approx A\frac{e^{-jkr}}{r}(e^{+jka} - e^{-jka})$$

$$= 2jA\frac{e^{-jkr}}{r}\sin(ka)$$

$ka = k\frac{s}{2}\cos\theta$

Assume $ks \ll 1$ compact source

$$\sin k\Delta$$

$$= \sin\left(\underbrace{\frac{ks}{2}\cos\theta}_{\text{very small}}\right)$$

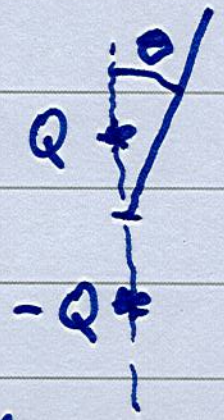
$$\simeq \frac{ks}{2}\cos\theta$$

$\sin\theta \quad \theta \ll 1$

$\downarrow$

θ

$$\tilde{p}(r) = 2jA \frac{e^{-ikr}}{r} \frac{ks}{2} \cos\theta$$



$$A = j\rho_0 c \frac{kQ}{4\pi}$$] monopole derivation

Q = volume source strength

$$\tilde{p}(r) = -\rho_0 c k^2 (Qs) \frac{e^{-ikr}}{4\pi r} \cos\theta$$

sound field radiated by a pt. dipole

$Qs = D$ dipole source strength = dipole moment

Assumptions

- compact source $ks \ll 1$
- receiver is in the far field

$$a \ll r \qquad s \ll r$$

For a monopole

$$\tilde{p}(r) = j\rho c \frac{kQ}{4\pi r} e^{-jkr}$$

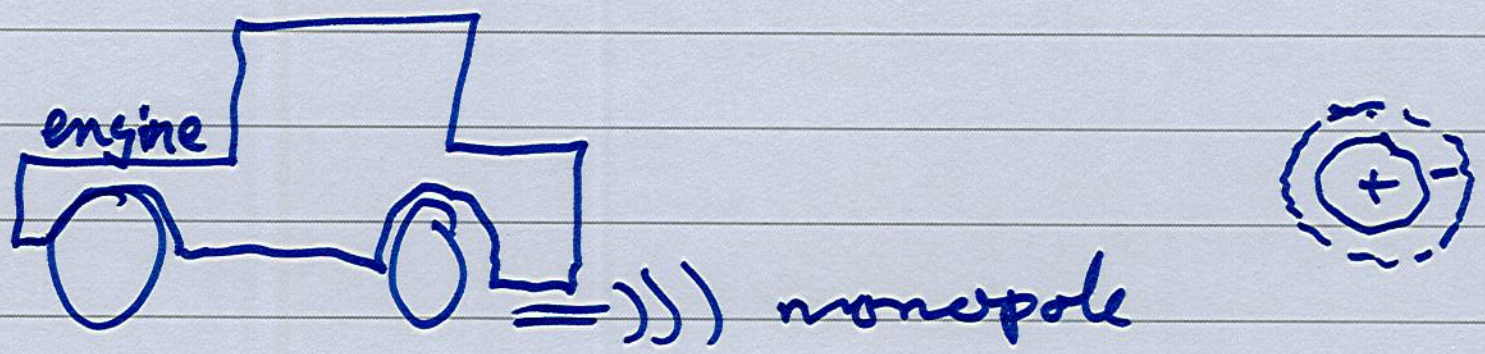
For a dipole

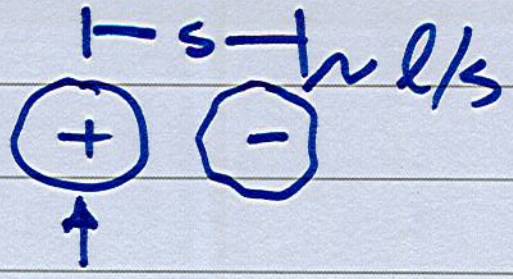
$$\tilde{p}(r) = -\rho c k^2 (QS) \frac{e^{-jkr}}{4\pi r} \cos\theta$$

$$\frac{|\tilde{p}(r)|_{dipole}}{|\tilde{p}(r)|_{monopole}} = ks\cos\theta \qquad ks \ll 1$$

for a given monopole source strength

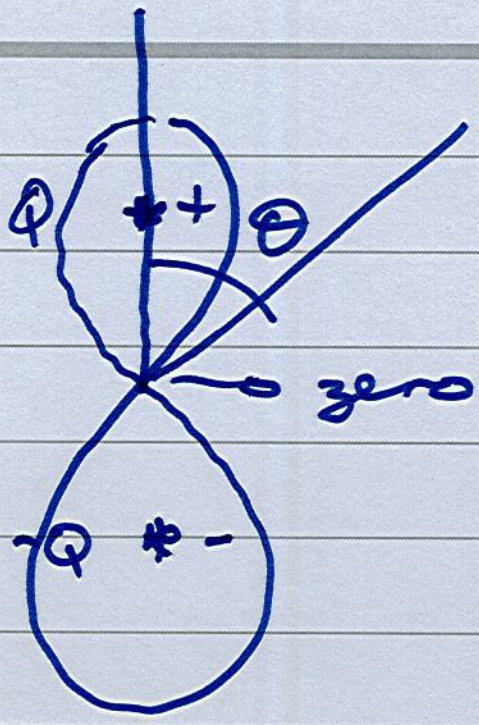
$$|\tilde{p}|_{dipole} \ll |\tilde{p}|_{monopole}$$





monopole + ~~Dea~~ turn into dipole

- reduced sound radiation because radiation efficiency is reduced.



pressure is a maximum when

$\theta = 0$ or π

pressure is a minimum when

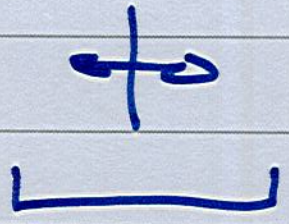
$\theta = \pi/2$

$$\tilde{p}(\theta=0) = -\tilde{p}(\theta=\pi)$$

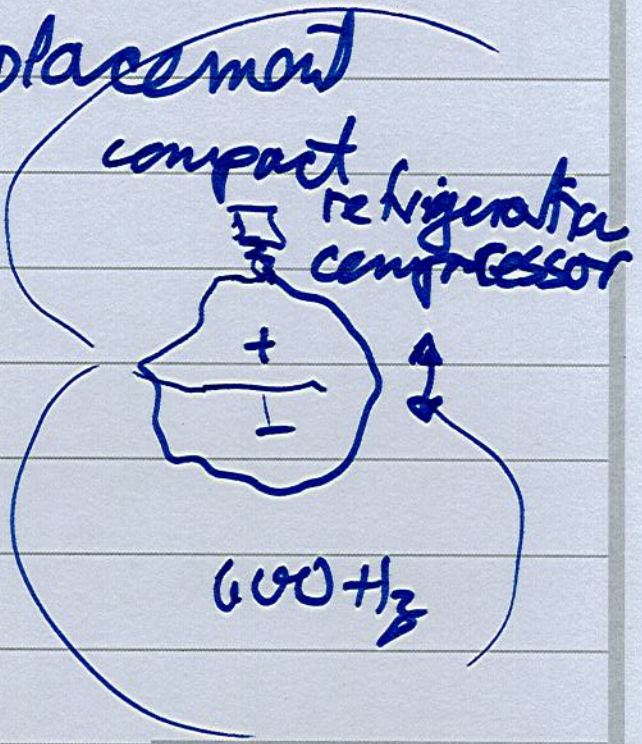
sound radiation is axisymmetric (independent of azimuth angle)

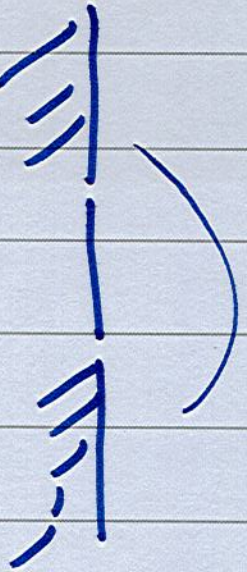
Dipole - no net volume displacement

unbaffled l/s



8" axial fan





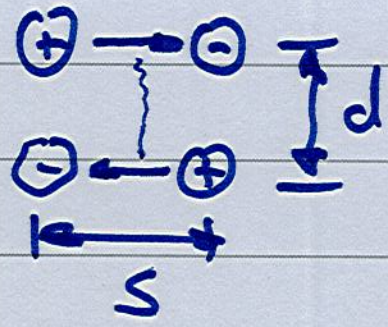
monopole

increase the sound radiation from a l/s by placing it in a baffle.

5.3.6 Quadrupoles, etc.

- Array of two dipole

lateral quadrupole

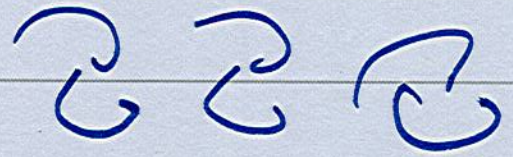


compact
$ks \ll 1$
$kd \ll 1$

quadrupole is used to represent a compact source that radiates ⇒ oscillatory rotation

- represents a point moment

turbulent flows



- many vortices interacting and changing rotation rate.

lateral quadrupoles are used to represent sound radiation from homogeneous turbulence

no boundaries (no interaction with solids)

M. J. Lighthill