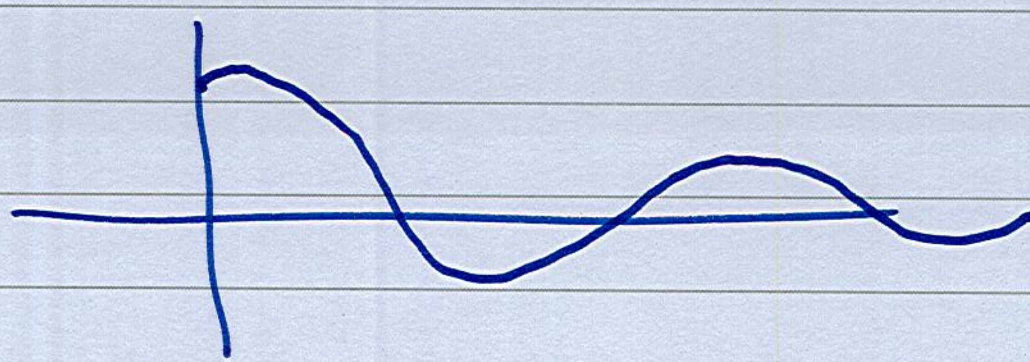
ME 513 Session 25 10/23/15

$$p(x,t) = A\boxed{e^{-\alpha x}}\,\underline{e^{-j\beta x}}\,e^{i\omega t}$$

$\cos\beta x - j\sin\beta x$



$$I = \frac{1}{2}\mathrm{Re}\{pu^*\}$$

$$= \frac{1}{2}\mathrm{Re}\{p\}\mathrm{Re}\{u^*\}$$

- Double panels

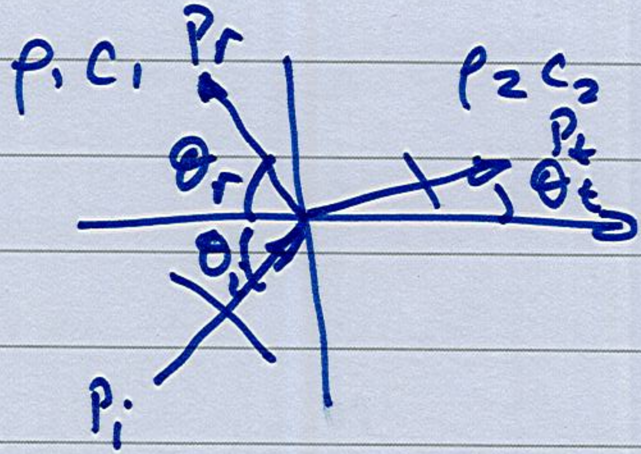
- relation between pressure & intensity reflection coefficients

$$T_I = \frac{1}{\zeta_{21}} |T|^2$$

$$= \frac{4\,\zeta_{21}}{(\zeta_{21} + 1)^2}$$

4.3 Oblique Incidence

4.3.1 Two-Fluid Case



need particle velocity $\perp$ to interface

$$\tilde{u}_{1x} = -\frac{1}{j\omega\rho_1}\frac{\partial \tilde{p}_1}{\partial x}$$

$$\hat{u}_{1x} = \frac{P_i}{\rho_1 c_1} \cos\theta_i \cdot e^{-i(k_{ix}x + k_{iy}y)}$$

$$- \frac{P_r}{\rho_1 c_1} \cos\theta_r \, e^{+i(k_{rx}x - k_{ry}y)}$$

$$\hat{p}_2 = P_t \, e^{-i(k_{tx}x + k_{ty}y)}$$

$$k_{tx} = k_2 \cos\theta_t$$

$$k_{ty} = k_2 \sin\theta_t$$

$$k_{tx}^2 + k_{ty}^2 = k_2^2$$

Normal velocity in region 2

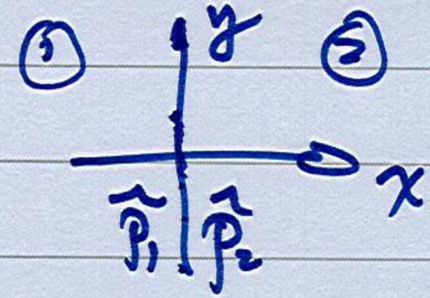
$$\tilde{u}_{2x} = -\frac{1}{j\omega\rho_2}\frac{\partial\tilde{p}_2}{\partial x}$$

$$= \frac{P_t}{\rho_2 c_2}\cos\theta_t \, e^{-i(k_{tx}x + k_{ty}y)}$$

P_i, P_r, P_t

$\div P_i \rightarrow R, T$

Apply B.C.'s



$$\hat{p}_1(0,y) = \hat{p}_2(0,y)$$

$$\underbrace{P_i \cancel{e^{-jk_{iy}y}} + P_r \cancel{e^{-jk_{ry}y}} = P_t \cancel{e^{-jk_{ty}y}}}$$

b.c. must be independent of y on an infinite interface

$$\underbrace{k_{iy} = \overbrace{k_{ry}}\ = k_{ty}}$$

$$\cancel{k_1} \sin\theta_i = \cancel{k_1} \sin\theta_r$$

$$\boxed{\theta_i = \theta_r}$$

angle of reflection = angle of incidence

$$k_{ry} = k_{ty}$$

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1}{k_2} = \left(\frac{c_2}{c_1}\right)$$ Snell's Law

use Snell's Law to calculate

θ_t given c_1, c_2 & θ_i

a

pressure b.c. at $x=0$

$$\tilde{p}_1 = \tilde{p}_2 \quad \text{at} \quad x=0$$

$$P_i + P_r = P_t \qquad \div P_i$$

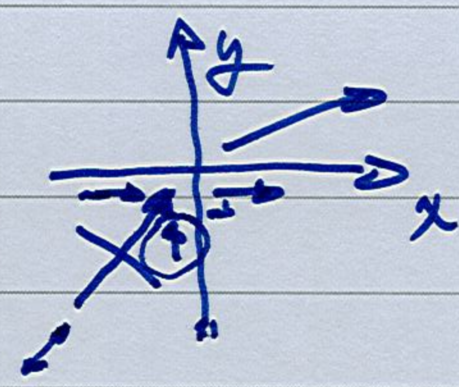
$$\underline{1 + R = T} \qquad (1)$$

$$T = \frac{P_t}{P_i} \qquad R = \frac{P_r}{P_i}$$

particle velocity B.C.

velocity $\perp$ to the interface must be continuous

(no concern about tangential velocity because of neglect of ~~velocity~~ viscosity)



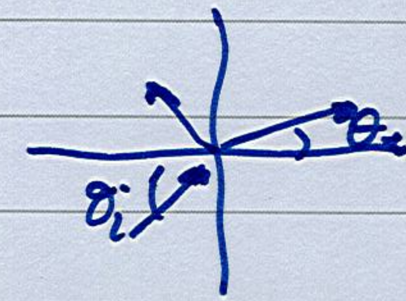
$$\hat{u}_{1x}(0,y) = \hat{u}_{2x}(0,y)$$

$$\frac{P_i}{\rho_1 c_1}\cos\theta_i - \frac{P_r}{\rho_1 c_1}\cos\theta_i = \frac{P_t}{\rho_2 c_2}\cos\theta_t \qquad \div P_i$$

$$1 - R = \frac{1}{\zeta_{21}}\frac{\cos\theta_t}{\cos\theta_i} T \qquad (2)$$

$$\zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$$ solve (1) & (2)

$$R = \frac{\zeta_{21} - \frac{\cos\theta_t}{\cos\theta_i}}{\zeta_{21} + \frac{\cos\theta_t}{\cos\theta_i}}$$



$\theta_i = \theta_t = 0$ normal incidence case

$$T = \frac{2\zeta_{21}}{\zeta_{21} + \frac{\cos\theta_t}{\cos\theta_i}}$$

$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1}$$

$\theta_i \to 0$

$$T \to \frac{2\zeta_{21}}{\zeta_{21} + 1}$$

Write R & T in terms purely of θ_i

we know $\sin^2\theta_t + \cos^2\theta_t = 1$

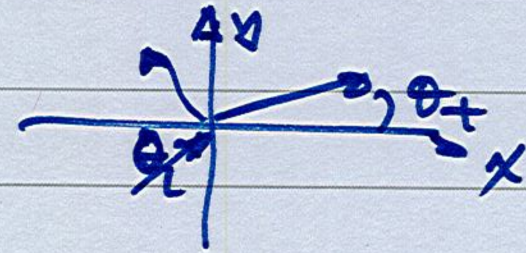
$$\cos\theta_t = \pm\sqrt{1 - \sin^2\theta_t}$$

from Snell's Law $\sin\theta_t = \frac{c_2}{c_1}\sin\theta$

$$\cos\theta_t = \pm\sqrt{1 - \left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}$$

sub into R & T

Notes:



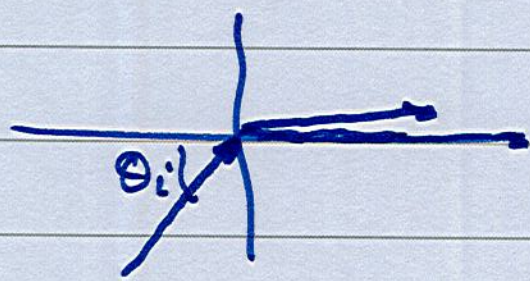
1. From Snell's Law

when $\frac{c_2}{c_1} < 1$ $\qquad$ $\sin\theta_t < \sin\theta_i$

$\theta_t < \theta_i$

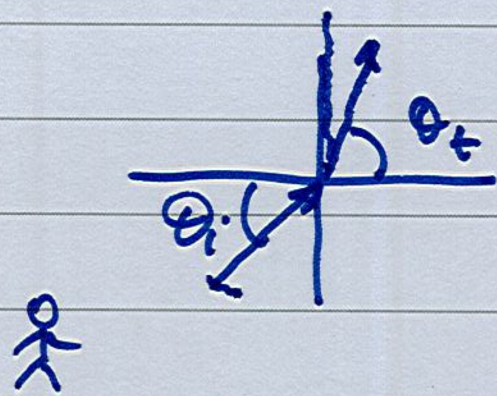
"sound refracts towards the normal"

when $c_2 < c_1$



2. if $c_2 > c_1$

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i$$



$\theta_t > \theta_i$ sound refracts away from the normal.

maximum possible <u>real</u> value of θ_t is $\frac{\pi}{2}$

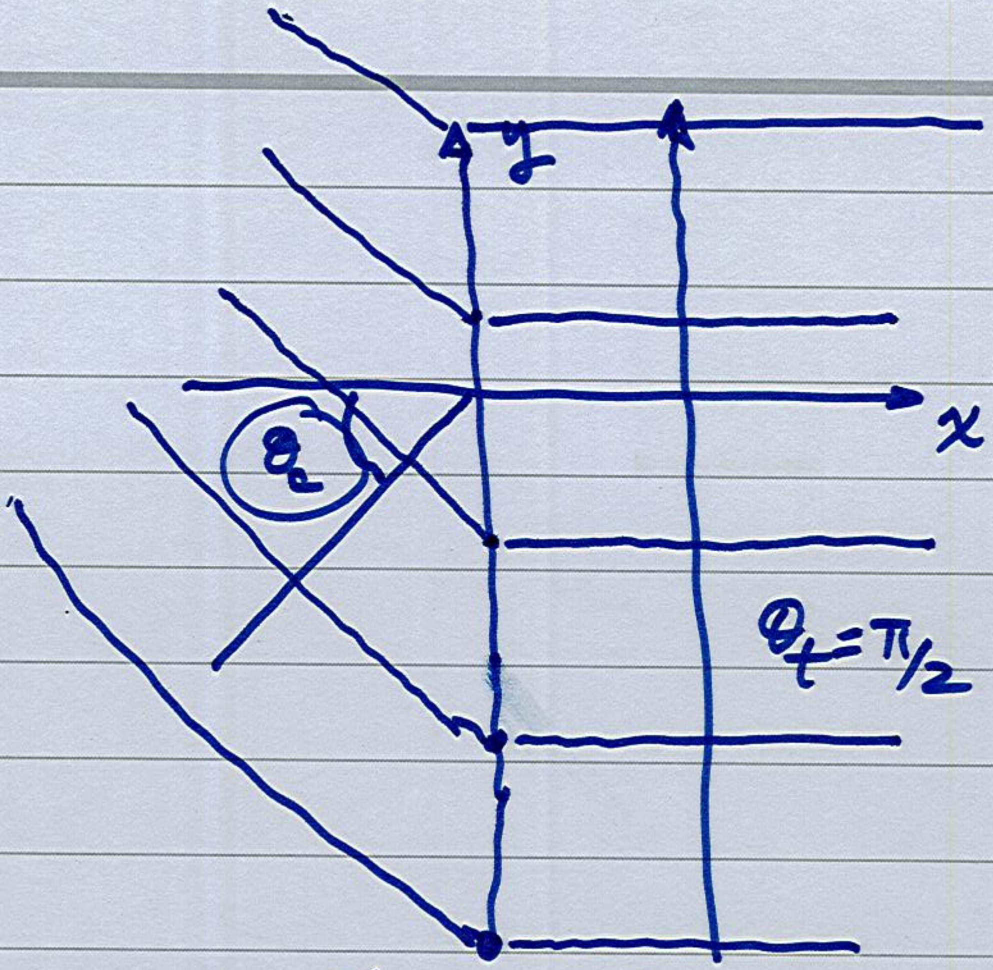
critical incidence angle θ_c

that is the θ_i that causes

$$\theta_t \rightarrow \underline{\underline{\pi/2}}$$

$$\sin\theta_t = 1 = \frac{c_2}{c_1}\sin\theta_c$$

$$\left.\sin\theta_c = \frac{c_1}{c_2}\right]$$ Definition of θ_c



$c_2 > c_1$

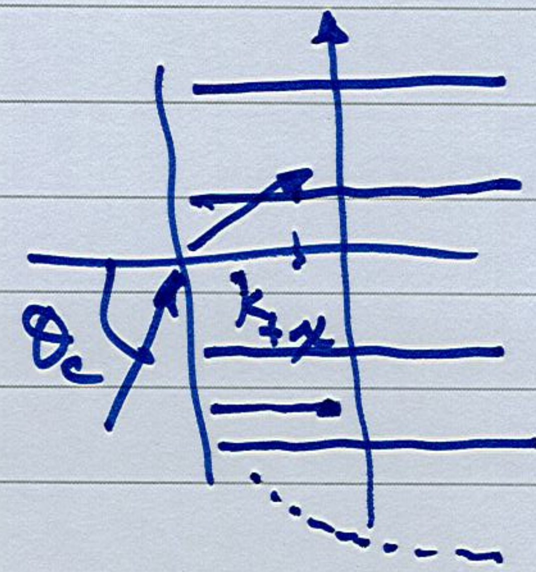
wave system when $\theta_i = \theta_c$

at θ_c - ~~⇒~~ sound field in region ② propagates $\parallel$ to interface

Total Internal Reflection

$|R| \to 1$ $\quad$ $|T| \to 2$

Transmitted field



$$\hat{p}_t = P_t e^{-i(k_{tx}x + k_{ty}y)}$$

$$k_{ty} = k_{iy} = k_1 \sin\theta_i$$

$$k_{tx} = k_2 \underline{\cos}\,\theta_t$$

$$= \pm k_2 \sqrt{\underbrace{1 - \left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}_{=1 \text{ when } \theta_i = \theta_c}}$$

when $\theta_i = \theta_c$ $k_{tx} = \pm k_2 \sqrt{1-1} = 0$