MIE 513 Session 5 ~~8~~ 9/2/15 1

Student Chapters

ASA AES

~~Schuyler~~

Riley Shannon

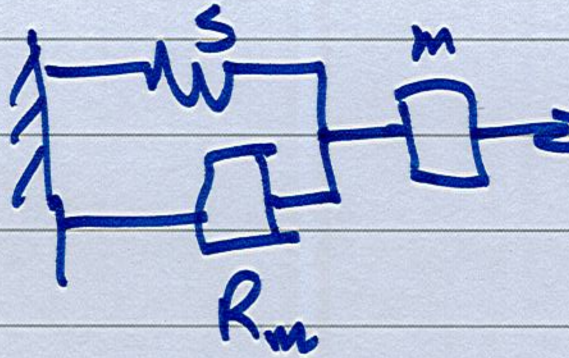
Sonja Adams

Callout 7:00 PM Pao

Basement 157

8

- mechanical impedance



$\tilde{f}_e(t) = F_e^{i\omega t}$

$$\tilde{Z}_m = \frac{\hat{f}_e}{\tilde{u}}$$

$j\omega \frac{m}{z}$ $\quad -j\frac{s}{\omega}$

if $\tilde{Z}_m$ is known

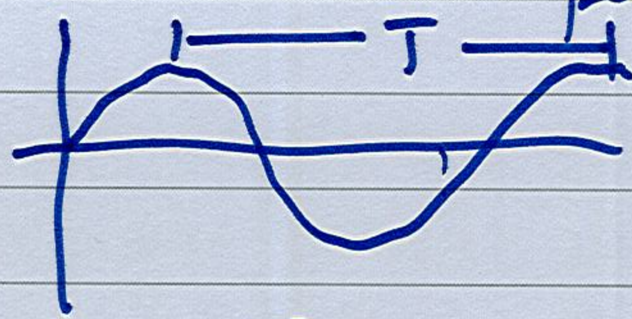
$$\tilde{u} = \frac{\hat{f}}{\tilde{Z}_m}$$

Resonance $\quad \mathrm{Im}\{\tilde{Z}_m\} \quad \omega_0$

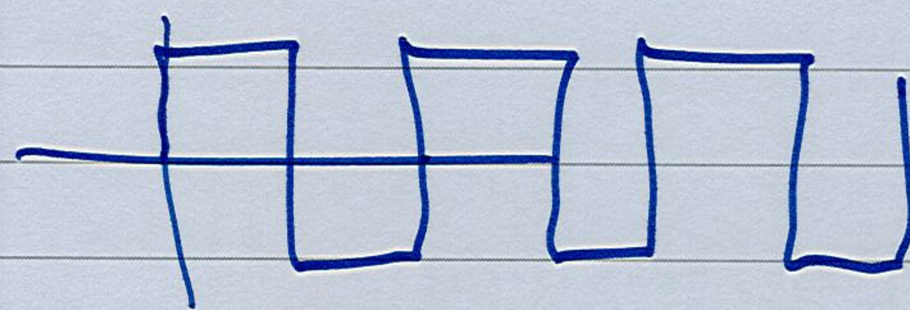
$\omega \rightarrow \omega_0$

1.5 Fourier Analysis - ME 579

periodic - signal repeats itself in T



sine



square

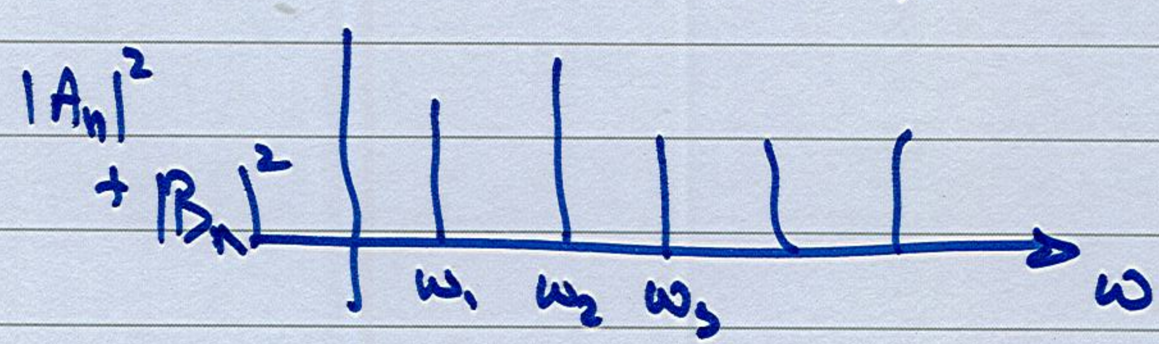
Any ~~period~~ single-valued periodic function can be represented **exactly** as a sum of sinusoids periodic in T

$f(t)$ periodic, SV

$$f(t) = \frac{1}{2}A_0 + A_1 \cos\omega_1 t + A_2 \cos\omega_2 t + \dots$$
$$+ B_1 \sin\omega_1 t + B_2 \sin\omega_2 t + \dots$$

$$\omega_1 = \frac{2\pi}{T} \qquad \omega_2 = 2\omega_1$$

- represented by contributions at discrete frequencies



$\omega_1 = \frac{2\pi}{T}$ fundamental - first harmonic

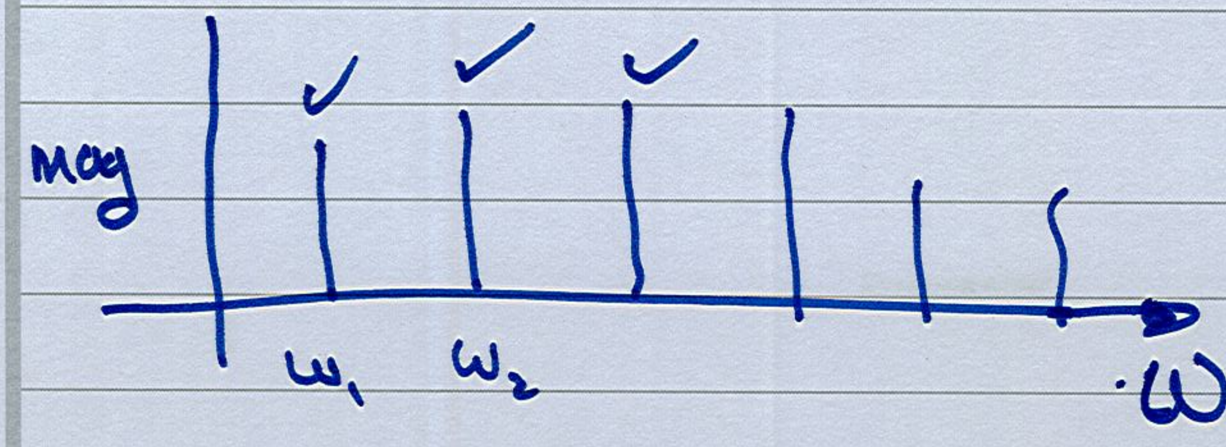
$\omega_2 = 2\omega_1$ 2nd harmonic

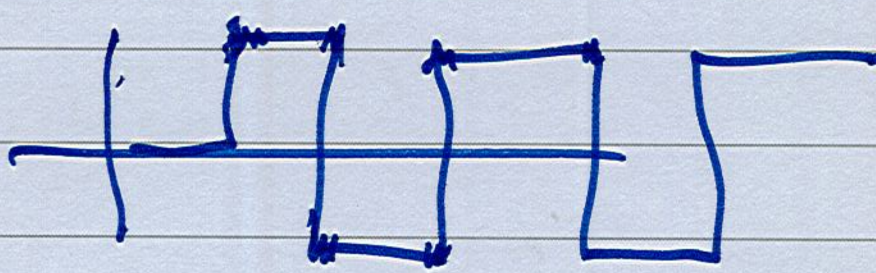
$\omega_3 = 3\omega_1$ 3rd harmonic

$$A_n = \frac{2}{T}\int_0^T f(t)\cos(\omega_n t)\,dt$$

$n = 1, 2, 3 \ldots\ldots$

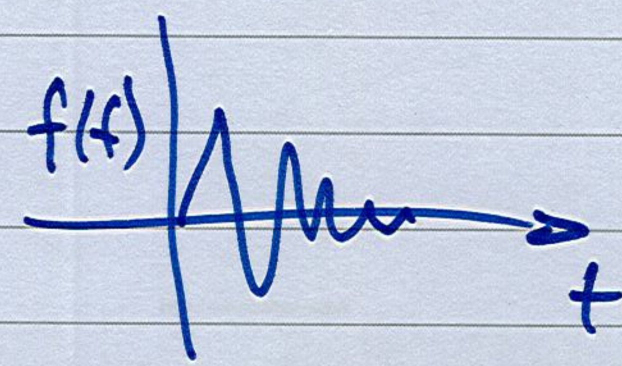
$$B_n = \frac{2}{T}\int_0^T f(t)\sin(\omega_n t)\,dt$$



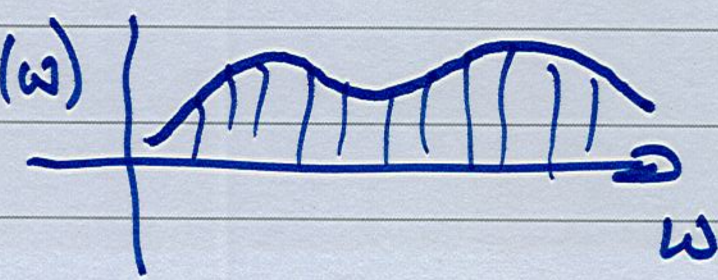


Gibb's phenomenon

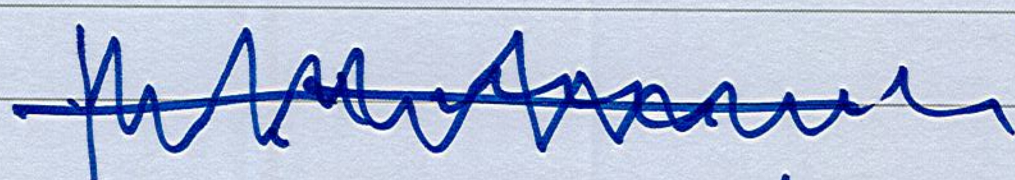
transient



Fourier transforms



Continuous - non-periodic

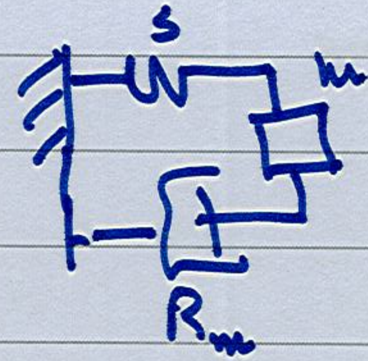


- power spectrum

Summary

Physical system

SDOF



mathematical model

To oscillate: mass & stiffness

Approach to problem solving

1. Governing Equations
   - force
   - motion ($f = ma$)

2. Combine $\rightarrow$ 2nd order ODE

3. Identify possible solutions expressed in a convenient form

4. Find the solutions that satisfy the <u>boundary</u> conditions

Free Response - responds at natural freq.

ω_0

Forced Response

- $f_e(t) = F e^{j\omega t}$

- responds at the driving frequency

$$x(t) = \hat{A} e^{j\omega t}$$

Resonance

- system is driven at a natural frequency

$$\mathrm{Im}\{\hat{z}_m\} = 0 \qquad \hat{z}_m = \frac{\hat{f}_e}{\hat{u}}$$

mechanical impedance

$$\hat{z}_m = R_m + j\left(\omega m - \frac{s}{\omega}\right)$$

2. Vibrating String

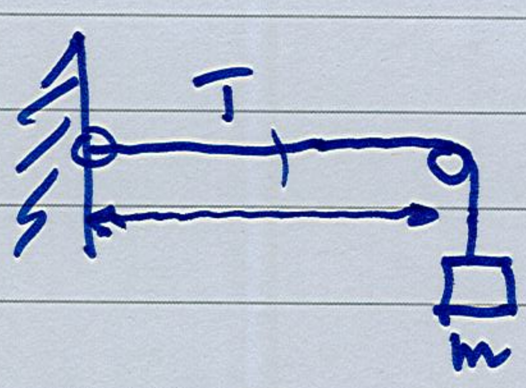
- vibration of extended systems
- derive a wave equation

- transverse wave propagation
    - particle motion $\perp$ to the direction of wave propagation
- phase speed
    - particle velocity
- wave number - spatial freq.

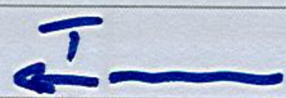
- application of b.c.'s
- characteristic impedance
- <u>standing waves</u>

  vs. <u>propagating waves</u>

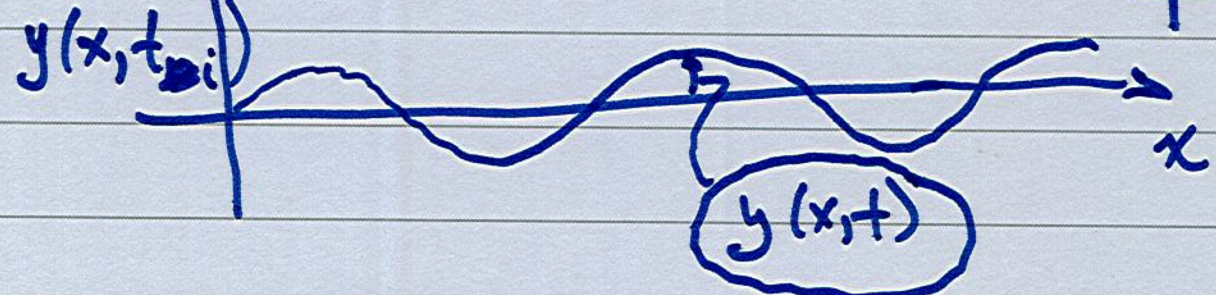
2.1 Derivation of a wave equation
- governing transverse vibration
of a tensioned string



ρ_L mass/unit length
linear density
kg/m



T - tension
force



$y(x,t)$ - instantaneous transverse displacement

<u>Assumptions</u>