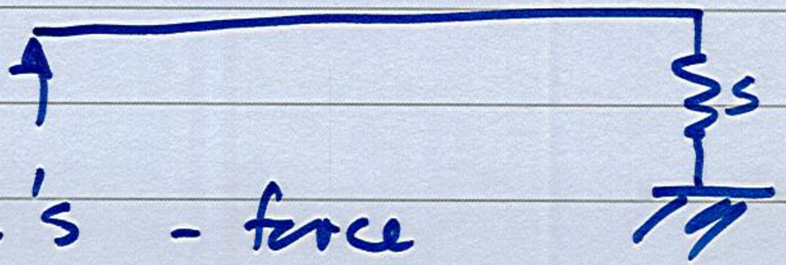
ME 313 Session 10 9/16/15

1

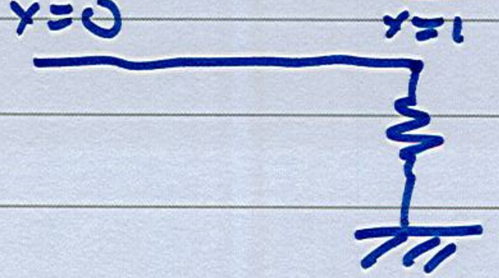


BC's
- force
- fixed
- stiffness
- mass

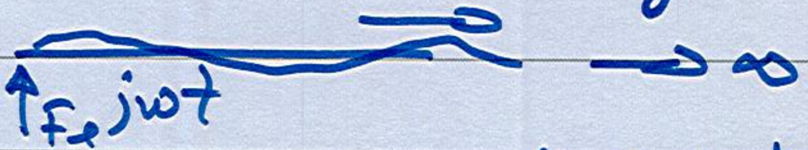
$$\left.\frac{dy}{dx}\right|_{x=0} = \left(\frac{s}{T}\right) y\Big|_{x=0}$$

$$y\big|_{x=0} = \left(\frac{T}{\cancel{s}}\right) \left.\frac{dy}{dx}\right|_{x=0} \to 0$$

$$y\big|_{x=0} = 0$$

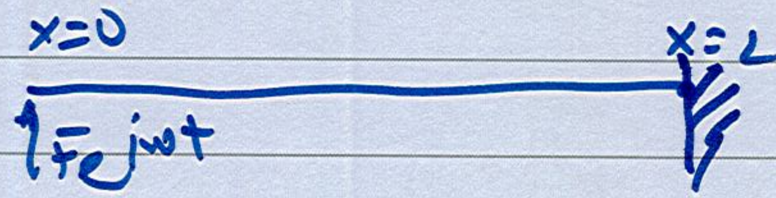


sign change



$$y(x,t) = \frac{F}{jkT} e^{j(\omega t - kx)}$$

$$z_{m0} = \rho_L c$$



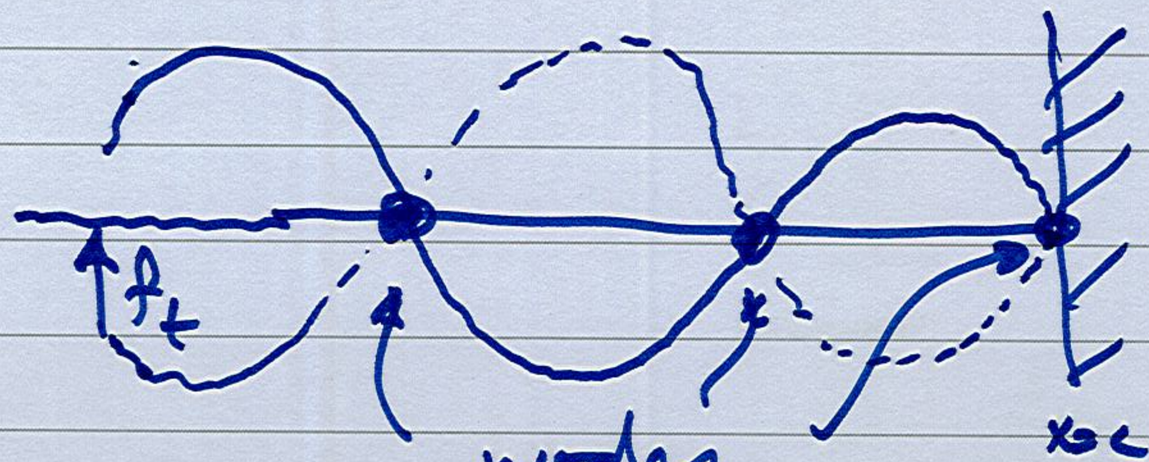
$$y(x,t) = \frac{F}{2jkT\cos kL}\left[e^{j(\omega t + k(L-x))} - e^{j(\omega t - k(L-x))}\right]$$

+ve $\rightarrow$ −ve $\leftarrow$

$$y(x,t) = \left(\frac{F}{kT}\right)\frac{\sin k[L-x]}{\cos kL}\, e^{j\omega t}$$

standing wave representation

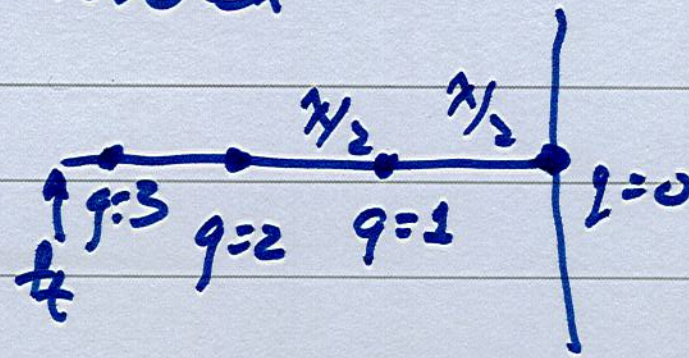
Standing wave - superposition of two propagating waves that interfere with each other



nodes

- displacement is always equal to zero

(iv) location of the nodes



$$\sin k(L - x_q) = 0$$

$$k(L - x_q) = q\pi \qquad q = 0, 1, 2, 3 \ldots$$

$$x_0 = L$$

$$x_q = L - q\frac{\pi}{k} \qquad k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

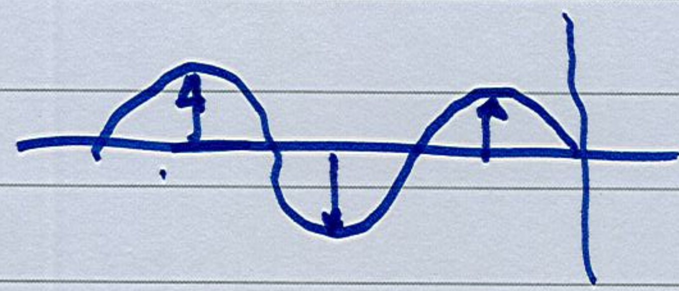
$$x_q = L - q\left(\frac{\lambda}{2}\right)$$

nodal points are at

L − integer number of $(\lambda/2)$'s away from termination

As the frequency increases,
the nodes move to the right
since λ is decreasing

(v) Antinodes



The points of
maximum
displacement half-way
between the nodes

(vi) $y(x,t) = \left(\dfrac{F}{kT \underline{\cos kL}}\right) \underline{\sin k} \underline{(L-x)} e^{j\omega t}$

$k = \dfrac{\omega}{c}$

response is largest
at the frequency at which
$\cos kL \to 0$
resonance condition

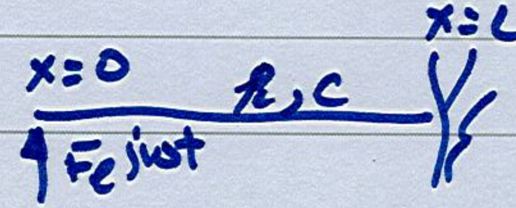
$\cos kL = 0$

$k_n L = \left(\dfrac{2n-1}{2}\right)\pi \qquad n = 1, 2, 3\ldots\ldots$

$k_n = \dfrac{\omega_n}{c} = \dfrac{2\pi f_n}{c}$ ~ natural freqs.

$f_n = \left(\dfrac{2n-1}{4}\right)\left(\dfrac{c}{L}\right) \qquad n = 1, 2, 3\ldots\ldots$

(vii) Input impedance



$$Z_{m0} = \frac{\text{input force}}{\text{velocity at the drive point}} = \frac{F_e e^{j\omega t}}{(j\omega)\dfrac{F_e e^{j\omega t}}{kT}\,\dfrac{\sin k[L-\underset{\to 0}{x}]}{\cos kL}}$$

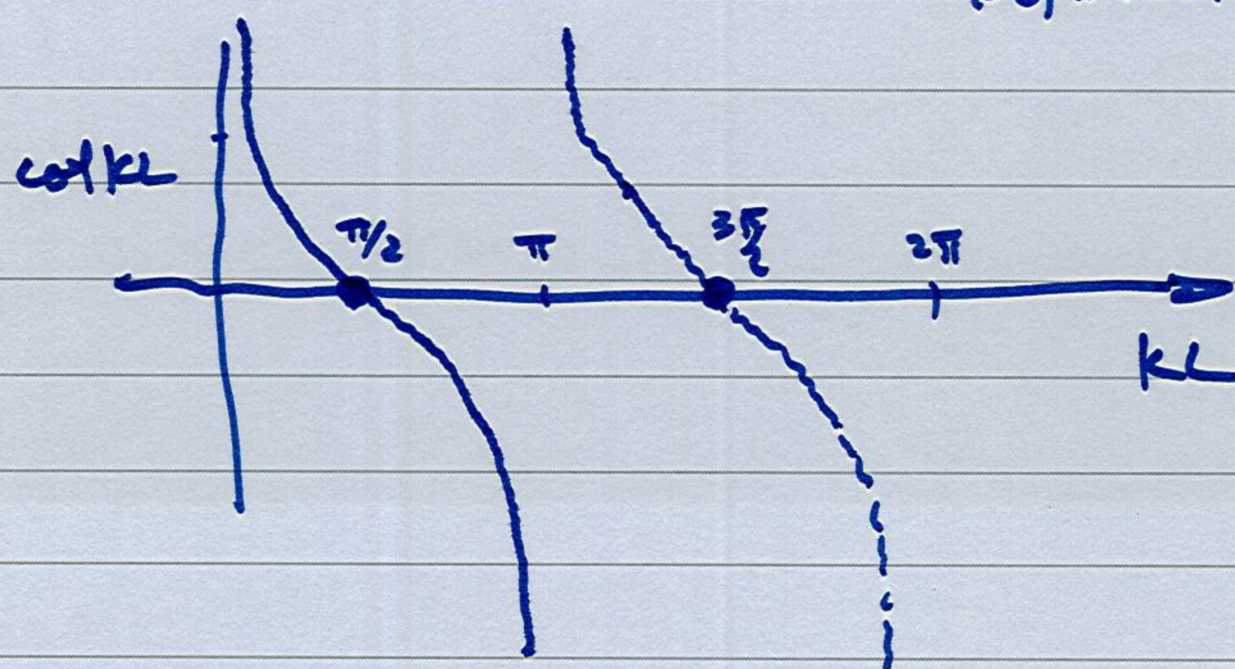
$k = \frac{\omega}{c}$ $\qquad T = \rho_L c^2$

$$\boxed{Z_{m0} = -j\,\underbrace{\rho_L c}_{\text{medium}}\,\underbrace{\cot kL}_{\text{geometry}}}$$

purely imaginary.

$$Z_{m0} = -j\rho_L c \underline{\cot kL}$$

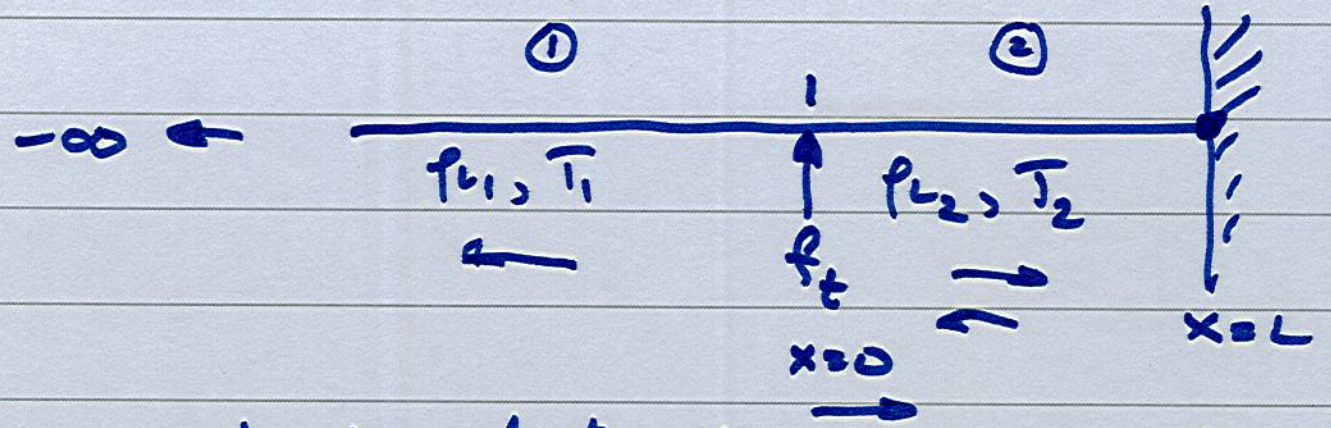
$\mathrm{Im}\{Z_{m0}\} \to 0$ defines resonance & natural frequencies



$(kL)_1 = \frac{\pi}{2}$ $\quad (kL)_2 = \frac{3\pi}{2}$ $\quad \ldots$

$\to$ natural frequencies

2.4.3 Strings with multiple segments



$f_t = F e^{j\omega t}$

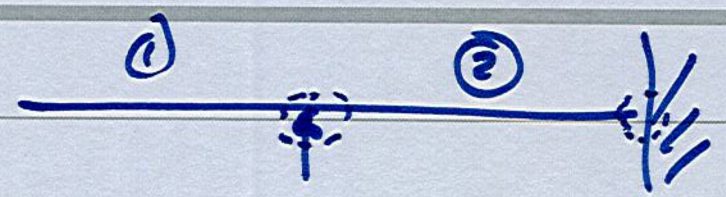
must treat the two segments as distinct strings - coupled through boundary conditions

- even if it is one string

wave eqn for each segment

$$\frac{\partial^2 y_1}{\partial x^2} - \frac{1}{c_1^2}\frac{\partial^2 y_1}{\partial t^2} = 0 \qquad \frac{\partial^2 y_2}{\partial x^2} - \frac{1}{c_2^2}\frac{\partial^2 y_2}{\partial t^2} = 0$$

$$c_1 = \sqrt{T_1/\rho_{L1}} \qquad c_2 = \sqrt{T_2/\rho_{L2}}$$



In region ① $x \leq 0$ $k_1 = \frac{\omega}{c_1}$

$$y_1(x,t) = \cancel{A e^{j(\omega t - k_1 x)}} + \boxed{B} e^{j(\omega t + k_1 x)}$$

+ve -ve

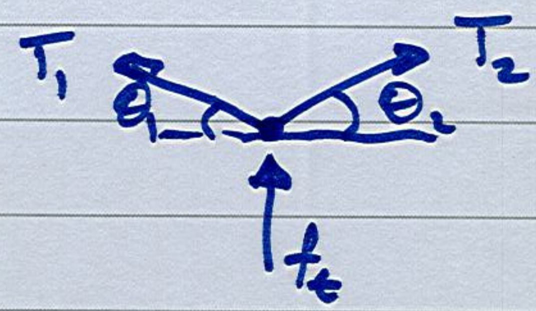
In region ② $0 \leq x \leq L$

$$y_2(x,t) = C e^{j(\omega t - k_2 x)} + D e^{j(\omega t + k_2 x)}$$ $k_2 = \omega / c_2$

3 unknowns

3 B.C.'s – 2 displacement
– 1 force

(iii) Force b.c. at $x=0$



$$\sum f_y = 0 = f_t + T_2 \sin\theta_2\big|_{x=0} + T \sin\theta_1\big|_{x=0}$$