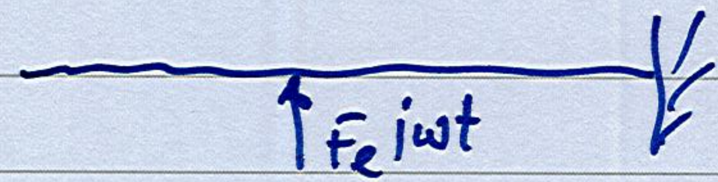
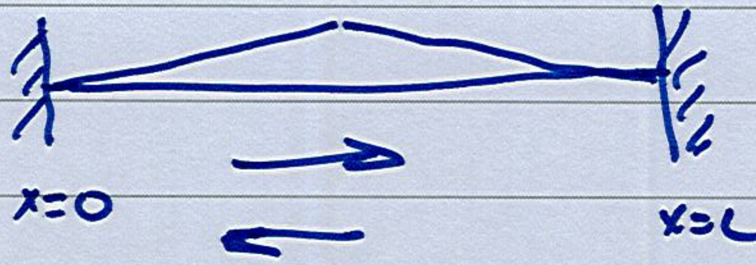


Forced Vibration
- external force



Free Vibration
- no external force
- motion results from initial conditions

Characteristic Eqn

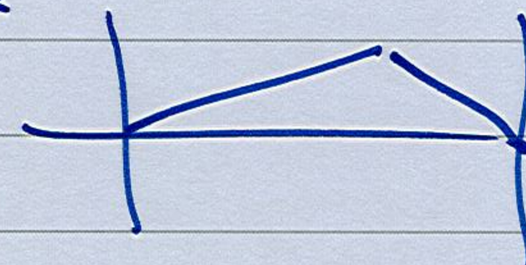
$\sin k_n L = 0$

solve~~s~~ for allowed k_n's & ω_n's

mode shapes - satisfy the b.c.'s

Total soln - weighted sum of modes that satisfy the initial conditions

$$y(x,t) = \sum_{n=1}^{\infty} \underbrace{\tilde{a}_n}_{a_n + jb_n} \sin k_n x \, e^{i\omega_n t} \quad *$$



$Re\{y(x,0)\}$ is known

Fourier Series

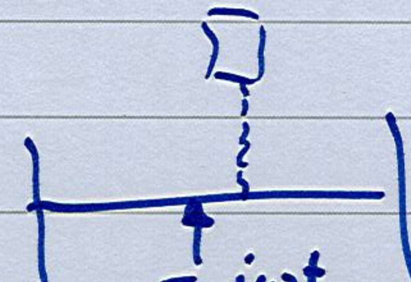
$$a_n = \frac{2}{L} \int_0^L Re\{y(x,0)\} \sin k_n x$$

3

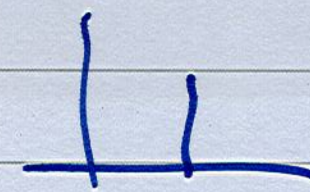
$$\tilde{a}_n = a_n + jb_n$$

Use initial velocity of the string to solve for b_n's

$$u(x,t) = \frac{\partial}{\partial t} \sum_{n=1}^{\infty} \tilde{a}_n \sin k_n x \, e^{j\omega_n t}$$



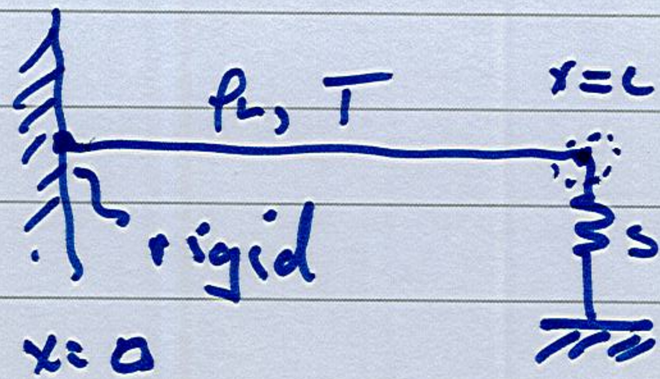
$$\mathrm{Re}\{u(x,0)\} \nearrow \downarrow b_n$$



$$\tilde{a}_n = a_n + j b_n$$

spectrum

2.5.4 Other B.C.'s



$$y(x,t) = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx)}$$

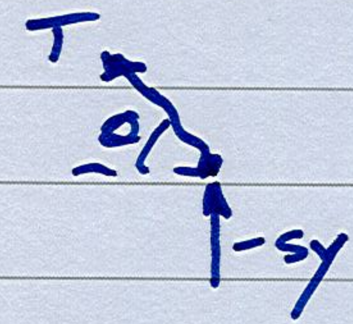
$k = \omega/c$

$c = \sqrt{T/\rho_L}$

at $x=0$ $\quad y(0,t) = 0$

$B \rightarrow -A$

$$y(x,t) = -2jA \sin kx\, e^{j\omega t}$$



at $x = L$

$\sum f_y = 0$

$T \sin\theta \big|_{x=L} - sy \big|_{x=L} = 0$

$-T \dfrac{\partial y}{\partial x}\Big|_{x=L} - sy\big|_{x=L} = 0$

$\dfrac{dy}{dx} = -2jkA \cos kx \, e^{j\omega t}$

$$+\cancel{Zj}kT\cancel{Ae^{j\omega t}}\cos kL + \cancel{Zj}s\cancel{Ae^{j\omega t}}\sin kL$$

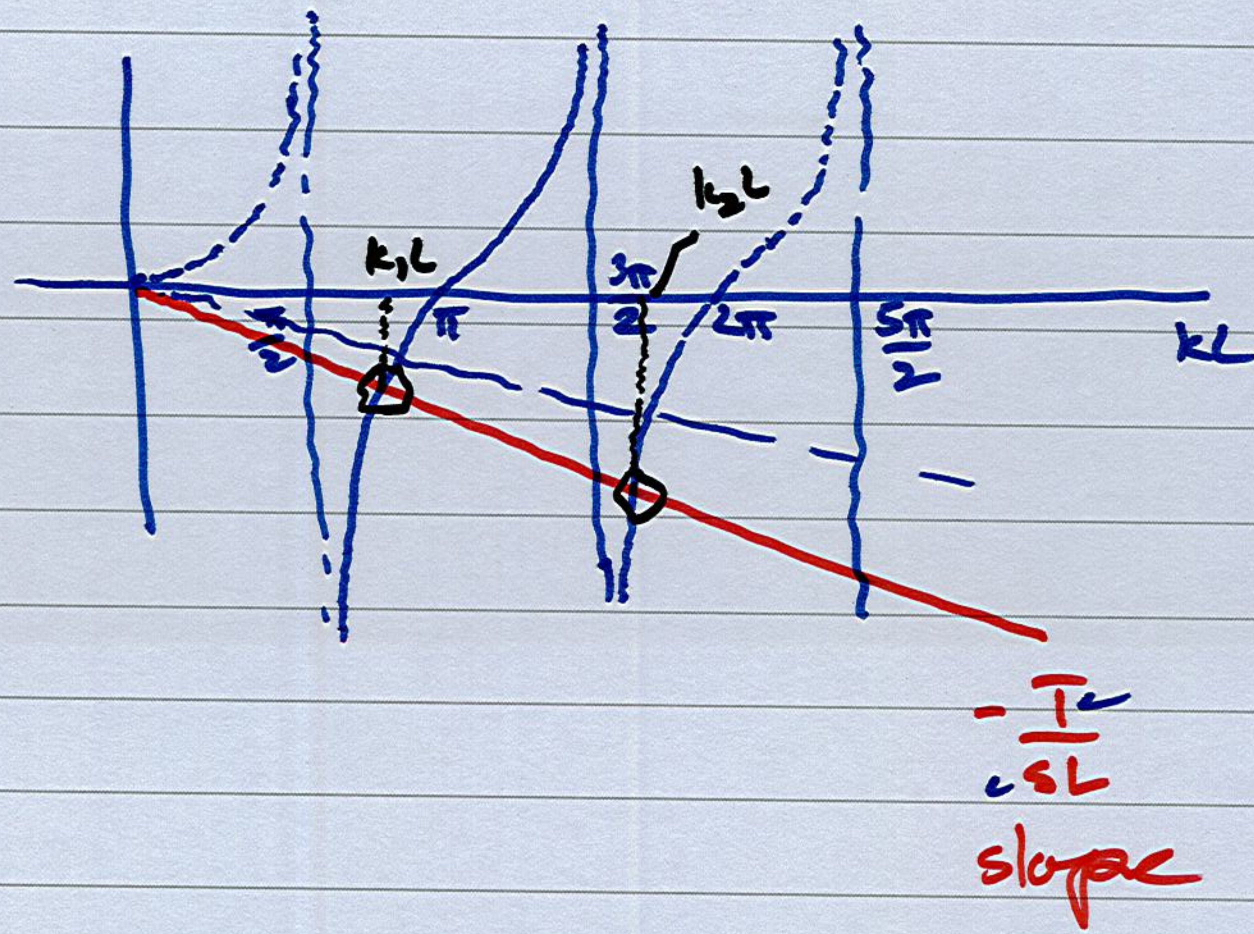
$$k_nT\cos k_nL = -s\sin k_nL$$

$$-\frac{k_nT}{s} = \tan(k_nL)$$

$$\boxed{-(k_nL)\left(\frac{T}{sL}\right) = \tan(k_nL)}$$

Characteristic Eqn.

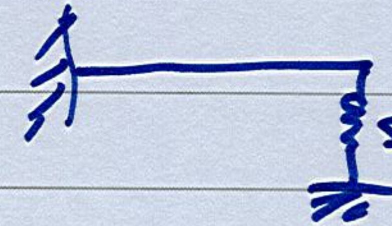
$$-(k_nL)\left(\frac{T}{SL}\right) = \tan k_n L$$



$k_1L \to k_1$

$\to \omega_1$

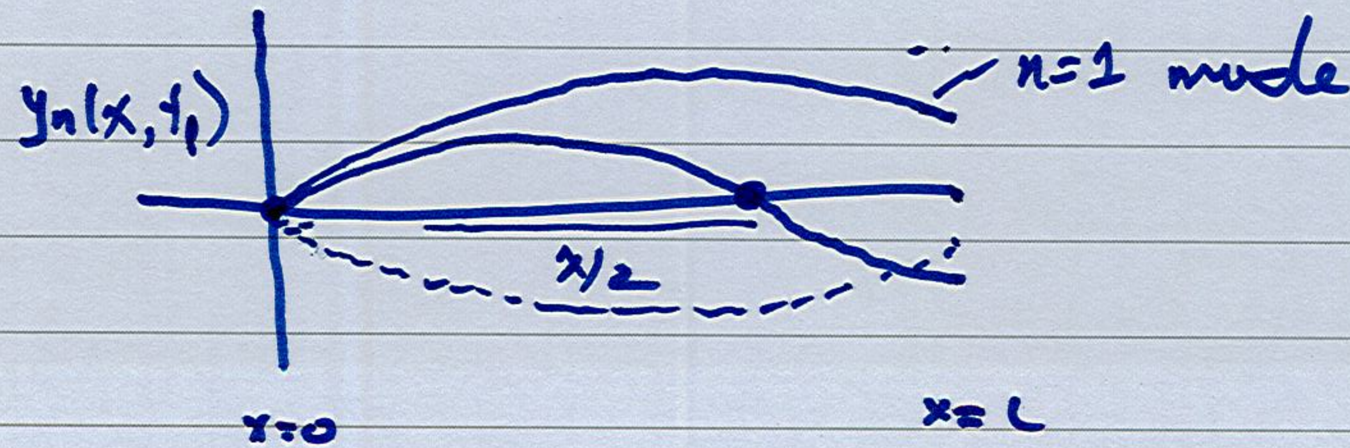
$\to f_1$



Mode shape

$$y_n(x,t) = -2jA_n \underbrace{\sin k_n x}\, e^{j\omega_n t}$$

n th ~~mo~~ mode



$$y(x,t) = \sum_{n=1}^{\infty} \text{modes} \qquad k_1 \rightarrow k_\infty$$

Summary

- Derivation of a wave equation
  (modeling)

  - restoring force ✓
  - equation of motion ✓

  $\Big]$ wave equation

  - inertia
  - stiffness

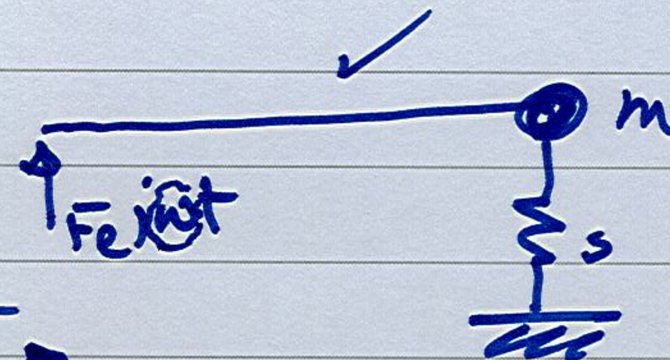
- Wave propagation

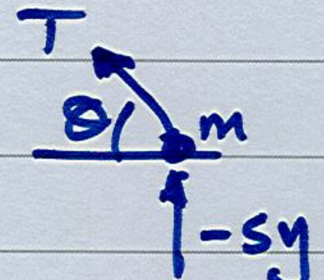
  $c = \sqrt{T/\rho_L}$ harmonic solution $e^{j\omega t}$

  e^{-jkx}

k = wave number
= spatial freq.

- Boundary Conditions
  - fixed
  - stiffness
  - mass
  - force





$\sum f_y = ma\big|_{x=l}$

Forced Response

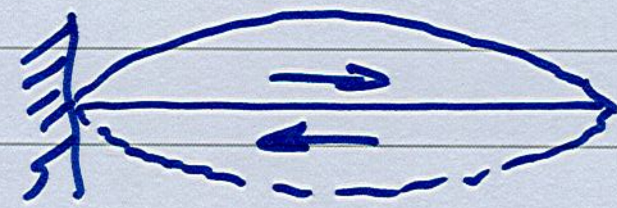
$$y(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$$

Impedance

$\left(\frac{\text{Force}}{\text{velocity}}\right)$

- Z_{mo}
- $\rho_L c$

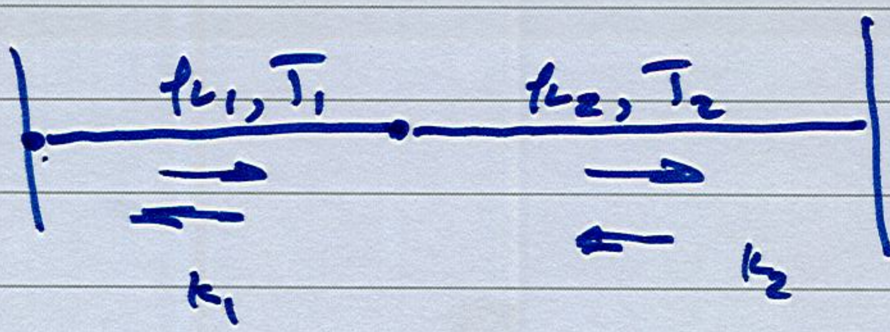
- standing wave
- propagating wave solution



Free Vibration

- natural frequencies
- mode shapes

$\Big]$ characteristic equation



5.1 $\rightarrow$ 5.13