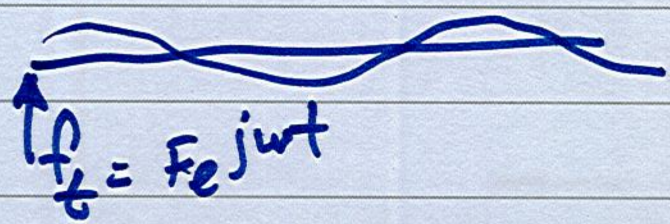
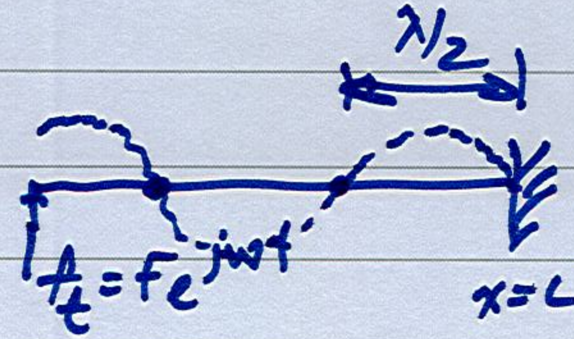


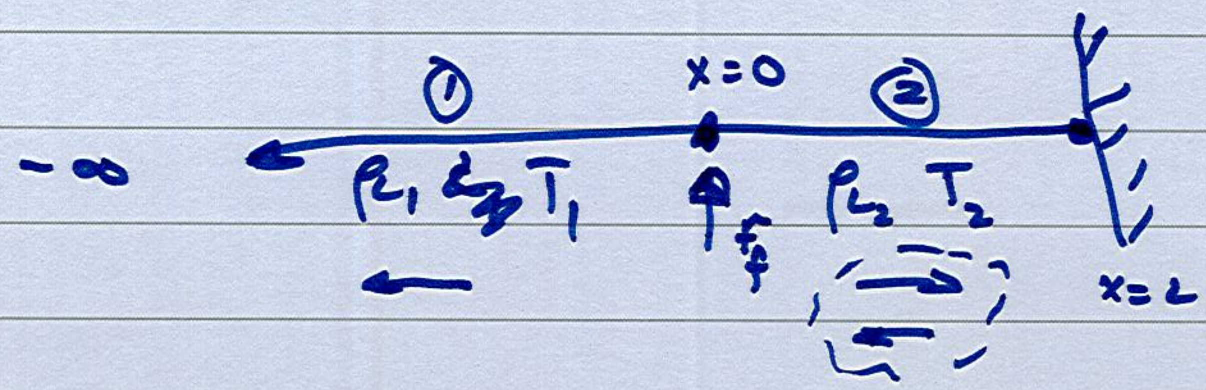
$$u(x,t) = \frac{F}{\rho_L c} e^{j\omega t} e^{-jkx}$$

$$z_{mo} = \rho_L c$$



$$z_{mo} = -j \rho_L c \, \underline{\cot kL}$$

2.4.3

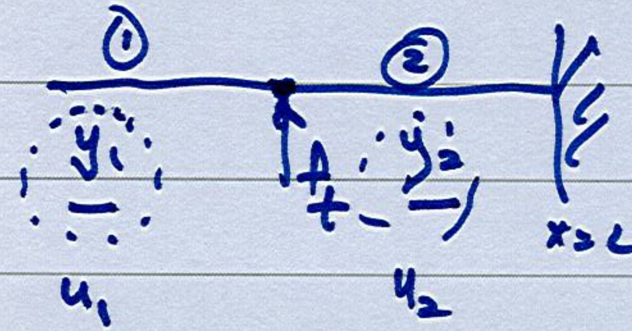


$$f_t + T_2 \frac{dy_2}{dx}\bigg|_{x=0} - T_1 \frac{dy_1}{dx}\bigg|_{x=0} = 0 \qquad (3)$$

substitute assumed soln's
into (1), (2) & (3)

B, C, D

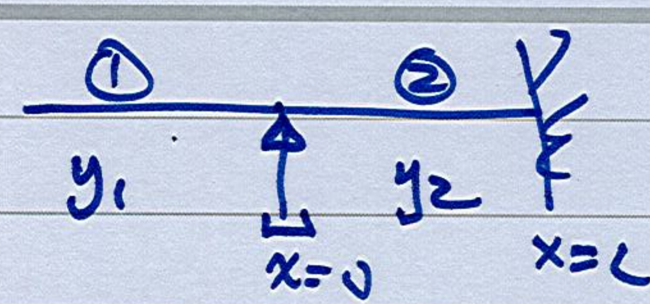
$$z_{m0} = \frac{f_t}{u_1|_{x=0}} = \frac{f_t}{u_2|_{x=0}}$$



$$= \underline{\rho_L c_1} - j\,\rho_{L_2} c_2 \cot k_2 L$$

- impedances of the string segments add in series
- velocity is shared between the two segments

use either $u_1|_{x=0}$ or $u_2|_{x=0}$



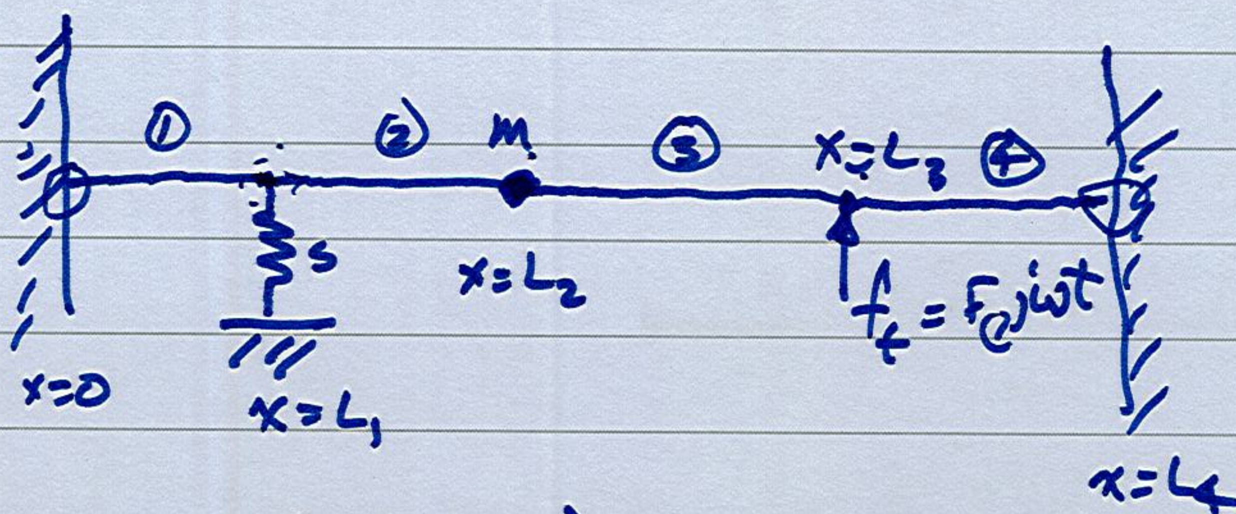
Notes :

(i) solution for y_1 applies only in the region $x \leq 0$

(ii) solution for y_2 applies only in the region $0 \leq x \leq L$

(iii) same approach even if two segments have the same properties

(iv) same approach can be extended to any number of segments



$$y_1(x,t) = Ae^{j(\omega t - k_1 x)} + Be^{j(\omega t + k_1 x)} \qquad k_1 = \frac{\omega}{c_1} \qquad c_1 = \sqrt{T_1/\rho_{L1}}$$

$$0 \le \cancel{0}\,x \le L_1$$

$y_2(x,t) = C \sim + D \sim$ k_2

$y_3(x,t) = E \sim + F \sim$ k_3

$y_4(x,t) = G \sim + H \sim$ k_4

8 unknowns

General Approach – multi-segments

- write the general sol'n for each segment
- write the b.c's
- substitute soln's into b.c's & solve

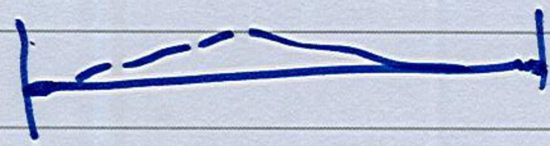
$$\begin{bmatrix} & & \\ & 8 \times 8 & \\ & \text{coefficient} & \\ & & \end{bmatrix} \begin{bmatrix} A \\ \vdots \\ H \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

forcing vector

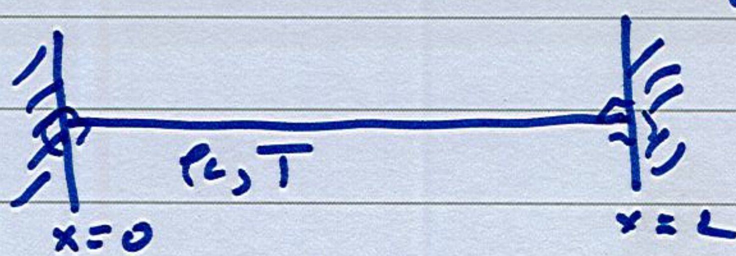
2.5 Normal Modes of Finite Strings

Free Vibration

- when a string is "forced" into vibration by initial conditions



2.5.1 Characteristic Eqn



$$y(x,t) = A e^{j(\omega t - kx)} + B e^{j(\omega t + kx)}$$

solution to wave eqn

b.c. at $x = 0$ $y(0,t) = 0$

$A + B = 0$

$\underline{B = -A}$ (1)

b.c. at $x = L$ $y(L,t) = 0$

$\left(A e^{-ikL} + B e^{+ikL}\right) \cancel{e^{i\omega t}} = 0$

$A\left(e^{-ikL} - e^{+ikL}\right) = 0$

$-2j \sin kL$

$$\boxed{-2jA\sin kL = 0} \qquad (2)$$

if $A = 0$ displacement $= 0$ for all times

$$\boxed{\sin k_n L = 0}$$

characteristic eqn

$k_n L = n\pi \qquad n = 1, 2, 3 \ldots\ldots$

$k_n = \frac{n\pi}{L} \qquad \underbrace{k = \frac{\omega}{c} = \frac{2\pi f}{c}}$

$f_n = \frac{n}{2}\frac{c}{L} \qquad n = 1, 2, \ldots$

allowed natural frequencies

$$L = \frac{n}{2}\frac{c}{f_n} = \underline{\frac{n}{2}\lambda_n}$$

$c = f\lambda$

at the natural frequencies
the string length is an
integral number of
half wavelengths
(fixed boundary at
both ends)

2.5.2 mode shape

$$y(x,t) = \left(Ae^{-jkx} + Be^{jkx}\right)e^{j\omega t}$$

first b.c. $\rightarrow$ $B = -A$

$$y(x) = -2jA \sin kx$$

$$y_n(x,t) = \underbrace{\underbrace{-2j\cdot A_n}_{c_n}\;\underbrace{\sin k_n x}_{\text{mode shape}}\; e^{j\omega_n t}}_{\text{normal mode}}$$

$n = 1, 2, 3 \ldots$

modes

- individual solutions of the wave equation that satisfy the boundary condition

$k_x^{y,z}$ allowed wavenumbers

$\omega_n = 2\pi f_n$ natural freqs

C_n = modal amplitude

complete solution = sum of the possible solutions that satisfy the initial conditions

2.5.3 Total Solution

Total solution = superposition of possible solutions

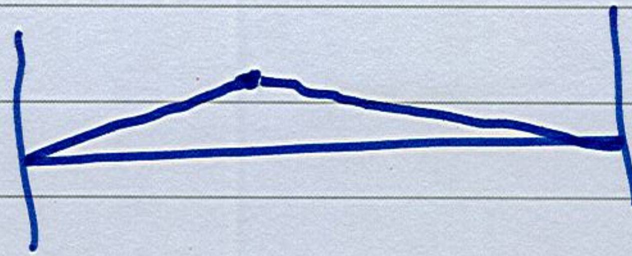
$$y(x,t) = \sum_{n=1}^{\infty} \tilde{a}_n \underline{\sin k_n x} \, \underbrace{e^{j\omega_n t}}_{\cos\omega_n t + j\sin\omega_n t}$$

$$\tilde{a}_n = a_n + jb_n$$

Real Displacement of The nth mode

$$\mathrm{Re}\{y_n(x,t)\} = (a_n \cos\omega_n t - b_n \sin\omega_n t)\sin k_n x$$

say $Re\{y(x,0)\}$ = known



at $t=0$

$$\underline{Re\{y(x,0)\}} = \sum_{n=1\ldots}^{\infty} \tilde{a}_n \sin k_n x$$