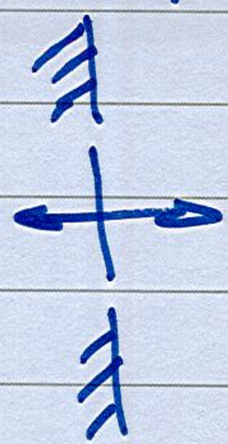
5 Sound Generation and Radiation in 3-D

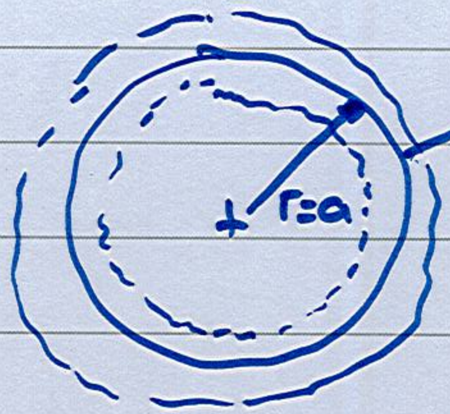
- Compact - simple
  - monopole]
  - dipole
  - quadrupole

- Non-compact source



piston
in a baffled

5.2 Sound Radiation from Pulsating Sphere



$$u_0 e^{j\omega t} \quad \tilde{p} = \frac{A}{r} e^{-jkr} \; \cancel{+ \frac{B}{r} e^{+jkr}} \rightarrow 0$$

spherically symmetric

5.2.2 Boundary Conditions

set the radial particle velocity at $r=a$ (in fluid) to $u_0 e^{j\omega t}$

surface radial velocity $\underbrace{\hat{u}_r(a)}_{\text{fluid velocity}} = \underbrace{u_0}_{\text{surface velocity}}$

$$-\nabla \hat{p} = j\omega \rho_0 \hat{u}_r$$

$$\hat{u}_r = -\frac{1}{j\omega\rho_0} \nabla \hat{p}$$

spherical
symmetry case

$$\frac{\partial}{\partial\theta} \quad \frac{\partial}{\partial\phi} = 0$$

$\hat{u}_r, \hat{u}_\theta, \hat{u}_\phi$

Ξ

purely radial

- no pressure gradient in the θ or ϕ directions

$$\underset{\sim}{\tilde{u}_r(r)} = -\frac{1}{j\omega\rho_0}\frac{\partial \tilde{p}}{\partial r} = \frac{\hat{A}}{\rho_0 c}\frac{e^{-jkr}}{r}\left(1 - \frac{j}{kr}\right)$$

nearfield.

$U_0 e^{i\omega t}$ at $r = a$

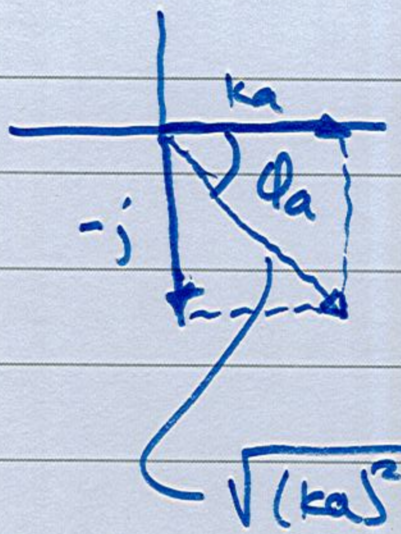
$\tilde{u}_r(a) = U_0$ b.c.

solve for $\tilde{A}$

$$\underset{\sim}{\tilde{A}} = \rho_0 c\, a U_0 e^{jka} \frac{1}{1 - \frac{j}{ka}}$$

$$\tilde{A} = \rho_0 c a U_0 e^{jka} \frac{ka}{(ka - j)}$$

$$(ka - j) = \sqrt{(ka)^2 + 1}\, e^{-j\phi_a}$$

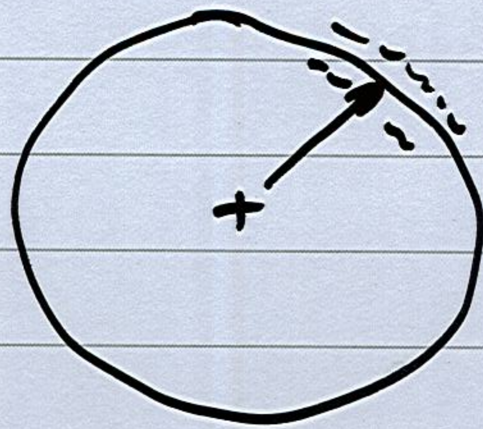


$$= \rho_0 c a U_0 e^{jka} \frac{\overbrace{ka}^{}}{\underbrace{\sqrt{(ka)^2+1}}_{\cos\phi_a}\; e^{-j\phi_a}}$$

$$= \rho_0 c a U_0 e^{jka} \cos\phi_a\, e^{+j\phi_a}$$

$$\tilde{p}(r) = \left(\rho_0 c a U_0 e^{jka} e^{j\phi_a} \cos\phi_a\right) \frac{e^{-jkr}}{r}$$

pulsating sphere
of radius a



<u>Volume Velocity source</u>

- changes volume

small volume velocity source
= monopole.

Problem 2.

An incident transverse wave propagates in the positive-x direction along a uniform tensioned string (the tension in the string is T and its mass per unit length is ρ_L) and reflects from a mass-spring termination at $x = 0$.

(i) Give an appropriate assumed solution for the displacement field in the region $x < 0$.

(ii) Draw a free body diagram of the forces acting at the string termination at $x = 0$.

(iii) Give in equation form the boundary conditions that apply at $x = 0$.

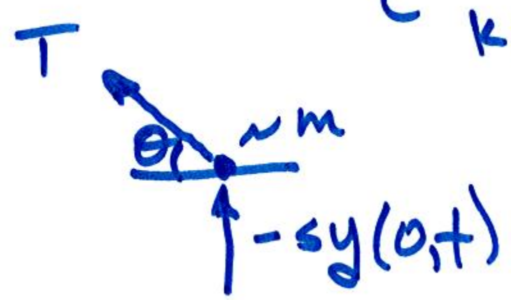
(iv) Use the boundary conditions in conjunction with the assumed solution to solve for the reflection coefficient at the termination: i.e., find the ratio of the complex amplitudes of the waves traveling in the negative and positive x-directions.

(v) Show that the magnitude of the reflection coefficient is always equal to unity for this type of termination.

(i) $y(x,t) = A e^{i(\omega t - kx)} + B e^{i(\omega t + kx)}$

$k = \omega/c \qquad c = \sqrt{T/\rho_L}$

(ii)



$-\frac{\partial y}{\partial x}$

(iii) $\Sigma f_y = ma \quad \left(\Sigma f_y = T\sin\theta\big|_{x=0} - sy\big|_{x=0}\right.$

$= m \frac{\partial^2 y}{\partial t^2}\Big|_{x=0}$

(iv) $\left(\frac{B}{A}\right)$ (v) $\frac{B}{A} = \frac{A -}{A +}$

Problem 3.

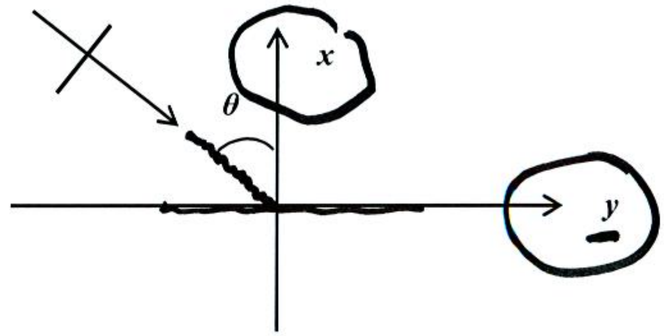
A plane sound wave is propagating in *free-space* in the direction shown in the sketch below.

(i) Give a complete expression for the sound pressure field, defining quantities (such as the wave numbers, for example) as necessary.

(ii) Derive by using the linearized momentum equation an expression for the vector particle velocity.

(iii) Give an expression for the specific acoustic impedance normal to the surface $x = 0$.

(iv) Derive an expression for the time-averaged acoustic intensity field in the x-direction, and show that the energy flow crossing the surface $x = 0$ goes to zero when θ goes to $\pi/2$.



$$A e^{+jk_x x} e^{-jk_y y} e^{j\omega t}$$

$$k_x = k\cos\theta \qquad k = \frac{\omega}{c}$$

$$k_y = k\sin\theta$$

$$u_x = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial x} \qquad I_x = \frac{1}{2}\mathrm{Re}\{\tilde{p}\tilde{u}_x^*\}$$

$$u_y = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial y} \qquad z_x = \frac{\hat{p}}{\hat{u}_x}$$

Problem 3.

When a plane sound wave transmits into a second fluid region at an angle of incidence greater than the critical angle, the sound field has the form

– in the second region

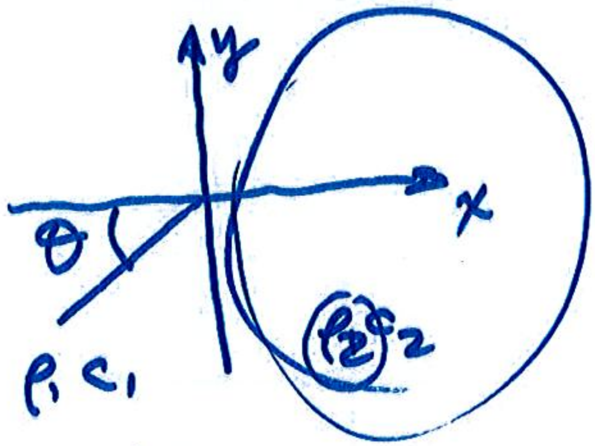
$$p_2(x,y,t) = Ae^{-\gamma x}e^{-jk_y y}e^{j\omega t}$$

where A is complex, γ and k_y are real, x is the coordinate normal to the interface between the two fluid media (and is positive into the second medium), y is the coordinate parallel to the interface, and the second medium has density, ρ_2 and speed of sound, c_2.

(i) Derive by using the linearized momentum equation an expression for the vector particle velocity field in the second medium.

(ii) Derive an expression for the vector, time-averaged acoustic intensity field in the second medium, and show that there is no energy flow normal to the interface in the second medium.

(iii) Sketch the spatial dependence of the sound field in the second medium.



$$\frac{p}{\rho c}$$

(i)
$$u_x = -\frac{1}{j\omega\rho_2}\frac{\partial p_2}{\partial x}$$
$$= -\frac{1}{j\omega\rho_2}\left(-\gamma A e^{-\gamma x}e^{-jk_y y}e^{j\omega t}\right)$$
$$= +\frac{\gamma A}{j\omega\rho_2}e^{-\gamma x}e^{-jk_y y}e^{j\omega t}$$

$$u_y = -\frac{1}{j\omega\rho_2}\frac{\partial p_2}{\partial y}$$

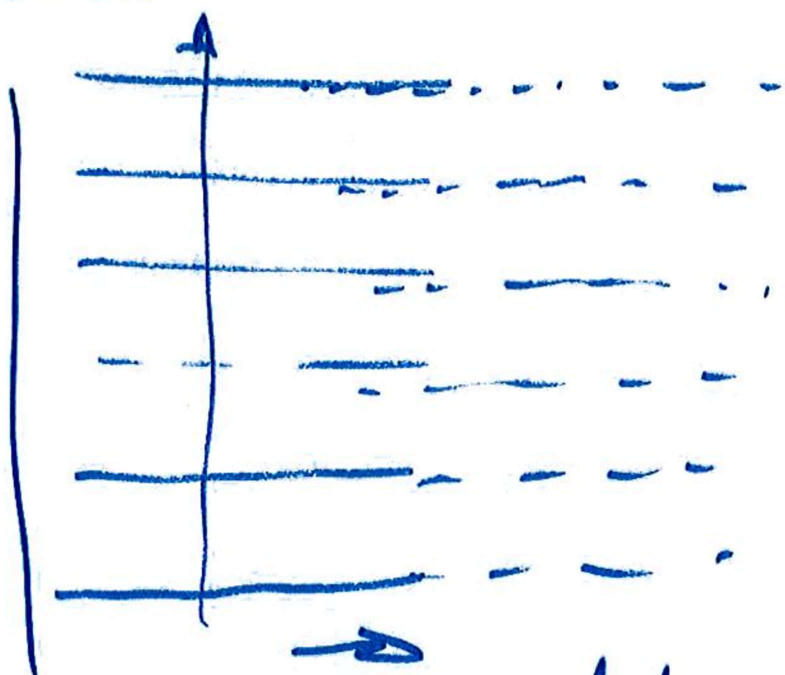
$$\bar{u}_2 = u_x\underset{\sim}{\bar{i}} + u_y\underset{\sim}{\bar{j}}$$ ✓
$$\bar{u}_2 = u_x + u_y$$

$$\bar{I} = \frac{1}{2} Re\{\tilde{p}_2 \tilde{u}_2^*\}$$

$$p = A e^{-\gamma x} e^{-ik_y y}$$

$$I_x = \frac{1}{2} Re\{\tilde{p}_2 \tilde{u}_{2_x}^*\}$$

$$I_y = \frac{1}{2} Re\{\hat{p}_2 u_{2_y}^*\}$$



exponential decay

$e^{-\gamma x}$