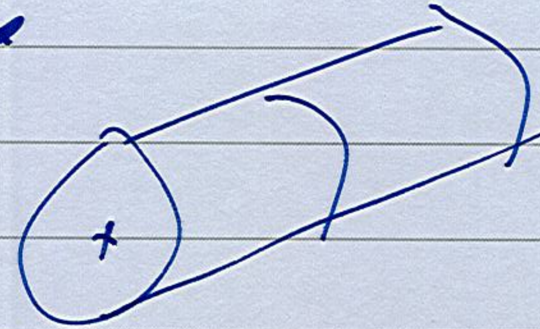
Homework 3

Cylindrical Waves

$|p| \propto \frac{1}{r^{1/2}}$

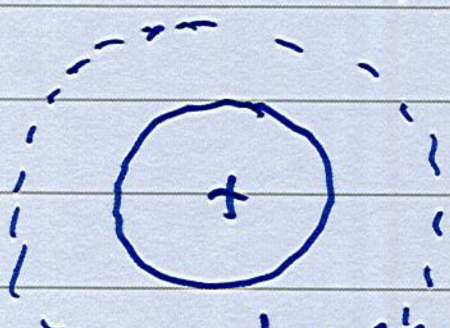


Specific Acoustic Impedance

$\tilde{z} = \frac{\tilde{p}}{\tilde{u}}$

freely prop plane

$\tilde{z} = \rho_0 c$



$\tilde{z} = \underline{j\omega\rho_0 r}$

$kr \ll 1$

3.5 Acoustic Intensity

In Mechanics

Force × distance = Work (Joules)

Force × velocity = Power (Watts)

In Acoustics

$$\underbrace{\text{pressure} \times \text{velocity}}_{\text{Intensity}} = \frac{\text{Power}}{\text{unit area}} \; (\text{Watts}/m^2)$$

Instantaneous Intensity

$$= p(t)\,\bar{u}(t) = \bar{I}_t(t)$$

Intensity is a vector quantity

Intensity – time-averaged acoustic intensity
– time-averaged rate of energy flow through a unit area

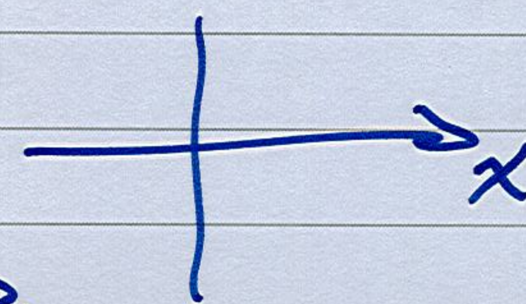
For a periodic signal

$$\bar{I} = \frac{1}{T}\int_0^T \underbrace{p(t)\,\bar{u}(t)}_{\text{real}}\,dt$$

Intensity = real

T = period

+ve going plane wave



$$\frac{p_+}{u_+} = \rho_0 c \qquad u_+ = \frac{p_+}{\rho_0 c}$$

$$I_x = \frac{1}{T}\int_0^T \frac{p_+^2}{\rho_0 c}\, dt$$

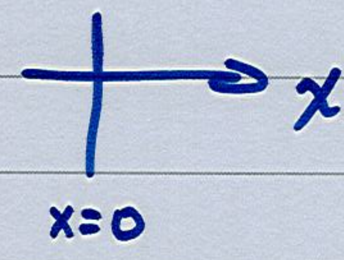
$$= \frac{1}{\rho_0 c} \underbrace{\left[\frac{1}{T}\int_0^T p_+^2(t)\, dt\right]}_{\text{mean square pressure}}$$

$$I_x = \frac{p_{rms}^2}{\rho_0 c}$$ +ve going plane wave

Consider the harmonic case

$$\tilde{p}_+ = A e^{j(\omega t - kx)}$$

$$\tilde{u}_+ = \frac{A}{\rho_0 c} e^{j(\omega t - kx)}$$



$x = 0$

$$\tilde{P}_+ = (A_r + jA_i)(\cos\omega t + j\sin\omega t)$$

$$\tilde{u}_+ = \frac{(A_r + jA_i)}{\rho_0 c}(\cos\omega t + j\sin\omega t)$$

$$\mathrm{Re}\{\tilde{P}_+\} = A_r \cos\omega t - A_i \sin\omega t$$

$$\mathrm{Re}\{\tilde{u}_+\} = \frac{A_r \cos\omega t - A_i \sin\omega t}{\rho_0 c}$$

$$I = \frac{1}{T}\int_0^T \mathrm{Re}\{\tilde{p}_+\}\,\mathrm{Re}\{\tilde{u}_+\}\,dt$$

$$= \frac{1}{\rho_0 c}\frac{1}{\cancel{T}}\left[\cancel{T}\frac{A_r^2}{2} + \cancel{T}\frac{A_i^2}{2}\right] \qquad \tilde{p}_+ = A e^{j\omega t}$$

$x = 0$

$$= \frac{1}{2\rho_0 c}\underbrace{\left[A_r^2 + A_i^2\right]}_{|A|^2 = |\tilde{p}_+|^2}$$

$$I = \frac{|\tilde{p}_+|^2}{2\rho_0 c}$$

mean square pressure

$$I = \frac{p_{rms}^2}{\rho_0 c}$$ for a freely prop plane wave – in the direction of wave prop

Complex Harmonic Signal

$$\boxed{I = \frac{1}{2} Re\left\{\tilde{p}_+ \hat{u}_+^*\right\}}$$

$$\left(e^{j\omega t}\right)^* \quad e^{-j\omega t}$$

$$= \frac{1}{2} Re\left\{A \cancel{e^{j\omega t}} \frac{A^*}{\rho_0 c} \cancel{e^{-j\omega t}}\right\}$$

$$= \frac{1}{2} Re\left\{\frac{AA^*}{\rho_0 c}\right\} = \frac{1}{2} Re\left\{\frac{|A|^2}{\rho_0 c}\right\}$$

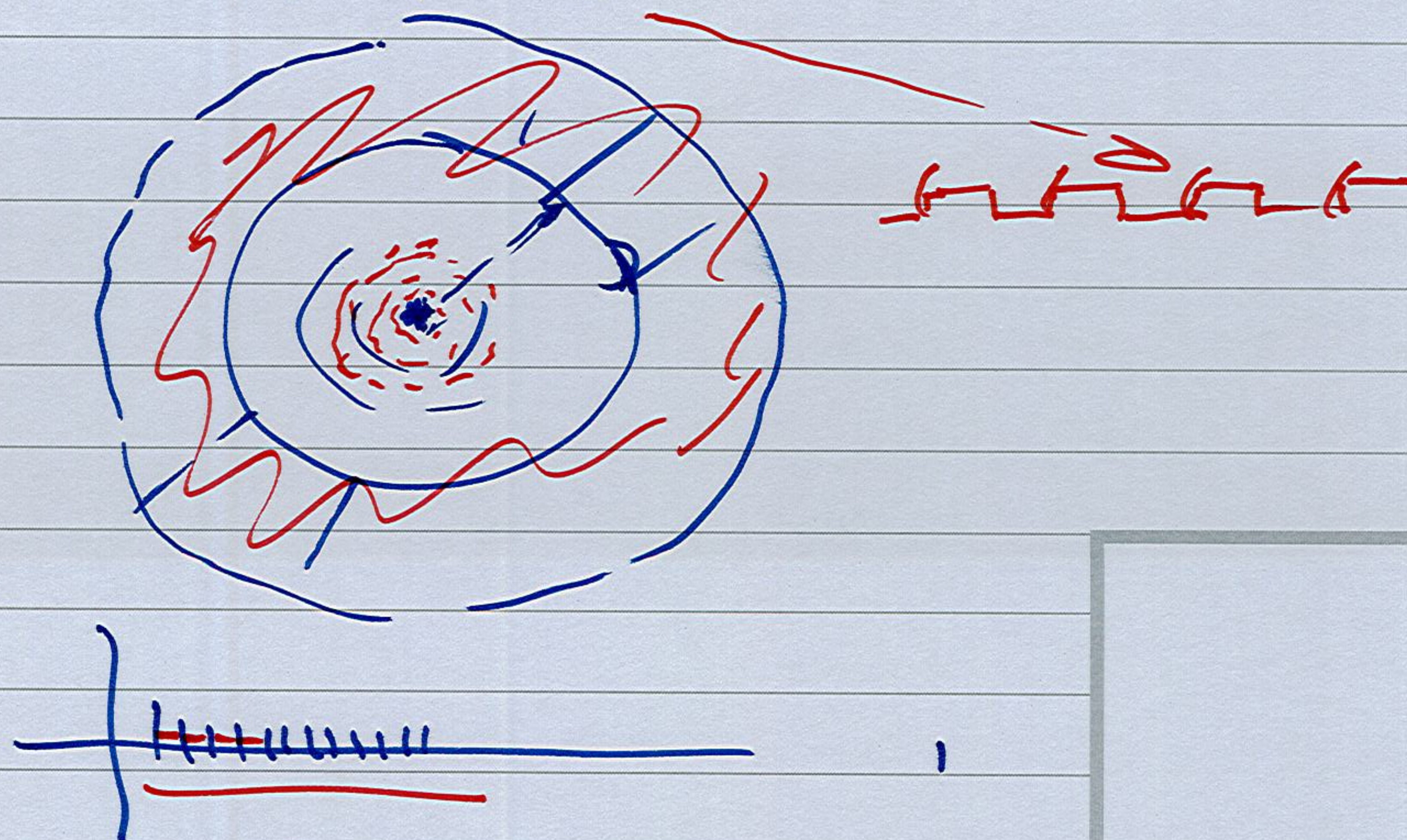
$$= \frac{1}{2}\frac{|A|^2}{\rho_0 c} \qquad |A| = |\tilde{p}_+|$$

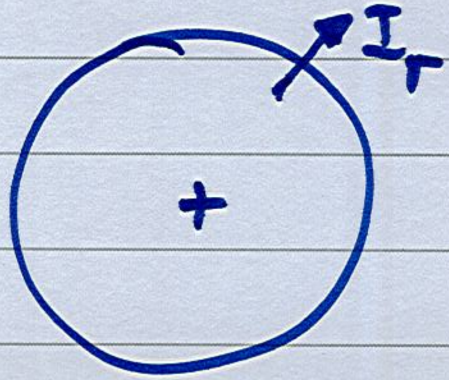
$$= \frac{1}{2}\frac{|\tilde{p}_+|^2}{\rho_0 c} \quad p_{rms}^2$$

$$I = \frac{1}{2}\mathrm{Re}\{\tilde{p}\,\tilde{u}^*\}$$

general expression for calculating acoustic intensity when using complex harmonic

Clapping Circle





spherically symmetric waves
- radiating into free-space

$$I_r = \frac{1}{2} \mathrm{Re}\{\tilde{p}_+ \tilde{u}_{r_+}^*\}$$

$$\tilde{u}_{r_+} = \frac{1}{\rho_0 c}\left(1 + \frac{1}{jkr}\right)\tilde{p}_+$$

$$\tilde{u}_{r_+}^* = \frac{1}{\rho_0 c}\left(1 - \frac{1}{jkr}\right)\tilde{p}_+^*$$

$$\tilde{p}_+ \tilde{u}_{r_+}^* = \frac{\tilde{p}_+ \tilde{p}_+^*}{\rho_0 c}\left(1 - \frac{1}{jkr}\right)$$

$$I_r = \frac{1}{2} \mathrm{Re}\{\tilde{p}_+ \tilde{u}_{r_+}^*\} = \frac{|\tilde{p}_+|^2}{2\rho_0 c} \quad p_{rms}^2$$

$$\tilde{p}_+ = \frac{A}{r} e^{-ikr}$$

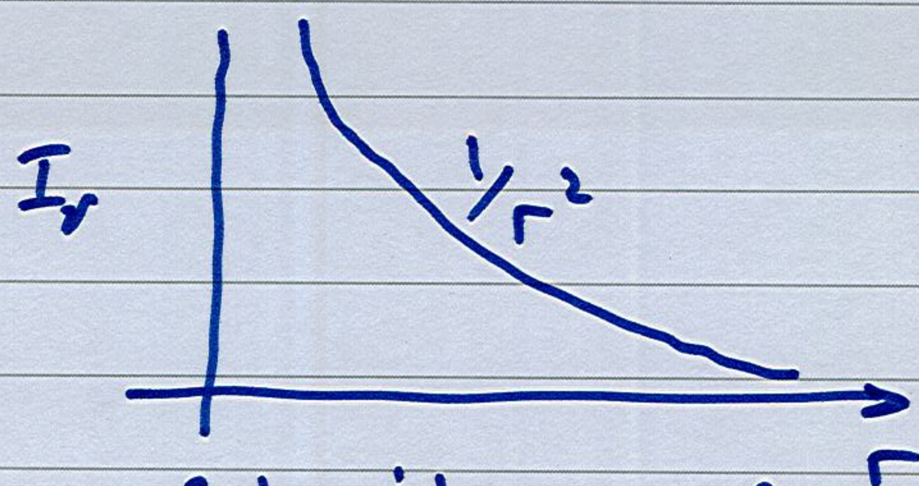
$$|\tilde{p}_+| \propto \frac{1}{r}$$

$$I_r = \frac{|\hat{p}_+|^2}{2\rho_0 c}$$

$$I_r \propto \frac{1}{r^2}$$

nearfield farfield

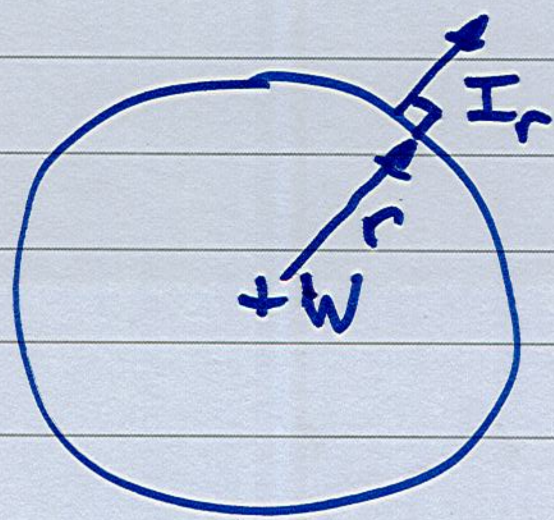
$kr \simeq 1$



* no intensity nearfield

Intensity $= \dfrac{\text{Sound Power}}{\text{unit area}}$

Sound Power - by integrating the normal intensity over a surface enclosing the source



$$W = \int_S \underset{\sim}{I_r} \, dS$$

spherical
sym
freely propagating

$$W = I_r \underbrace{\int dS}_{S = 4\pi r^2}$$

W = independant of measurement position

$$W = I_r (4\pi r^2)$$

$$I_r = \frac{W}{4\pi r^2}$$

Inverse ~~Sq~~ Square Law