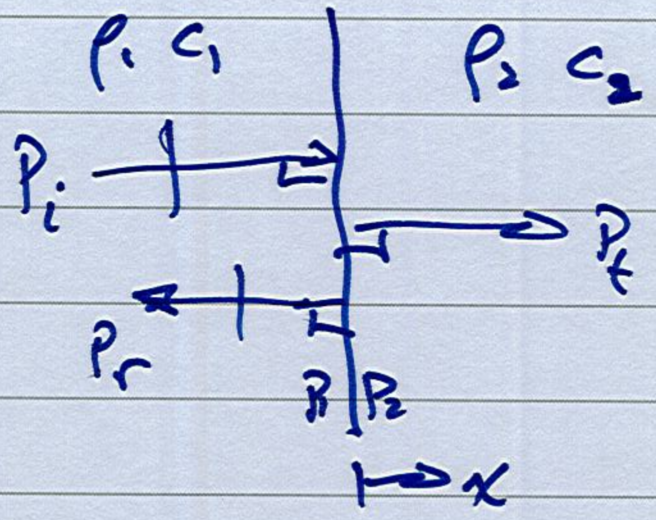
ME 513 Session 22 10/16/15

4.2 Normal Incidence



$\hat{\bar{p}}_1(0) = \hat{\bar{p}}_2(0)$ pressure continuity

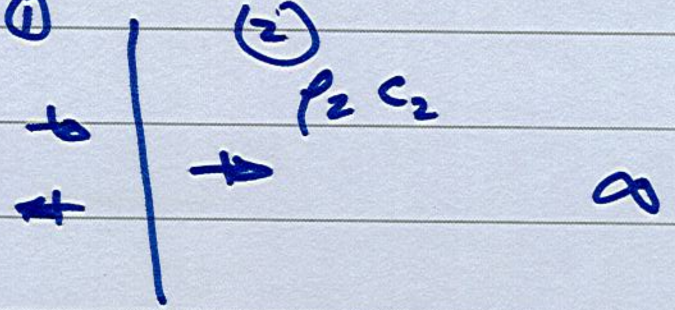
$\hat{u}_{1n}(0) = \hat{u}_{2n}(0)$ velocity continuity

$$\hat{\bar{z}}_1\Big|_{x=0} = \hat{\bar{z}}_2\cancel{(x)}\Big|_{x=0}$$

Application of Boundary Conditions

$\rho_1 c_1$

$$\hat{p}_1 = P_i e^{-jk_1 x} + P_r e^{+jk_2 x}$$



$$\hat{p}_2 = P_t e^{-jk_2 x} \qquad k_1 = \frac{\omega}{c_1} \qquad k_2 = \frac{\omega}{c_2}$$

Pressure b.c. at $x = 0$

$$\hat{p}_1(0) = \hat{p}_2(0)$$

$$\boxed{P_i + P_r = P_t} \quad (1)$$

$$\hat{u}_1 = \frac{P_i}{\rho_1 c_1} e^{-jk_1 x} - \frac{P_r}{\rho_1 c_1} e^{+jk_1 x}$$

$$\hat{u}_2 = \frac{P_t}{\rho_2 c_2} e^{-jk_2 x}$$

$$\hat{u}_1(0) = \hat{u}_2(0)$$

$$\boxed{\frac{p_i}{\rho_1 c_1} - \frac{p_r}{\rho_1 c_1} = \frac{p_t}{\rho_2 c_2}} \quad (2)$$

$$R = \frac{p_r}{p_i} \qquad T = \frac{p_t}{p_i}$$

plane wave <u>pressure</u>
reflection &
transmission
coefficients

Divide (1) & (2) by P_i

(1) $1 + R = T$

(2) $1 - R = \frac{\rho_1 c_1}{\rho_2 c_2} T = \frac{1}{\zeta_{21}} T \qquad \zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1} \Big]$

$$T = \frac{2\zeta_{21}}{\zeta_{21} + 1}$$

$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1} \Big]$$

(ii) If There is no dissipation in regions ① & ②

$$\frac{\rho_2 c_2}{\rho_1 c_1} = \zeta_{21} = \text{real}$$

$$T = \frac{2\zeta_{21}}{\zeta_{21} + 1}$$

R + T are purely real

$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1}$$

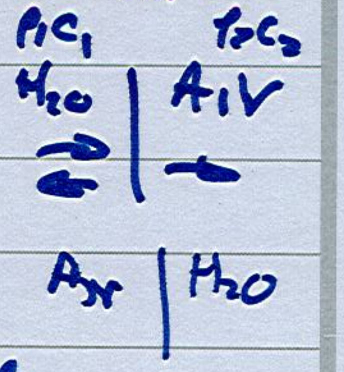
$\equiv$

R can be +ve or -ve depending on whether

$\zeta_{21} = \xi \rho_2 c_2 / \rho_1 c_1$

$\zeta_{21} > 1$ $\quad$ $\zeta_{21} < 1$

$\Leftarrow$



$\zeta_{21} > 1$ - <u>in phase reflection</u>

$\zeta_{21} < 1$ - <u>out of phase reflection</u>

(ii) ζ_{21} Air | Water

ρ_1 | $\rho_2 c_2 \sim 1.5 \times 10^6$ MKS Rayls

$(\rho_1 c_1)_{air} \simeq 415$ MKS Rayls.

$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1} \qquad \zeta_{21} \rightarrow \infty$$

$$\simeq 1$$

$$\tilde{P}_1(0) = P_i + P_r = P_i(1 + R)$$

$$= 2P_i$$

when $\zeta_{21} \gg 1$ pressure is doubled at the surface.

(iii) $\zeta_{21} >> 1$

air | water

$\tilde{R}$ | $\tilde{P}_2$ = twice The incident pressure

$$T = \frac{2\zeta_{21}}{\zeta_{21} + 1} \simeq 2$$

$R = 1 \qquad T = 2$

$$\underline{R + T \neq 1}$$

$$I_2 = \frac{(p^2_{rms})_2}{\rho_2 c_2}$$

$$\simeq 0$$

$$I_{1_i} = \frac{(p^2_{rms})_i}{\rho_1 c_1}$$

Essentially there is no energy transmission at a hard surface.

(iv) $R = \dfrac{\xi_{21} - 1}{\xi_{21} + 1}$

$R \rightarrow 0$ when $\xi_{21} \rightarrow 1$ $T \rightarrow 1$

~~Z_1~~ $\left.\begin{array}{l} \rho_1 = \rho_2 \\ c_1 = c_2 \end{array}\right]$

$\rho_1 c_1 = \rho_2 c_2$

$\left(\frac{\rho_1}{\rho_2}\right) = \left(\frac{c_2}{c_1}\right)$ if this condition is true

zero reflection + perfect transmission

(v) $\zeta_{21} << 1$ water ① | air ②

$\rho_1 c_1$ | $\rho_2 c_2$

$\hat{P}_1$ | $\hat{P}_2$

$$\zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$$

$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1} \simeq -1$$

out-of-phase reflection

$$\tilde{P}_1(0) = P_i + P_r = P_i(1 + \overset{-1}{\cancel{R}})$$

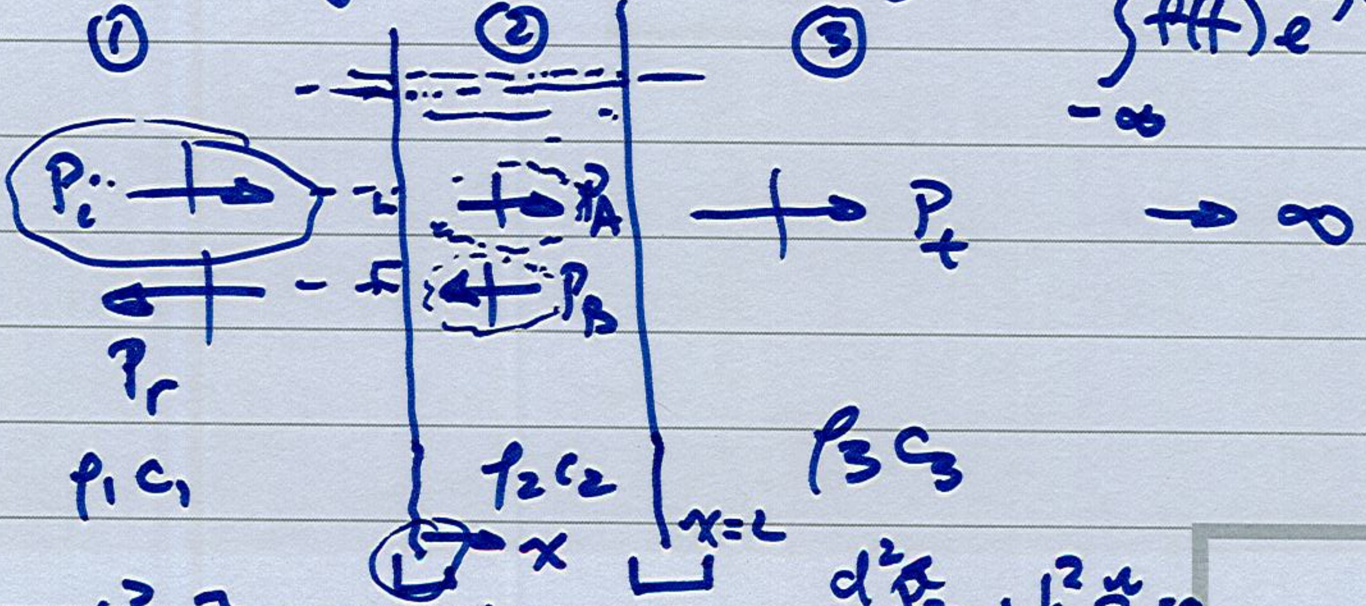
$$\simeq 0$$

pressure release surface

$$T = \frac{2\zeta_{21}}{\zeta_{21} + 1} \simeq 0$$

4.2.2 Normal Incidence sound transmission through a fluid layer

$$\int_{-\infty}^{\infty} f(t)\, e^{-j2\pi ft}\, dt$$



$$\frac{d^2\tilde{p}_1}{dx^2} + k_1^2\,\tilde{p}_1 = 0 \qquad \frac{d^2\tilde{p}_2}{dx^2} + k_2^2\,\tilde{p}_2 = 0 \qquad \frac{d^2\tilde{p}_3}{dx^2} + k_3^2\,\tilde{p}_3 = 0$$

$$k_1 = \omega/c_1 \qquad k_2 = \omega/c_2 \qquad k_3 = \omega/c_3$$

$$\tilde{p}_1(x) = P_i e^{-jk_1 x} + P_r e^{+jk_1 x}$$

$$\tilde{p}_2(x) = P_A e^{-jk_2 x} + P_B e^{+jk_2 x}$$

$$\tilde{p}_3(x) = P_t e^{-jk_3 x}$$

B.C.'s

at $x = 0$ $\quad \tilde{p}_1(0) = \tilde{p}_2(0) \quad (1)$

$\tilde{u}_{1x}(0) = \tilde{u}_{2x}(0) \quad (2)$

at $x = L$ $\quad \tilde{p}_2(L) = \tilde{p}_3(L) \quad (3)$

$\tilde{u}_{2x}(L) = \tilde{u}_{3x}(L) \quad (4)$

(1) $P_i + P_r = P_A + P_B \quad \div P_i$

$$\boxed{1 + R = A + B}$$

$\underset{\checkmark}{R} = \frac{P_r}{P_i}$ $\quad \overset{\checkmark}{A} = \frac{P_A}{P_i}$ $\quad \overset{\checkmark}{B} = \frac{P_B}{P_i}$

(2) $1 - R = \frac{1}{\zeta_{21}}(A - B)$ $\quad \zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$

(3) $Ae^{-jk_2L} + Be^{+jk_2L} = Te^{-jk_3L}$ $\quad \overset{\checkmark}{T} = P_t/P_i$

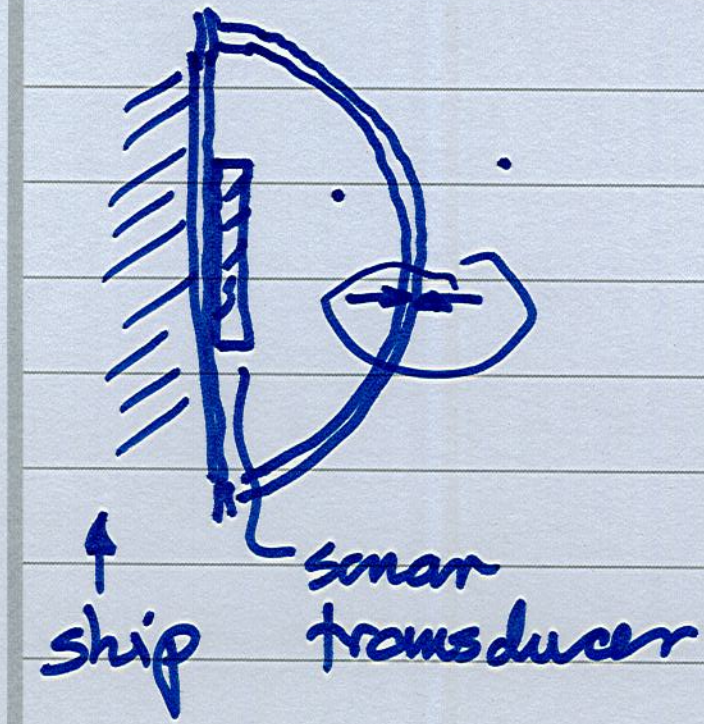
(4) $Ae^{-jk_2L} - Be^{+jk_2L} = \frac{T}{\zeta_{32}} e^{-jk_3L}$

$\zeta_{32} = \frac{\rho_3 c_3}{\rho_2 c_2}$

$$R = \frac{\left(1 - \frac{\rho_1 c_1}{\rho_3 c_3}\right) \cos k_2 L + j \left(\frac{\rho_2 c_2}{\rho_3 c_3} - \frac{\rho_1 c_1}{\rho_2 c_2}\right) \sin k_2 L}{\left(1 + \frac{\rho_1 c_1}{\rho_3 c_3}\right) \cos \underbrace{k_2 L} + j \left(\frac{\rho_2 c_2}{\rho_3 c_3} + \frac{\rho_1 c_1}{\rho_2 c_2}\right) \sin \underbrace{k_2 L}}$$

$$k_2 L = \frac{2\pi}{\lambda_2} L = 2\pi \left(\frac{L}{\lambda}\right)$$

non-dimensional layer depth


sonar transducer
ship