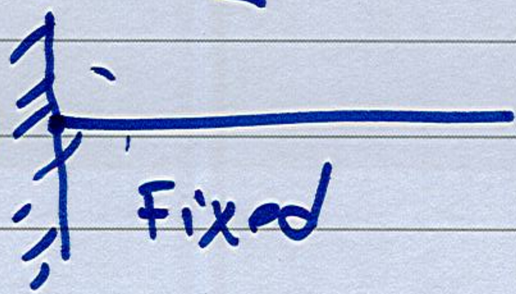
Chapter 2.1 → 2.11

$f_t = Fe^{j\omega t}$

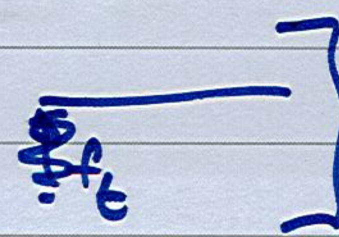
$\underset{\sim}{A}$, $\underset{\sim}{B}$

Boundary Conditions

$y(0,t) = 0$



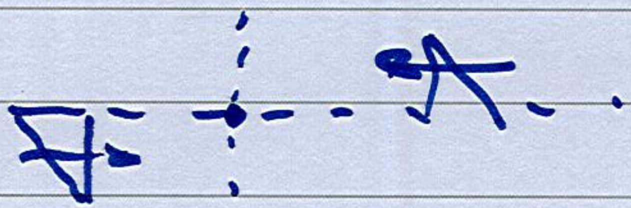
Fixed



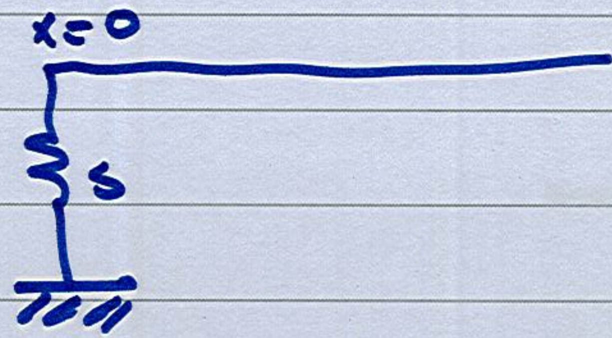
f_t

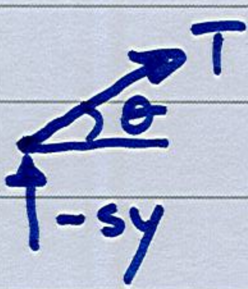
$\left.\frac{\partial y}{\partial x}\right|_{x=0} = 0$

m



2.3.4 Stiffness B.C.





$\sum f_y = 0$

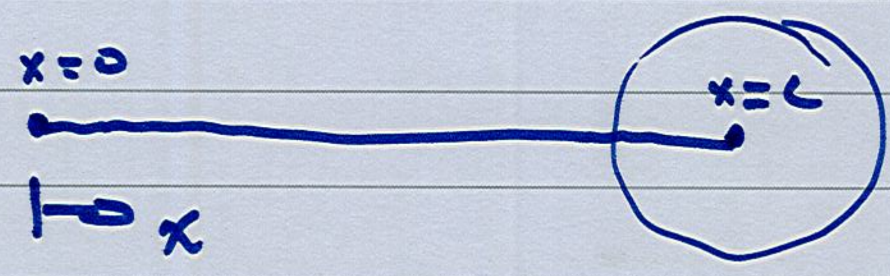
$T\sin\theta\big|_{x=0} - sy\big|_{x=0} = 0$

$T\frac{\partial y}{\partial x}\Big|_{x=0} - sy\big|_{x=0} = 0$

$$\boxed{\frac{\partial y}{\partial x}\Big|_{x=0} = \left(\frac{s}{T}\right)y\Big|_{x=0}}$$

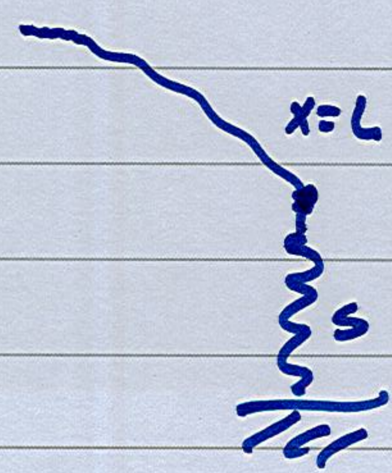
- fixed
- mass
- stiffness
- transverse force

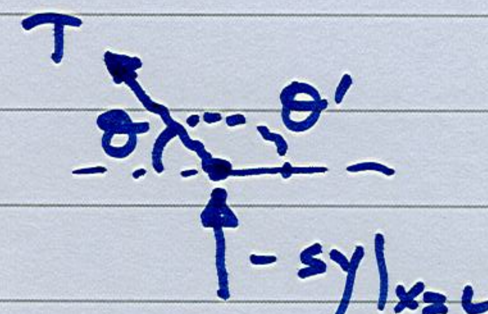
$x = 0$



2.3.5 B.C. Applied at the +ve x-end of string

e.g., stiffness





$$T \sin\theta'|_{x=L} - sy|_{x=L} = 0$$

$$\sin\theta' \approx -\frac{\partial y}{\partial x}$$

$$-T\frac{\partial y}{\partial x}\bigg|_{x=L} = sy\big|_{x=L}$$

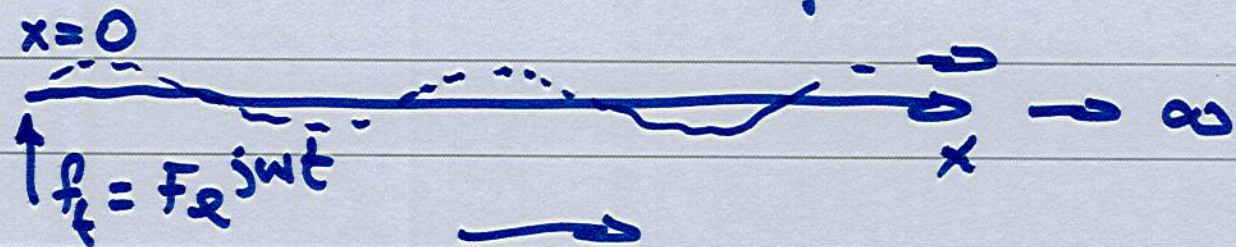
$$\frac{\partial y}{\partial x}\bigg|_{x=L} = -\frac{s}{T}y\big)_{x=L}$$

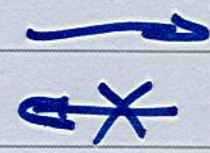
sign of the b.c. depends on position

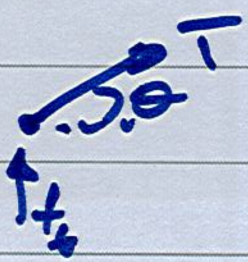
EXAM

2.4 Forced Vibration

2.4.1 Semi-infinite string







$\Sigma f_y = 0$

$$\left.\frac{dy}{dx}\right|_{x=0} = -\frac{f_t}{T}$$

$$y(x,t) = Ae^{j(\omega t - kx)} + \cancelto{0}{Be^{j(\omega t + kx)}}$$

nothing returns from infinity

$$y(x,t) = \hat{A}e^{j(\omega t - kx)}$$

$$\frac{\partial y}{\partial x} = -jkAe^{j(\omega t - kx)}$$

$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = -jk\underset{\xi}{A}\cancel{e^{j\omega t}} = -\frac{F\cancel{e^{j\omega t}}}{T}$$

$$A = \frac{F}{jkT}$$

$$\boxed{y(x,t) = \frac{F}{jkT} e^{i(\omega t - kx)}}$$

physical solution $Re\{y(x,t)\}$

Transverse Velocity

$$u(x,t) = \frac{\partial y}{\partial t} = \frac{\cancel{j}\omega F}{\cancel{j} k T} e^{j(\omega t - kx)}$$

$k = \frac{\omega}{c}$

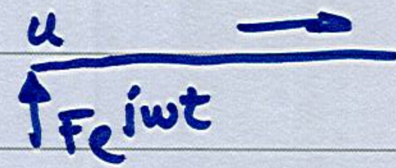
$$c = \sqrt{\frac{T}{\rho_L}} \longrightarrow T = \rho_L c^2$$

$$\underline{u}(x,t) = \frac{F}{\rho_L c} e^{j(\omega t - kx)}$$

$\rho_L c$ characteristic impedance

- "characteristic" of the medium & the wave type

Input Mechanical Impedance



Drive Point Impedance

$$z_{mo} = \frac{\text{complex Applied driving force at } x=0}{\text{Velocity at the drive point}},$$

$$= \frac{\cancel{Fe^{i\omega t}}\,\rho_L c}{\cancel{Fe^{i\omega t}}}$$

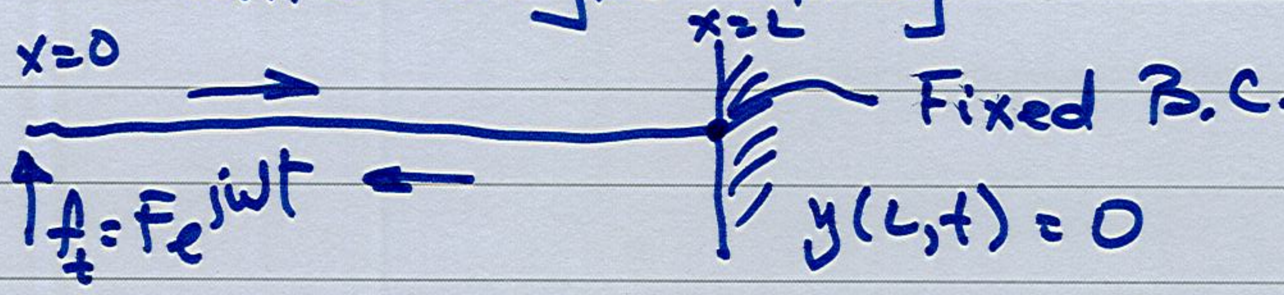
$$\boxed{z_{mo} = \rho_L c}$$

input mech imp = characteristic imp.

- real - resistive
- energy is carried away to infinity

In this case $\rho_L c = Z_{m0}$

2.4.2 Finite Length String



$$y(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$$

(i) $y(L,t) = \boxed{0 = Ae^{-jkL} + Be^{+jkL}}$ (1)

(ii) force b.c. at $x=0$

$$f_t = -T \frac{\partial y}{\partial x}\Big|_{x=0}$$

$$\frac{\partial y}{\partial x} = -jkA\, e^{j(\omega t - kx)} + jkB e^{j(\omega t + kx)}$$

$$F \cancel{e^{j\omega t}} = jkT(A-B) \cancel{e^{j\omega t}}$$

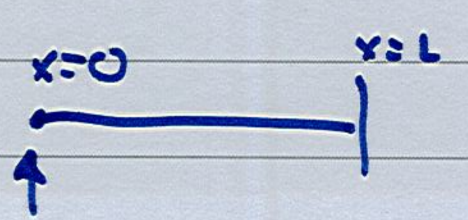
$$\boxed{jkTA - jkTB = F} \quad (2)$$

(iii) 2 simultaneous eqns in A & B

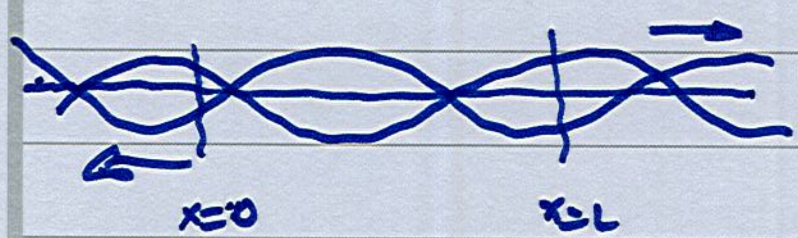
$$\begin{bmatrix} e^{-jkL} & e^{+jkL} \\ jkT & -jkT \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ F \end{bmatrix}$$

$$y(x,t) = \underbrace{\frac{F e^{jkL}}{2jkT \cos kL}}_{A} e^{j(\omega t - kx)} - \underbrace{\frac{F e^{-jkL}}{2jkT \cos kL}}_{B} e^{j(\omega t + kx)}$$

$$= \frac{F}{2jkT \cos kL}\left[e^{j(\omega t + k(L-x))} - e^{j(\omega t - k(L-x))} \right]$$



Express solution in terms of two propagating waves



+ superposition

- solution gives the "real" answer in this region alternatively

$$y(x,t) = \frac{F e^{j\omega t}\, 2j \sin k[L-x]}{2jkT \cos k[L-x]}$$

$$= \left(\frac{F}{kT}\right)\left[\frac{\sin k[L\cdot x]}{\underline{\underline{\cos kL}}}\right] e^{i\omega t}$$

standing wave representation
(separation of time & space)