ME 513 Session 18 10/5/15 1

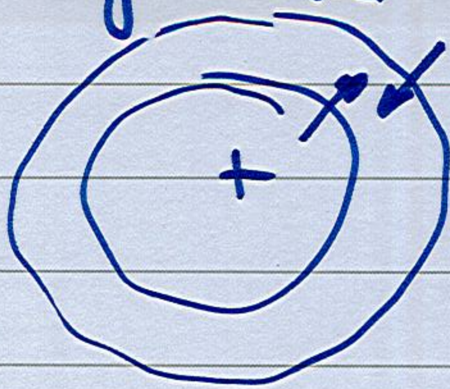
One Dimensional Solutions

Plane Wave $\tilde{p}(x,y,z,t) = A e^{j(\omega t \pm k_x x \pm k_y y \pm k_z z)}$

where $\underline{k^2 = k_x^2 + k_y^2 + k_z^2}$

$k = \frac{\omega}{c}$

Spherical



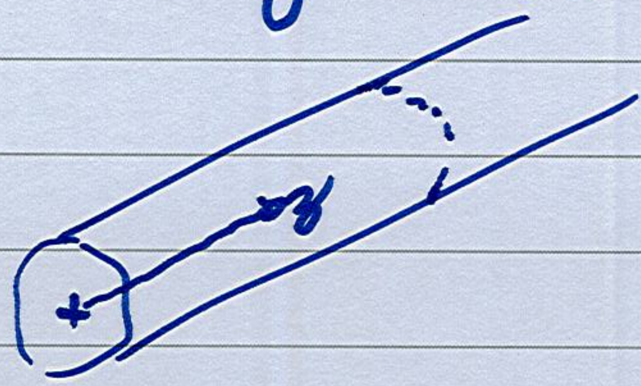
$\tilde{p}(r) = \frac{A}{r} e^{-jkr} + \frac{B}{r} e^{+jkr}$

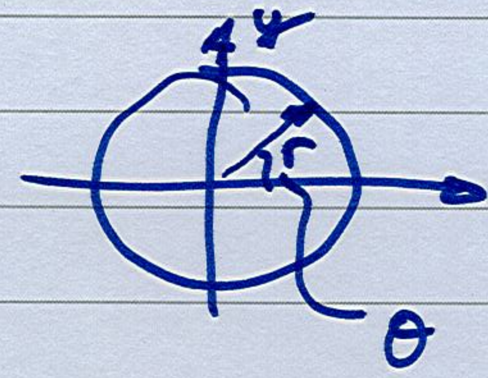
$|\tilde{p}| \propto \frac{1}{r}$

nearfield

$\tilde{u}_{r+} = \frac{1}{\rho_0 c}\left(1 + \frac{1}{jkr}\right)\tilde{p}_+$

3.3.3 Cylindrical Waves





$\tilde{p}(r, \theta, z)$

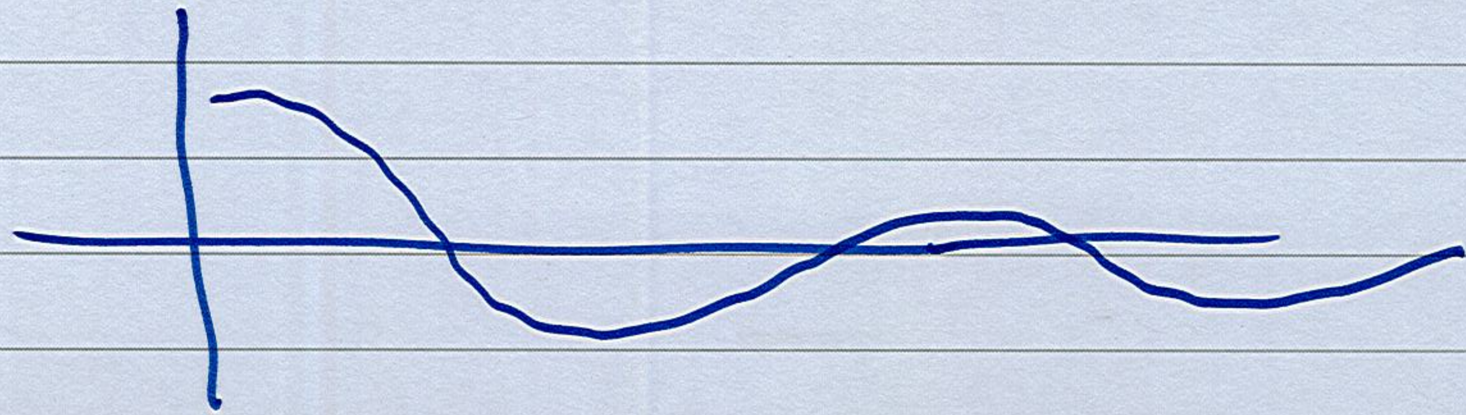
no variation with z or with θ cylindrical symmetry

$$\tilde{p}(r,\theta,z) = \underset{z}{R(r)}\,\cancel{\Theta(\theta)}\,\cancel{Z(z)}\,e^{j\omega t}$$

Outward Propagating harmonic
cylindrically symmetric sound Field

$$\tilde{p}(r,t) = A \underbrace{H_0^{(2)}(kr)}_{\text{Hankel}} e^{i\omega t}$$

$$H_0^{(2)}(kr) = \underbrace{J_0(kr) - j\,Y_0(kr)}_{\text{Bessel Functions}}$$



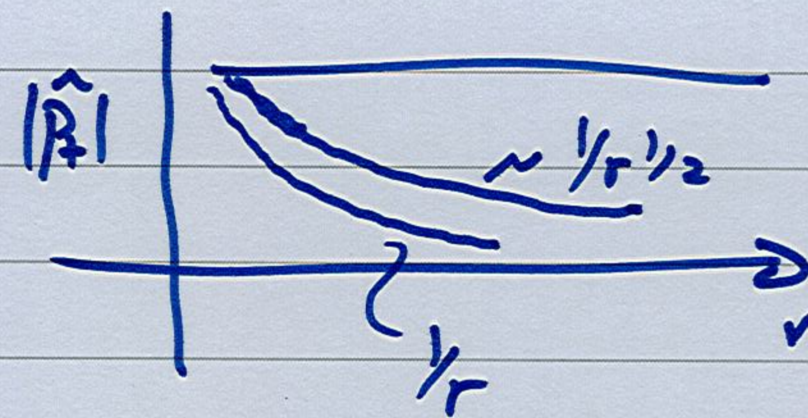
when ~~ks~~ $kr >> 1$ farfield

$$\tilde{p}(r) = A\left(\frac{2}{\pi k r}\right)^{1/2} e^{j(\omega t - kr + \frac{\pi}{4})}$$

$|\tilde{p}(r)| \propto \frac{1}{r^{1/2}}$ for a cylindrical wave in the farfield plane waves

$|\hat{p}_+|$ independent of position

spherical wave



$|\hat{p}_+| \propto \frac{1}{r}$

Cylindrical Wave

$|\hat{p}_+| \propto \frac{1}{r^{1/2}}$

3.4 <u>specific</u> Acoustic Impedance

$\underbrace{\text{specific}}$ per unit area

$$\tilde{z} = \frac{\text{acoustic pressure}}{\text{acoustic particle velocity}}$$

$$= \frac{\tilde{p}}{\tilde{u}} \quad \text{harmonic case}$$

Plane wave +ve x-direction

$$\tilde{p}_+ = A e^{-ikx}$$

$$\tilde{u}_+ = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}_+}{dx} = \frac{A}{\rho_0 c}e^{-jkx} = \frac{\tilde{p}_+}{\rho_0 c}$$

$$\tilde{z}_+ = \frac{\tilde{p}_+}{\tilde{u}_+} = \rho_0 c$$ Characteristic Impedance

415 Rayls

For a negative-going plane wave

$$\tilde{z}_- = \frac{\tilde{p}_-}{\tilde{u}_-} = -\rho_0 c$$

specific acoustic impedance is a function of direction more generally - function of position

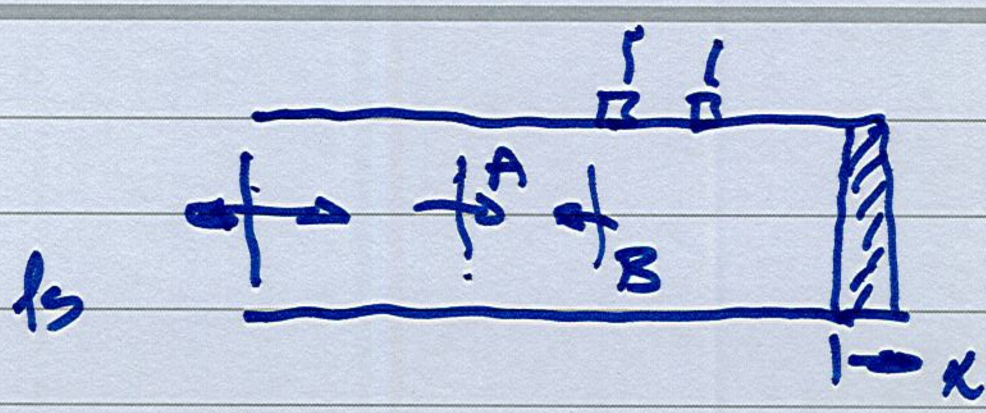
In general

$$\tilde{p}(x) = A e^{-jkx} + B e^{+jkx}$$

$$\tilde{u}(x) = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dx} = \frac{A}{\rho_0 c} e^{-jkx} - \frac{B}{\rho_0 c} e^{+jkx}$$

$$\tilde{z}(x) = \frac{\tilde{p}(x)}{\tilde{u}(x)} = \rho_0 c \, \frac{A e^{-jkx} + B e^{+jkx}}{A e^{-jkx} - B e^{+jkx}}$$

function of position



$\left(\frac{B}{A}\right)$ - plane wave reflection coefficient

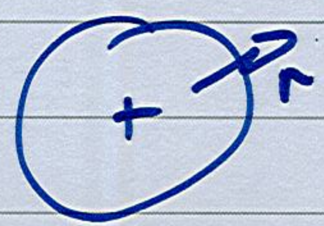
$$Z = \rho_0 c \, \frac{e^{-jkx} + \left(\frac{B}{A}\right) e^{+jkx}}{e^{-jkx} - \left(\frac{B}{A}\right) e^{+jkx}}$$

$$0 \leq \left|\frac{B}{A}\right| \leq 1$$

(i) $\frac{B}{A} \to 0 \quad Z = \rho_0 c$

(ii) $\frac{B}{A} \to 1 \quad Z \to i \rho_0 c \cot kL$

spherically symmetric case



$$\frac{A}{r} e^{-jkr}$$

$$\hat{u}_r = -\frac{1}{j\omega \rho_0} \frac{d\hat{p}}{dr}$$

$$\hat{u}_{r+} = \frac{1}{\rho_0 c}\left(1 + \frac{1}{jkr}\right)\hat{p}_+$$

$$\hat{z}_+ = \frac{\hat{p}_+}{\hat{u}_{r+}} = \frac{\rho_0 c}{1 + \frac{1}{jkr}}$$

Farfield
$kr \gg 1$

$$\hat{z} \longrightarrow \rho_0 c$$

nearfield

$kr \ll 1$

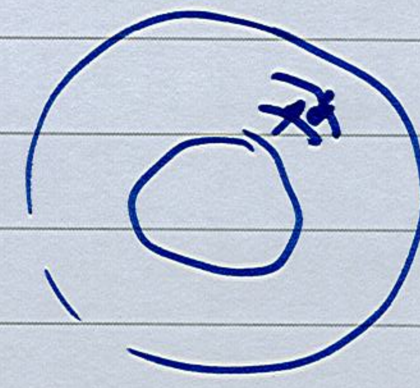
$\tilde{z} \rightarrow jkr\rho_0 c$ $\qquad k = \frac{\omega}{c}$

$\rightarrow j\frac{\omega}{\cancel{c}} r \rho_0 \cancel{c}$ — mass-like
- imaginary
- positive
- linearly prop to ω

$\tilde{p}$ & $\tilde{u}$ are out of phase

General Spherical Case

$$\tilde{p}(r) = \frac{A}{r} e^{-jkr} + \frac{B}{r} e^{+jkr}$$



$$\tilde{u}_r(r) = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dr} \rightarrow 4 \text{ terms}$$

$$\tilde{z}(r) = \frac{\tilde{p}(r)}{\tilde{u}(r)}$$ relatively complicated

Notes:

(i) Impedance is usually expressed in terms of

$$\tilde{z} = \underbrace{r}_{\text{specific acoustic resistance}} + j\,\underbrace{x}_{\text{specific acoustic reactance.}}$$

(ii) $\rho_0 c$ is the characteristic impedance

$\neq$ s.a. impedance

(iii) outward-going $\begin{pmatrix}\text{cylind} \\ \text{spherical}\end{pmatrix}$ waves

$kr \gg 1 \qquad \tilde{z} \rightarrow \rho_0 c$

3.5 Acoustic Intensity