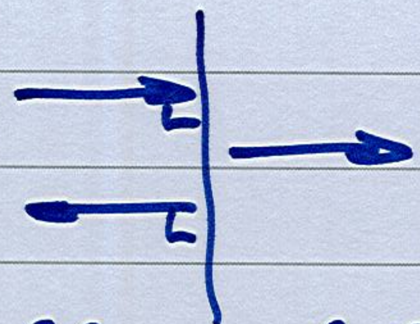
Nov. 6 Midterm

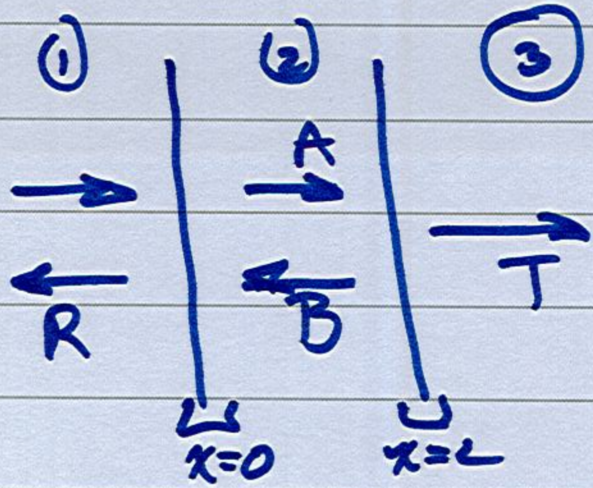


$\rho_1 c_1$ $\rho_2 c_2$

$T = \dfrac{2\zeta_{21}}{\zeta_{21}+1}$

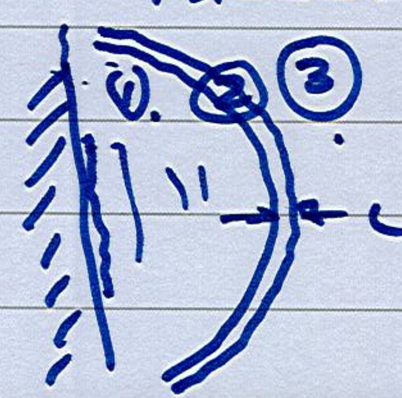
$\zeta_{21} = \dfrac{\rho_2 c_2}{\rho_1 c_1}$

$R = \dfrac{\zeta_{21}-1}{\zeta_{21}+1}$



$\zeta_{21} \gg 1$

$\zeta_{21} \ll 1$



$$R = \frac{\left(1 - \frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos k_2 L + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} - \frac{\rho_1 c_1}{\rho_2 c_2}\right)\sin k_2 L}{\left(1 + \frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos k_2 L + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} + \frac{\rho_1 c_1}{\rho_2 c_2}\right)\sin k_2 L}$$

Sonar Dome

$R = 0 \quad T = 1$ perfect transmission

choose $k_2 L = n\pi$

$$L = n\left(\frac{\lambda_2}{2}\right)$$

$\sin k_2 L = 0$

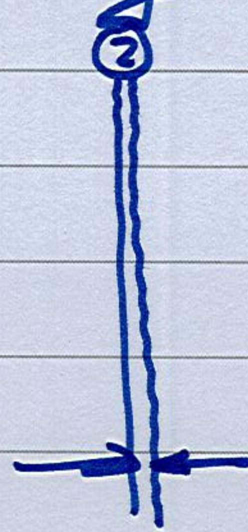
$\cos k_2 L = 1 \qquad \rho_1 c_1 = \rho_3 c_3$

$R \to 0 \qquad |T| \to 1$

Thin, heavy barrier case

① ② ③

$\rho_1 c_1$ $\rho_3 c_3 = \rho_1 c_1$



very thin compared to a wavelength, limp, heavy

$k_2 L \ll 1$

$\rho_2 c_2 \gg \rho_1 c_1$

$\sin k_2 L \to k_2 L$

$\cos k_2 L \to 1$

$$R \to \frac{\left(1 - \cancelto{0}{\frac{\rho_1 c_1}{\rho_3 c_3}}\right) 1 + i\left(\frac{\rho_2 c_2}{\rho_3 c_3} - \cancelto{0}{\frac{\rho_1 c_1}{\rho_2 c_2}}\right) k_2 L}{\left(1 + \cancelto{1}{\frac{\rho_1 c_1}{\rho_3 c_3}}\right) 1 + i\left(\frac{\rho_2 c_2}{\rho_3 c_3} + \cancelto{0}{\frac{\rho_1 c_1}{\rho_2 c_2}}\right) k_2 L}$$

$$R = \frac{i \frac{\rho_2 \cancel{c_2}}{\rho_3 c_3} \overbrace{\frac{\omega}{\cancel{c_2}} L}^{k_2 L}}{2 + i \frac{\rho_2 \cancel{c_2}}{\rho_3 c_3} \frac{\omega}{\cancel{c_2}} L}$$

① ② ρ_2 ③

$\rho_2 L = m_s$ mass/unit area

$\rho_1 c_1 = \rho_3 c_3 = \rho_0 c$

ambient characteristic impedance

Thin heavy barrier

$$\boxed{R = \frac{j\omega m_s}{2\rho c + j\omega m_s}}$$

important physical parameter of the wall is m_s

$$\left.\begin{array}{l} \omega \to \infty \\ m_s \to \infty \end{array}\right\} R \to 1$$

$\omega \to 0 \quad R \to 0 \quad (T \to 1)$

Normal Incidence Transmission Coefficient

$$T = \frac{2e^{jk_3L}}{\left(1+\frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos k_2L + j\left(\frac{\rho_2 c_2}{\rho_3 c_3}+\frac{\rho_1 c_1}{\rho_2 c_2}\right)\sin k_2L}$$

(i) Sonar Case

$k_2 L = n\pi$

$\sin k_2 L \to 0$

$\cos k_2 L \to 1$

$\rho_1 c_1 = \rho_3 c_3$

$$T \Rightarrow \frac{2e^{ik_3L}}{\pm 2}$$

$|T| = 1$ - perfect transmission except for a phase change

(ii) Thin, heavy barrier

$$\rho_1 c_1 = \rho_3 c_3 \qquad \frac{\rho_2 c_2}{\rho_1 c_1} \gg 1$$

$$k_2 L \ll 1$$

$$T \to \frac{2e^{jk_3L}}{2 + j\left(\frac{\rho_2 c_2}{\rho_3 c_3}\right)k_2 L}$$

$k_1 = k_3 = k$

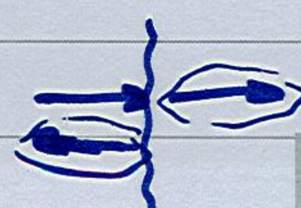
$\rho_1 c_1 = \rho_3 c_3 = \rho_0 c$

$\rho_2 L = m_s$

$$\boxed{T = \frac{2\rho_0 c\, e^{jkL}}{\underline{2\rho_0 c} + \underline{j\omega m_s}}}$$

$k = \frac{\omega}{c}$ air

Thin, limp, barrier



$\omega \to 0 \quad |T| \to 1$

"high frequency"

$\omega m_s \gg \rho_0 c$

In the high freq region

$$\boxed{|T| = \frac{2\rho_0 c}{\omega m_s}}$$

<u>mass law</u>

$$|T| \propto \frac{1}{m_s}$$

$$|T| \propto \frac{1}{\omega}$$

"limp" - flexural stiffness
$\equiv$ is assumed negligible

sheet of AL 0.05"

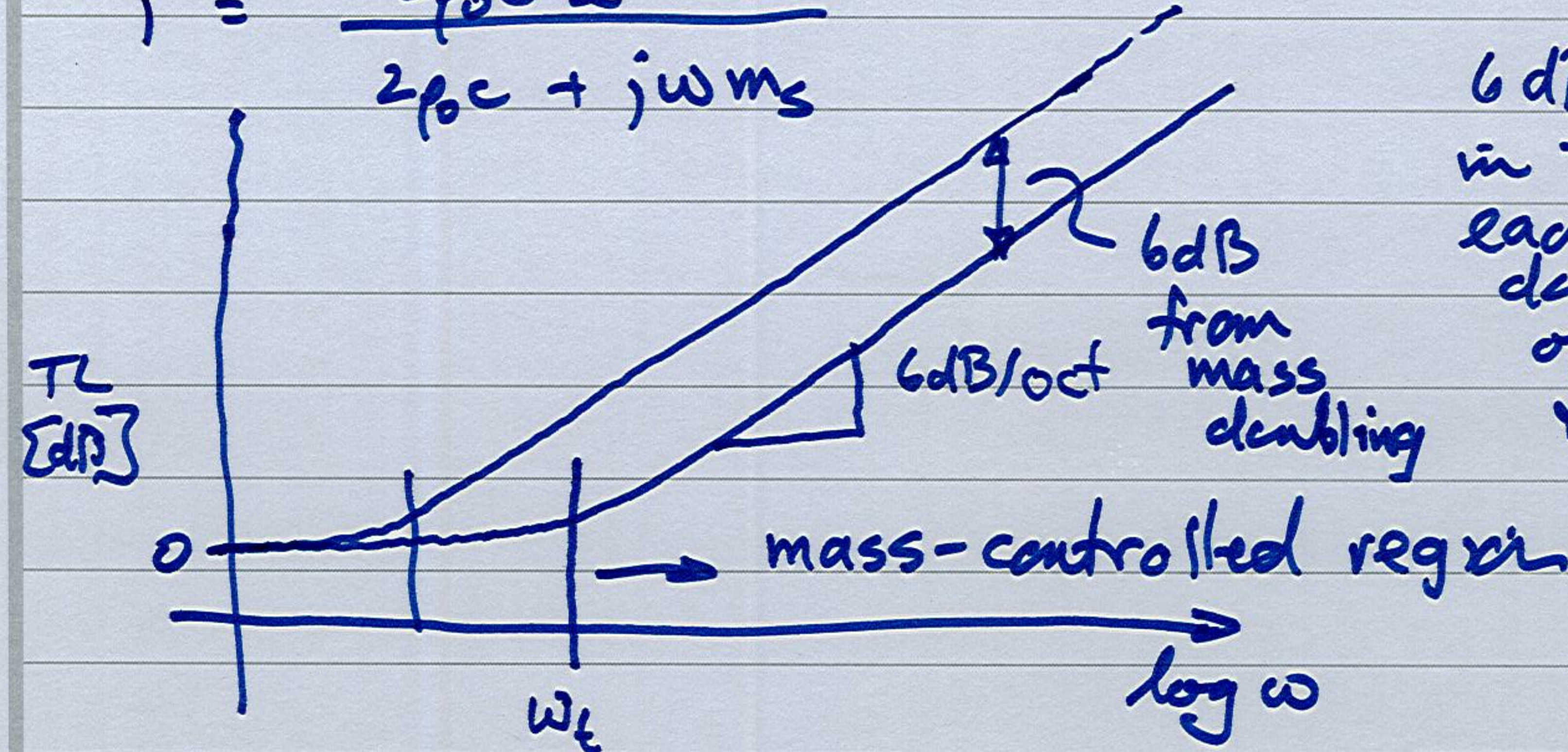
Transmission Loss

$$TL = 10 \log_{10} \frac{1}{|T|^2} \quad dB$$

$|T| \downarrow \quad TL \uparrow$

true only when the same medium is on both sides of the wall

$$T = \frac{2\rho_0 c\, e^{jkL}}{2\rho_0 c + j\omega m_s}$$



6 dB increase in TL for each doubling of freq + mass

$$2\rho_0 c \simeq \omega_t m_s$$

$$\omega_t \simeq \frac{2\rho_0 c}{m_s}$$