Homework Hints

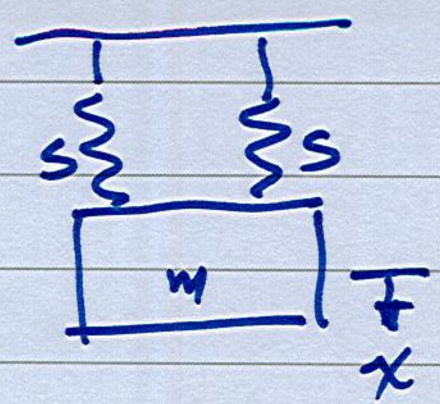
1.2.1 FBD → restoring force eqn

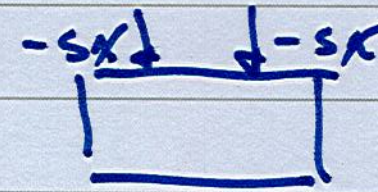
↓ combine with EOM

to create $\frac{d^2x}{dt^2} + \bigcirc\, x = 0$

ω_0^2

(a)





$f = -2sx$

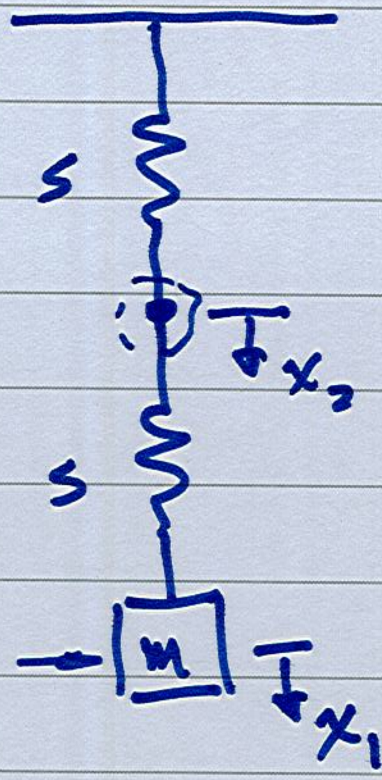
$$f = ma$$
$$= m\frac{d^2x}{dt^2}$$

$$-2sx = m\frac{d^2x}{dt}$$

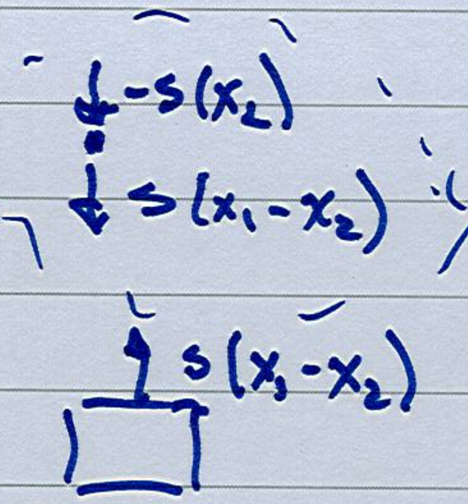
$$\frac{d^2x}{dt} + \frac{2s}{m}x = 0$$

$$\omega_0^2 = \frac{2s}{m}$$

(c)



$\Sigma F = 0$



$f = ma$ $\quad m\frac{d^2x_1}{dt} = -s(x_1 - x_2)$

1.3.2 $x = x_0 \cos \omega_0 t + \frac{u_0}{\omega_0} \sin \omega_0 t$]

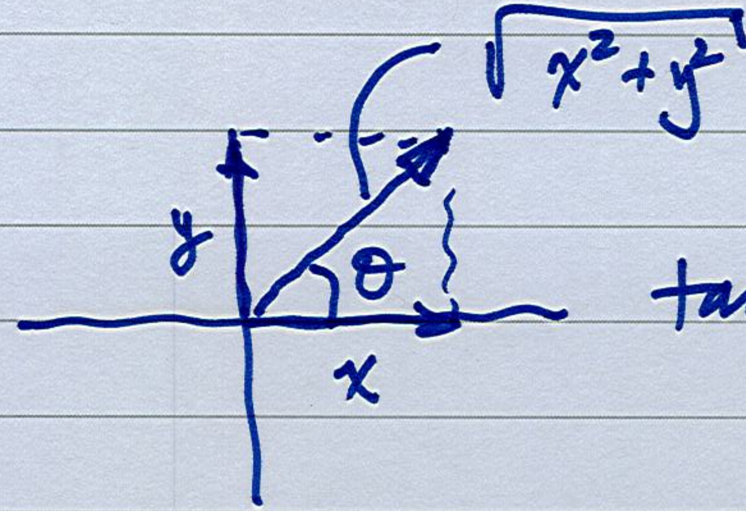
$\omega_0 = 5$ rad/s

$x_0 = 0.03$ m

$u_0 = 0$

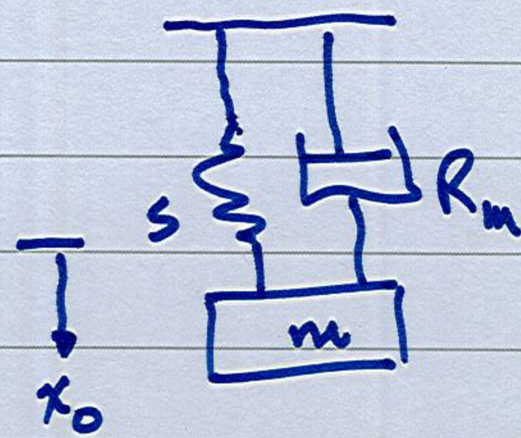
1.5.2

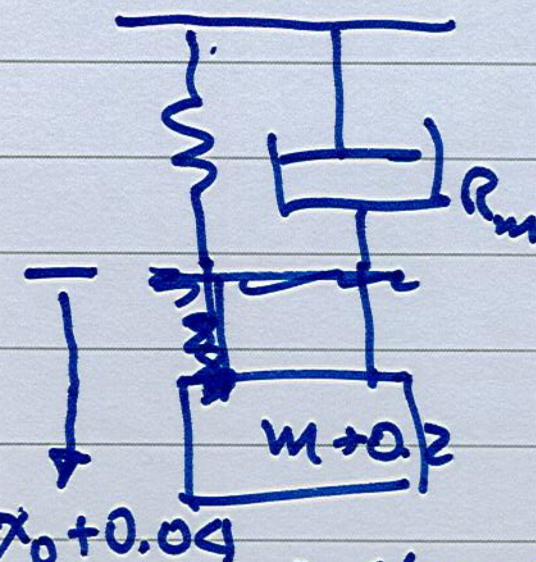
(a) $\sqrt{x+jy}$

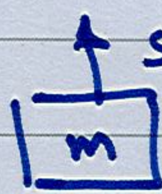


$\tan\theta = \frac{y}{x}$

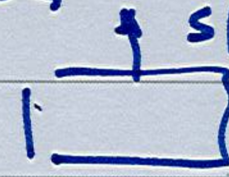
1.6.1







$sx_0 = mg$



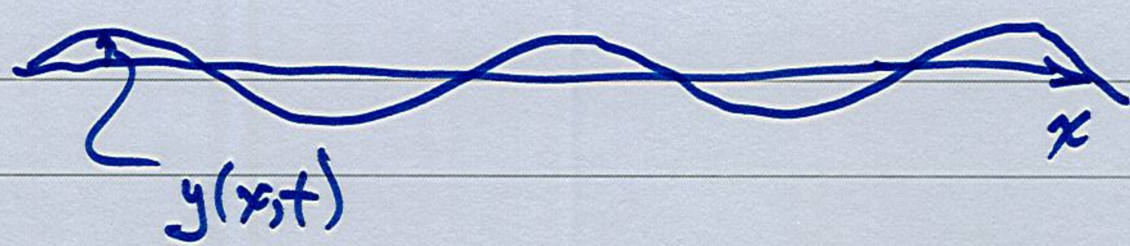
$s(x_0+0.04) = (m+0.2)g$

see page 9 R_m

$$x = A e^{-\beta t}(\cos \omega_d t + \phi)$$

magnitude decreases by e^{-1} in $1\,s$ $\beta t = 1$ when $t = 1$

Transverse Vibration of a Tensioned String



$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0 \qquad c = \sqrt{T/\rho_L}$$

$$y(x,t) = \underbrace{y_1(ct - x)}_{+ve\ x} + \underbrace{y_2(ct + x)}_{-ve\ x}$$

waves propagate without changing shape - nondispersive

2.2.2 Harmonic Single Frequency solution

Assume a seperable form of solution

$$\left.\begin{array}{l} y(x,t) = Y(x)\,e^{j\omega t} \\[2ex] \dfrac{\partial^2 y}{\partial x^2} - \dfrac{1}{c^2}\dfrac{\partial^2 y}{\partial t^2} = 0 \end{array}\right]\text{sub}$$

$$\frac{d^2 Y}{dx^2}\cancel{e^{j\omega t}} + \frac{\omega^2}{c^2} Y \cancel{e^{j\omega t}} = 0$$

governs spatial behavior

define $k^2 = \dfrac{\omega^2}{c^2}$ $\quad k = \dfrac{\omega}{c}$ wave number

$$\boxed{\frac{d^2Y(x)}{dx^2} + k^2 Y(x) = 0} \qquad (4)$$

Scalar Helmholtz Eqn.

SDOF $\frac{d^2x}{dt^2} + \omega_0^2 x = 0 \qquad x = Ae^{\pm j\omega t}$

$Y(x) = Ae^{\pm jkx}$ two solutions

solution for the spatial dependence of single freq transverse vibration of a string

$$y(x) = Ae^{\pm ikx} \qquad Ae^{-jkx} \qquad Ae^{+jkx}$$

$$y(x,t) = A_1 e^{+ikx} e^{i\omega t} + A_2 e^{-ikx} e^{i\omega t}$$

$$= A_1 e^{+j(kx+\omega t)} + A_2 e^{+j(-kx+\omega t)}$$

$k = \frac{\omega}{c}$

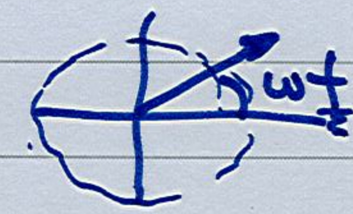
$$= \underbrace{A_1 e^{+jk(x+ct)}}_{y_2(ct+x)} + \underbrace{A_2 e^{+jk(-x+ct)}}_{y_1(ct-x)}$$

$y_2(ct+x)$ $y_1(ct-x)$

−ve going wave   +ve going wave

what is k ?

Recall $e^{j\omega t}$ $e^{i\phi}$ $\phi = \omega t$



$$\frac{d\phi}{dt} = \omega$$

ω: rate of change of phase with time

$\omega = \frac{2\pi}{T}$ (rad/s) $f = \frac{\omega}{2\pi}$ (Hz)

period

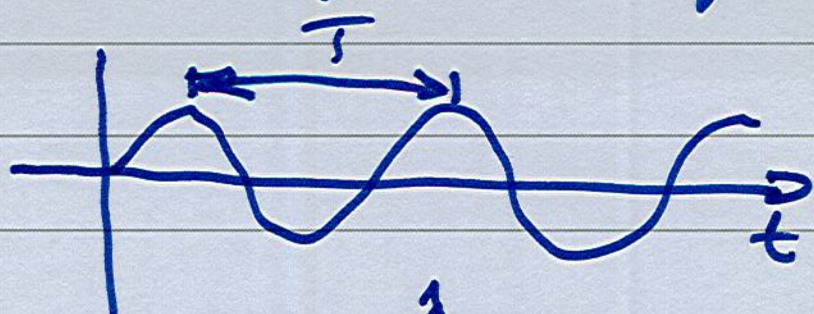
Temporal frequency

$e^{+ikx} \rightarrow e^{i\phi} \qquad \phi = kx$

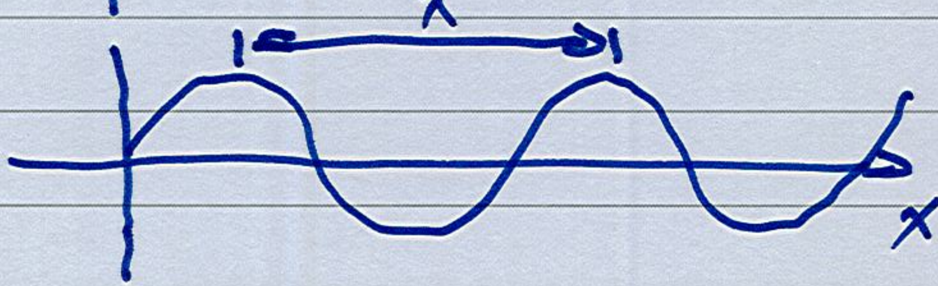
$$\frac{d\phi}{dx} = k$$

k: rate of change of phase with position

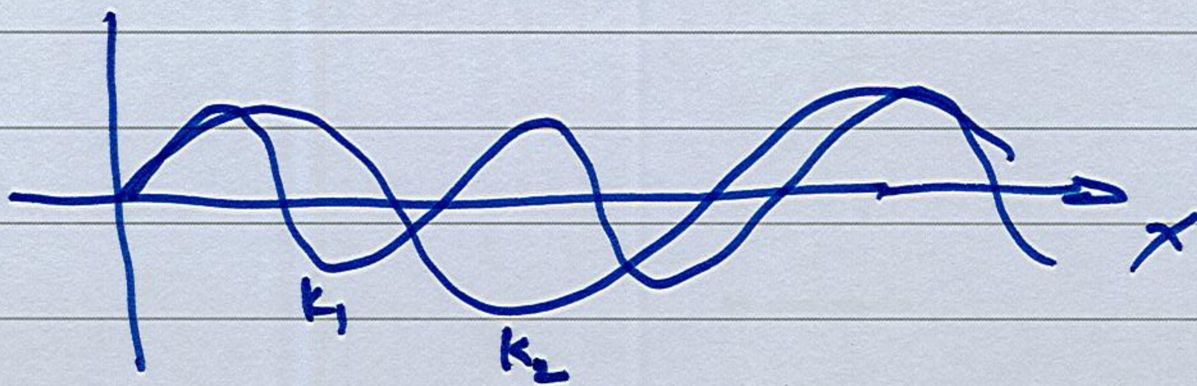
$k \rightarrow$ spatial frequency.



$\omega = \frac{2\pi}{T}$



$k = \frac{2\pi}{\lambda}$



$k_1 > k_2$

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

$c = f\lambda$

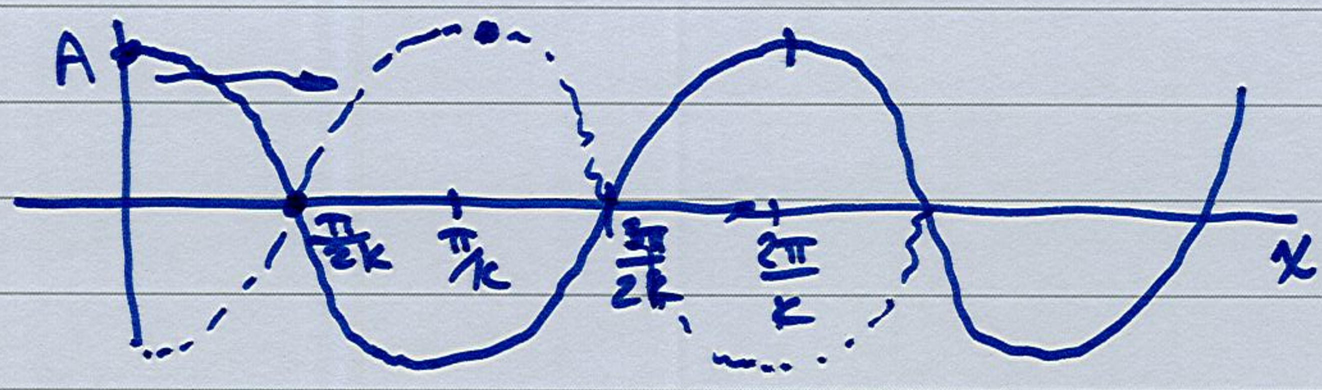
$\lambda = \frac{c}{f}$

$$\boxed{k = \frac{2\pi}{\lambda} = \frac{\omega}{c}}$$

exactly equivalent

plot $y = Ae^{j(\omega t - kx)}$ $\quad \cos(\omega t - kx) - j\sin(\omega t - kx)$

$\mathrm{Re}\{y\}$ at $t = 0$ when A $\quad \cos(-kx) = \cos(kx)$



$kx = \frac{\pi}{2}$

$x = \frac{\pi}{2k}$

$t = \frac{T}{2}$

when $t = T$ wave pattern have advanced by one wavelength.