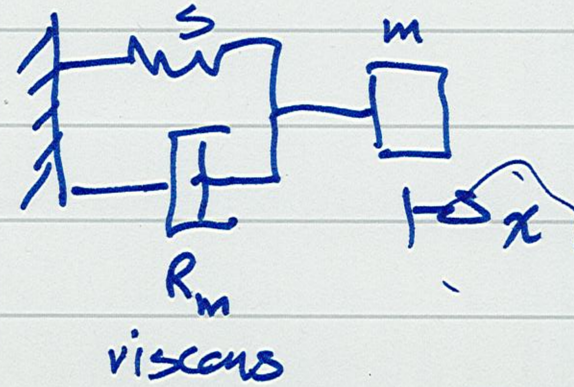
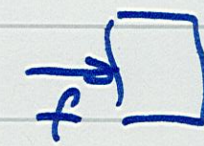
MB 513 Session 3 8/28/15 1

- governing eqns
- wave equation
- possible solutions
- boundary conditions



Restoring Force

$$f = -sx - R_m \frac{dx}{dt}$$



EOM

$$f = m\frac{d^2x}{dt^2}$$

$$\tilde{x} = \tilde{A}e^{j\gamma t}$$

$$\frac{d^2x}{dt^2} + \left(\frac{R_m}{m}\right)\frac{dx}{dt} + \omega_0^2 x = 0$$

$$\gamma = \frac{-\left(\frac{R_m}{m}\right) \pm \sqrt{\left(\frac{R_m}{m}\right)^2 - 4\omega_0^2}}{2}$$

$$\beta = \frac{R_m}{2m}$$

$$\gamma = -\beta \pm \sqrt{\underbrace{\beta^2 - \omega_0^2}}$$

ω_d = damped natural freq

Usually - underdamped

$$\beta < \omega_0$$

$$\gamma = -\beta \pm j\sqrt{\underbrace{\omega_0^2 - \beta^2}_{\text{positive}}}$$

$$\omega_0^2 - \beta^2 = \omega_d^2$$

$$\gamma = -\beta \pm j\omega_d$$

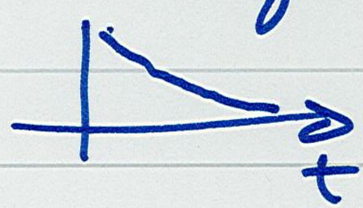
complete solution

$$\tilde{x} = \tilde{A}_1 e^{-\beta t} e^{+j\omega_d t} + \tilde{A}_2 e^{-\beta t} e^{-j\omega_d t}$$

$$= \underbrace{e^{-\beta t}}_{\text{exponential decay}} \underbrace{\left[\tilde{A}_1 e^{+j\omega_d t} + \tilde{A}_2 e^{-j\omega_d t}\right]}_{\text{oscillatory term}}$$

at $t = 0$
$\tilde{x} = \tilde{A}_1 + \tilde{A}_2$

as $t \to \infty$
$\tilde{x} \to 0$



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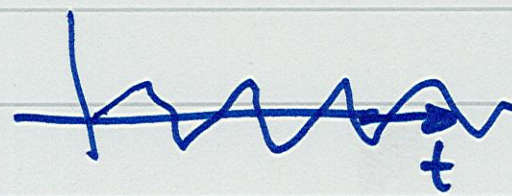
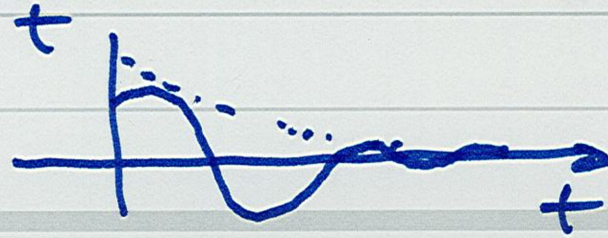
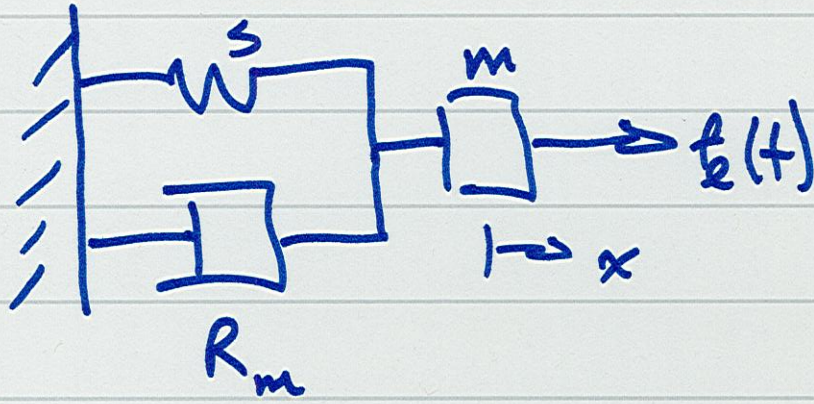


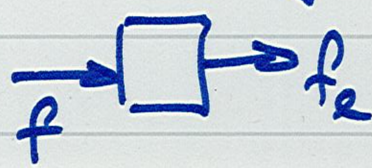
$Re\{\tilde{x}(t)\}$



1.3 Forced Oscillation



1.3.1 Governing Eqns



EOM

$$f + f_e = m\frac{d^2x}{dt^2} \quad (1)$$

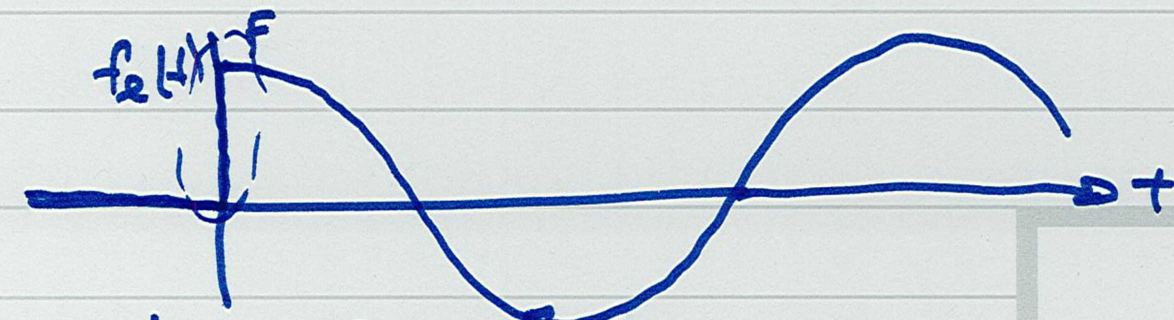
Restoring force

$$f = -sx - R_m\frac{dx}{dt} \quad (2)$$

$$\boxed{m\frac{d^2x}{dt^2} + R_m\frac{dx}{dt} + s\,x = f_e(t)} \qquad (3)$$

inhomogeneous ODE

Example $\quad f_e(t) = F\cos \underline{\underline{\omega}} t \quad t \geq 0$



Laplace Transform

$\rightarrow$ algebraic solution in transform space.

$x = x_{\text{transient}} + \underline{x_{\text{steady state}}}$

~ homogeneous eqn

for real systems
since $R_m > 0$ always

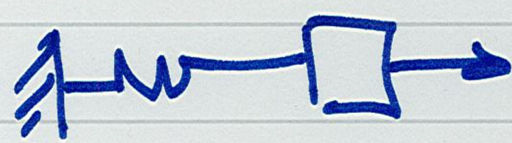
transient sol'n decays to negligible
values as t increases

1.3.2 Steady State Solution

Assume complex form for the driving force

$\tilde{\hat{f}}_e = Fe^{j\omega t}$ ↑ driving frequency.

$Re\{\tilde{\hat{f}}_e(t)\} = F\cos\omega t$ if F = real



$\tilde{\hat{f}}_e(t) = Fe^{j\omega t}$

Linear Systems

- in a steady state the responds at the driving freq.

ω - forcing freq
- you control

ω_0 - natural freq
- property of a system

Assumed solution

$$\hat{f}_e = F e^{j\omega t} \qquad \hat{x} = \hat{A} e^{j\omega t}$$

sub into eqn (3)

$$-\omega^2 m \hat{A} \cancel{e^{j\omega t}} + j\omega R_m \hat{A} \cancel{e^{j\omega t}} + s \hat{A} \cancel{e^{j\omega t}} = F e^{j\omega t}$$

$$\hat{A} = \frac{F}{-\omega^2 m + j\omega R_m + s}$$

$$= \frac{F}{j\omega\left(R_m + j\left(\omega m - \frac{s}{\omega}\right)\right)}$$

$$\tilde{x} = \tilde{A} e^{j\omega t}$$

$$= \frac{F e^{j\omega t}}{j\omega\left(R_m + j\left(\omega m - \frac{s}{\omega}\right)\right)}$$

steady-state response

Physical Displacement

$$x(t) = \mathrm{Re}\{\tilde{x}(t)\}$$

Velocity: $\hat{u} = \frac{d\tilde{x}}{dt} = j\omega\tilde{x}$

Acceleration: $\hat{a} = \frac{d^2\tilde{x}}{dt^2} = \frac{d\hat{u}}{dt} = -\omega^2\tilde{x} = j\omega\hat{u}$

$$\hat{u} = \frac{F e^{j\omega t}}{R_m + j\left(\omega m - \left(\frac{s}{\omega}\right)\right)}$$

progressive plane

$$\left|\frac{p}{u}\right| = \rho_0 c$$

$$|u| = \frac{|p|}{\rho_0 c} \quad \text{say} \quad p = 1\ \text{Pa}$$

94 dB

$$= 0.0025\ \text{m/s}$$

$$\omega |x| = |u|$$

$$|x| = \frac{|u|}{\omega} = \frac{2.5 \times 10^{-3}}{2\pi \times 1 \times 10^{3}}$$

f=1000 Hz

$$\simeq 0.5 \times 10^{-6}\ \text{m}$$

1.3.3 Mechanical Impedance

Define $\tilde{Z}_m = \dfrac{\text{complex driving force}}{\text{complex velocity}}$ at the driven point

$$= \frac{\tilde{f}_e}{\tilde{u}}$$

Damped SDOF

$$\tilde{Z}_m = \frac{\cancel{F e^{j\omega t}}}{\dfrac{\cancel{F e^{j\omega t}}}{R_m \cancel{E} + j\left(\omega m - \frac{s}{\omega}\right)}}$$

$$= R_m + j\left(\omega m - \frac{s}{\omega}\right)$$

$$\tilde{z}_m = R + jX$$

mechanical resistance (R) mechanical reactance (X)

$$R = \mathrm{Re}\{\tilde{z}_m\}$$

$$X = \mathrm{Im}\{\tilde{z}_m\}$$

$$\tilde{z}_m = \underline{R_m} + j\left(\omega \underline{m} - \frac{\underline{s}}{\omega}\right)$$