ME 513 Session 26 10/26/2015

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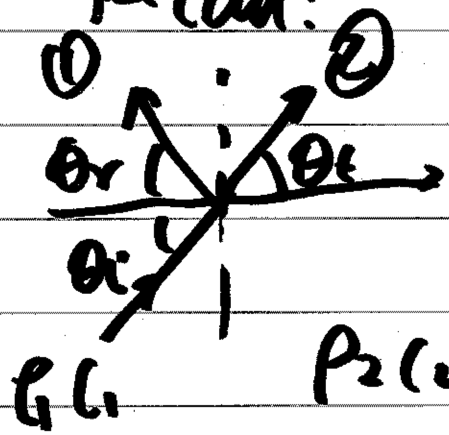
Recall:



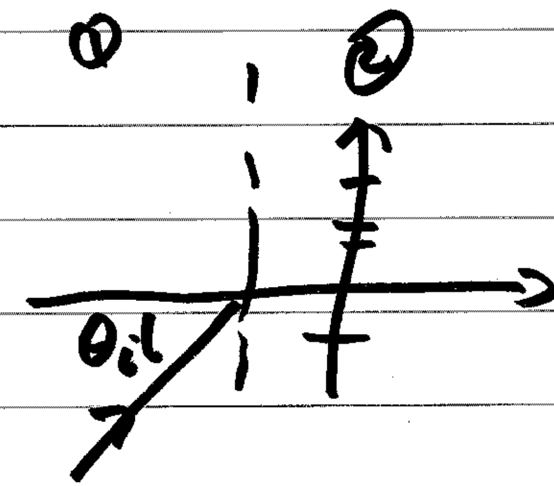
Snell's law:

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{c_2}{c_1}$$

$$R = \frac{\zeta_{21} - \frac{\cos\theta_t}{\cos\theta_i}}{\zeta_{21} + \frac{\cos\theta_t}{\cos\theta_i}}$$

$$T = \frac{2\zeta_{21}}{\zeta_{21} + \frac{\cos\theta_t}{\cos\theta_i}}, \quad \zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$$

Critical angle: θ_c :



$C_2 > C_1$

when $\theta_i = \theta_c$, $\underline{\theta_t = \frac{\pi}{2}}$

$$\tilde{P}_t = P_t e^{-j(k_{tx}x + k_{ty}y)}$$

$k_{tx} = k_2 \cos\theta_t$, $k_{ty} = k_2 \sin\theta_t$

$\cos\theta_t \to 0$

$k_{tx} = 0$

→ no prop. wave in x-direction

if $\theta_i > \theta_c$, $\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i > 1$

$$\sin\theta_t = \frac{e^{j\theta_t} - e^{-j\theta_t}}{2j} > 1$$

θ_t – complex number

$$k_{tx} = \pm k_2 \sqrt{1 - \underbrace{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}_{>1}}$$

$$= \pm \underbrace{k_2 \sqrt{-1}}_{j} \underbrace{\sqrt{\left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i - 1}}_{>0}$$

$$= \pm j\gamma, \quad \gamma = \sqrt{\left(\frac{c_2}{c_1}\right)^2 \sin\theta_i - 1}$$

transmitted sound field:

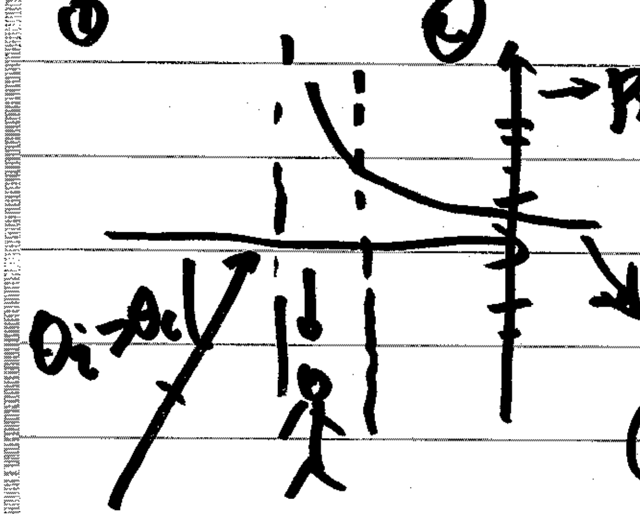
$$\tilde{p}_t = P_t e^{-jk_{tx}x} e^{-jk_{ty}y} \quad , \quad k_{tx} = \pm j\gamma$$

$$= P_t \underline{e^{\pm\gamma x}} \, \underline{e^{-jk_1 \sin\theta_i y}} \quad k_{ty} = k_2 \sin\theta_t = k_1 \sin\theta_i$$

allows all values $0 < \theta_i < \frac{\pi}{2}$

Exponential growth or decay of sound pressure

→ Wave prop. in y direction



→ Prop. wave in y- direction

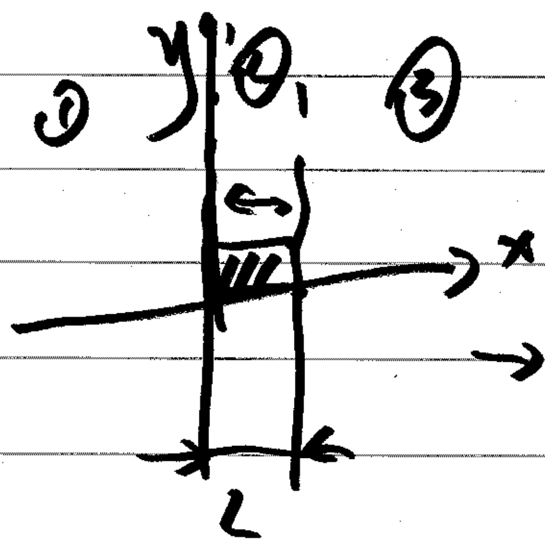
→ Exponential decay in x direction (discarded growth possibility on physical grounds).

You can hear sound in the near field.

$\because \theta_i > \theta_c \quad \theta_i \nearrow$

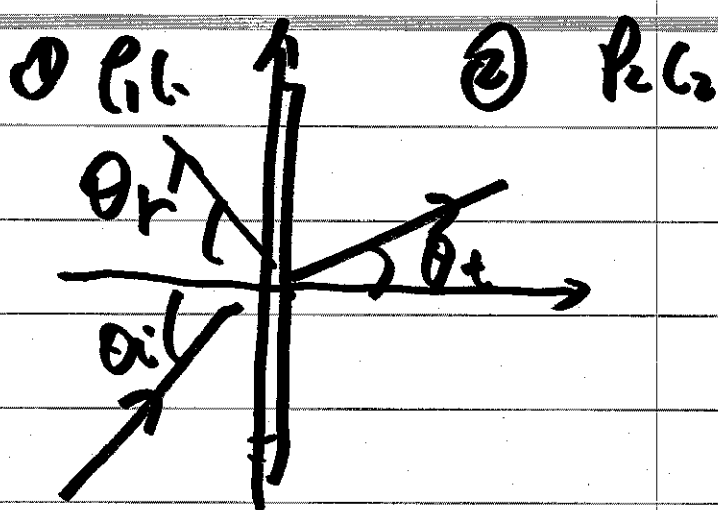
$\Rightarrow \gamma \uparrow \Rightarrow$ decay rate $\uparrow$

4.3.2 Reflection and transmission at a limp. thin panel.



$k_2 L \ll 1$

1. no wave in x-direction
2. no wave in y direction
3. only consider. rigid body ~~trans~~ transverse motion



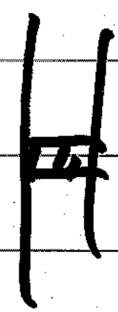
$\rho_1 = \rho_2$. $c_1 = c_2$

m_s – mass / unit area of ~~pe~~ the panel

~~w(y)~~ $w(y)$ – transverse displacement

Limp. (Locally reacting)

- no free wave in the panel



In region ①:

$$\tilde{p}_1 = P_i e^{-j(k_x x + k_y y)} + P_r e^{+j(k_x x - k_y y)}$$

$$\theta_i = \theta_r = \theta_t = \theta, \quad k_x = k\cos\theta, \quad k_y = k\sin\theta$$

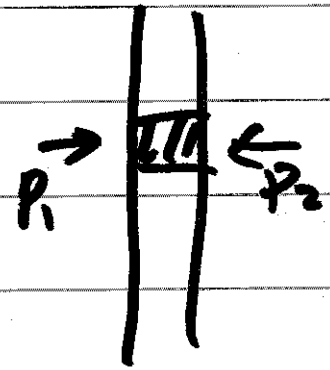
$$\tilde{u}_{1x} = \cancel{\frac{P_i}{\cos\theta}} \frac{P_i}{\rho_0 c}\cos\theta\, e^{-j(k_x x + k_y y)} - \frac{P_r}{\rho_0 c}\cos\theta\, e^{+j(k_x x - k_y y)}$$

In region ②:

$$\tilde{p}_2 = P_t e^{-j(k_x x + k_y y)}$$

$$\tilde{u}_{2x} = \frac{P_t}{\rho c}\cos\theta\, e^{-j(k_x x + k_{xy} y)}$$

EoM of the panel



$\Xi\, dy$ $\quad F = (P_1 - P_2)\, dy$

$= ma$

$$\Rightarrow (P_1 - P_2)\Big|_{x=0} dy = m_s\, dy\, \frac{\partial^2 w}{\partial t^2}$$

B.C. ① $\left(u_{1x} = u_p = u_{2x}\right)\Big|_{x=0}$

② $(P_1 - P_2)\Big|_{x=0} = m_s \left.\dfrac{\partial u_{1x}}{\partial t}\right|_{x=0}$

$= m_s\, j\omega\, u_{1x}\Big|_{x=0}$

Subs the assumed solutions into B.C.

$$\begin{cases} P_i - P_r = P_t \\ P_i + P_r - P_t = \dfrac{j\omega m_s}{\rho_0 c} \cos\theta \, P_t \end{cases}$$

$$\Rightarrow R = \frac{P_r}{P_i} = \frac{j\omega m_s}{\dfrac{2\rho_0 c}{\cos\theta} + j\omega m_s}$$

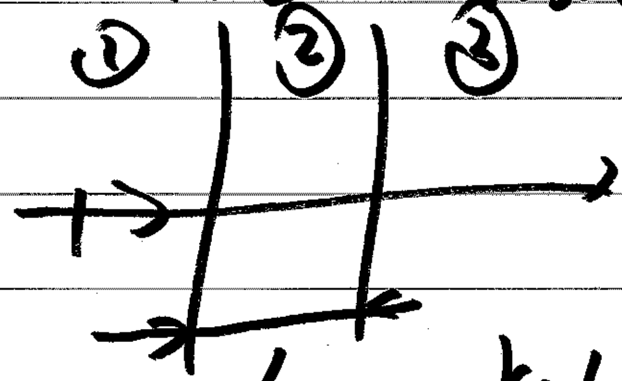
$$T = \frac{P_t}{P_i} = \frac{\dfrac{2\rho_0 c}{\cos\theta}}{\dfrac{2\rho_0 c}{\cos\theta} + j\omega m_s}$$

Special cases:

(I) $\theta_i \to 0$ Normal incidence

$$R = \frac{j\omega m_s}{2\rho_0 c + j\omega m_s}, \quad T = \frac{2\rho_0 c}{2\rho_0 c + j\omega m_s}$$

- the same results as



$k_2 L \ll 1$

(II) $m_s \to 0$ light panel

$R \to 0$ $\quad$ $\cancel{P}\,T \to 1$

– panel disappears

(III) $m_s \gg 1$ massive panel

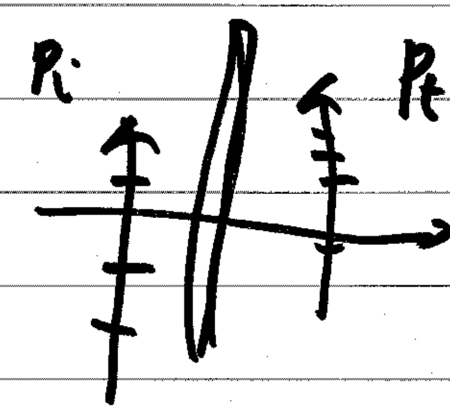
$R \to 1$ $\quad$ $T \to \frac{2\rho_0 c}{\cos\theta} \frac{1}{j\omega m_s}$

$T \propto \frac{1}{\omega}$, $T \propto \frac{1}{m_s}$

– mass law ($\omega m_s \gg \frac{2\rho_0 c}{\cos\theta}$)

(IV) Grazing Incidence Case:

$\theta_i \to \frac{\pi}{2}$, $\cos\theta \to 0$



$\underline{R \to 0 \quad , T \to 1}$