ME 513 Session 30 11/4/15 1

Final Exam

Weds 12/16/15

8:00 - 10:00 AM

ARMS B071

Note misprint correction

page 11 11/9/15

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Problem 3.

A sound pressure field in air has the form

$$p(x, y, z, t) = Ae^{-j\beta x}e^{-\alpha y}e^{j\omega t}$$

where A is complex and β and α are real.

(i) Derive an expression for the vector particle velocity associated with this pressure field.

(ii) By calculating the vector intensity of this field, show that there is no energy flow in the y direction.

(iii) In sketch form, illustrate the spatial variation of the sound field.

$$\hat{u}_x = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial x} = -\frac{1}{j\omega\rho_0}\left(-j\beta A^* e^{+j\beta x} e^{-\alpha y} e^{-j\omega t}\right)$$

$$\hat{u}_y = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial y} = -\frac{1}{j\omega\beta}\left(-\alpha A e^{-j\beta x} e^{-\alpha y} e^{j\omega t}\right)$$

$$\bar{u} = \hat{u}_x \bar{i} + \hat{u}_y \bar{j}$$

$A = a + jb$

$A^* = a - jb$

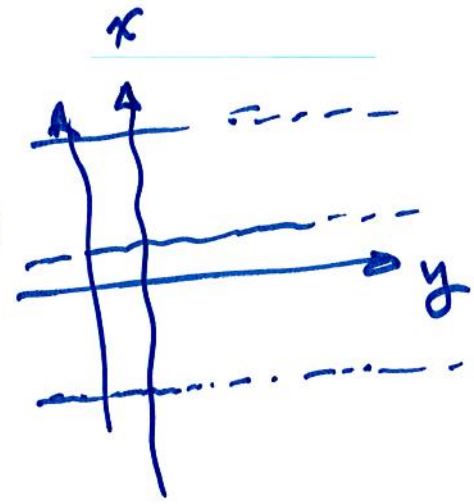
$$\bar{I} = I_x \hat{i} + I_y \bar{j}$$

$$I_x = \frac{1}{2}\mathrm{Re}\{\hat{p}\, u_x^*\}$$

$$I_y = \frac{1}{2}\mathrm{Re}\{\underbrace{\hat{p}\, \tilde{u}_y^*}_{j}\} = 0$$

A A^*

$|A|^2$



Problem 3.

The spatial variation for the first non-planar mode in a two-dimensional hard-walled channel of width L is:

$$p(x,y) = A_1 \cos\left(\frac{\pi y}{L}\right) e^{-jk_1 x}$$

$$where : k_1 = \Big[\underbrace{k^2 - \left(\frac{\pi}{L}\right)^2}_{-ve}\Big]^{\frac{1}{2}}$$

$k = \omega/c$ and A_1. is complex.

(i) Derive the vector particle velocity field by using the linearized momentum equation.

(ii) Calculate the vector intensity field in the channel, and show, in particular, that the y-component of the intensity is equal to zero.

(iii) Explain, based on your results, why the x-component of the intensity is also equal to zero when $k^2 < (\pi/L)^2$.

$$u_x = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial x} = -\frac{1}{j\omega\rho_0}\left(-\cancel{j}k_1 A_1 \cos\frac{\pi y}{L} e^{-jk_1 x}\right)$$

$$u_y = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial y} = -\frac{1}{j\omega\rho_0}\left(-\frac{\pi}{L} A_1 \sin\left(\frac{\pi y}{L}\right) e^{-jk_1 x}\right)$$

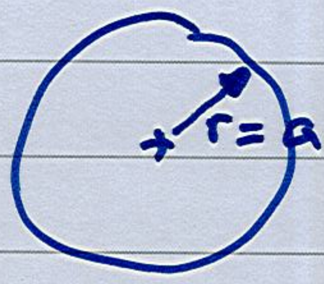
$$\bar{u} = \hat{u}_x \bar{i} + \hat{u}_y \bar{j}$$

$$I_x = \frac{1}{2}\mathrm{Re}\{\tilde{p}\,\tilde{u}_x^*\} = -k_1 \quad 0$$

$$I_y = \frac{1}{2}\mathrm{Re}\{\tilde{p}\,\tilde{u}_y^*\} = 0$$

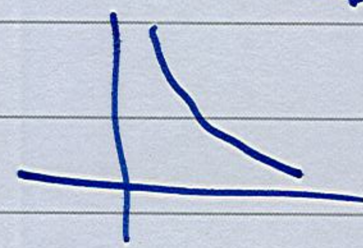
w

Sound Generation & Radiation in 3-D



$$\tilde{p}(r) = \left(\rho_0 c\, a\, \underline{\underline{U_0}}\, e^{+jka}\, e^{+j\phi_a} \cos\phi_a \right) \underline{\underline{\frac{e^{-jkr}}{r}}}$$

$$\cos\phi_a = \frac{ka}{\sqrt{(ka)^2+1}}$$



- $\frac{\tilde{p}\tilde{p}^*}{2} = p_{rms}^2$

$$L_p = 10 \log \frac{p_{rms}^2}{p_{ref}^2}$$

5.2.3 Impedance, Intensity & Sound Power.

Impedance: $$\frac{\tilde{p}(r)}{\tilde{u}_r(r)} = \tilde{z}(r) = \rho_0 c \frac{1}{1 - \frac{j}{kr}}$$

$$= \rho_0 c \frac{kr}{kr - j} \qquad \cos\phi_r = \frac{kr}{\sqrt{(kr)^2+1}}$$

$$= \rho_0 c \cos\phi_r \, e^{j\phi_r}$$

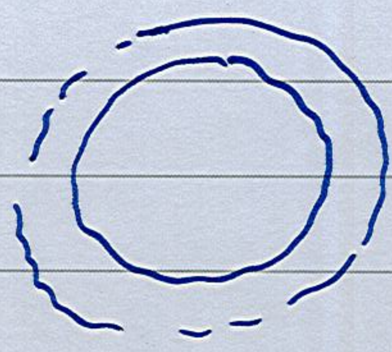
farfield $kr \gg 1$

$$\lim_{kr\to\infty} \tilde{z}(r) = \underline{\underline{\rho_0 c}}$$

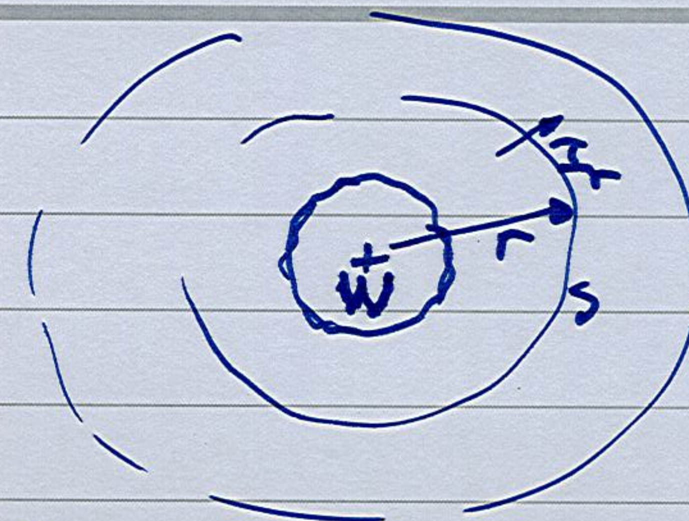
plane wave

$kr \ll 1$ $\qquad \tilde{z} \simeq +i \rho_0 c \, (\underline{k} r)$

mass-like impedance



$$I_r = \frac{1}{2}\mathrm{Re}\{\hat{p}(r)\,\hat{u}_r^*(r)\}$$



$$= \frac{1}{2}\left(\frac{a}{r}\right)^2 \rho_0 c\, U_0^2 \cos^2\phi_a$$

$I_r \propto \underset{\sim\sim}{\dfrac{1}{r^2}}$ Inverse Square Law

Sound Power

$$W = \int_S I_r\, dS = I_r \int_S dS$$

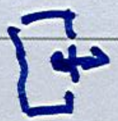
$$= 4\pi r^2 I_r$$

$$W = 2\pi a^2 \rho_0 c\, U_0^2 \cos^2\phi_a$$

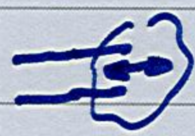
independent of r

5.3 Simple Sources

5.3.1 Point Monopole – compact source that changes volume.



volume sources



$$p = \tilde{A}\,\frac{e^{-ikr}}{r}$$

free field

sphere of arbitrary radius

$$\hat{A} = j\rho_0 c\, a\, \underline{U_0}\, e^{jka}\, \cos\theta_a$$

$$\frac{ka}{ka-j}$$

$4\pi a^2 u_0$ = Volume displaced by the sphere /s

$[m^3/s]$

$= Q =$ volume monopole $\}$ source strength

$$\hat{A} = j\omega\rho_0 (ka) \frac{Q}{4\pi (ka)^2} e^{jka} \frac{ka}{jka+1} \qquad ka = 2\pi \frac{a}{\lambda}$$

non-dimensional source radius

$$\lim_{(ka)\to 0} \hat{A} = j\omega\rho \frac{Q}{4\pi}$$

"point source"

point monopole

$$\tilde{p}(r) = j\omega\rho_0 \frac{Q}{4\pi r} e^{-jkr}$$

$$\boxed{\tilde{p}(r) = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr}}$$

omni-directional

Radial Intensity

$$I_r = \frac{1}{2}\left(\frac{a}{r}\right)^2 \rho_0 c \frac{Q^2}{(4\pi a^2)^2} \cos^2\phi_a$$

$$ka << 1$$

$\cos\phi_a \to ka$ for small ka

$$\lim_{(ka)\to 0} I_r = \frac{\rho_0 c}{2} k^2 \frac{Q^2}{\cancel{(4\pi r)^2}}$$

Should be $(4\pi r^2)^2$

$$Q = 4\pi a^2 U_0$$

$$I_r \propto a^4$$

- rapid increase in intensity with increasing source size
- small sources very poor radiators

Use a point monopole to represent any compact source that exhibits a periodic volume change, and which displace Q volume of fluid/s

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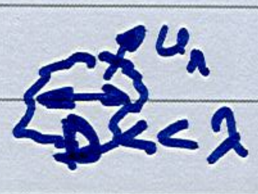
If a source is small compared to a wavelength in <u>all</u> dimensions

- The sound radiation does not depend on the spatial distribution of the source velocity



$$\underline{Q} = \int u_n(s)\, dS$$