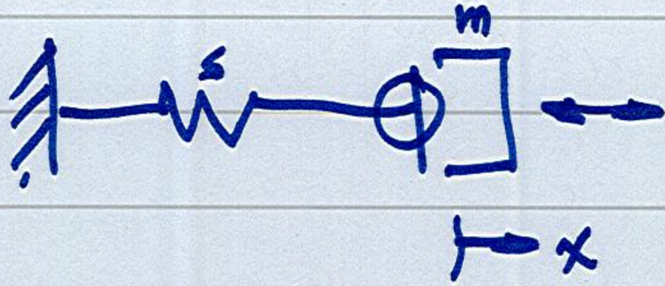
1. Fundamentals of Vibration



$f = m\frac{d^2x}{dt^2}$

$f = -sx$

$\frac{d^2x}{dt^2} + \omega_0^2 x = 0$

$\omega_0 = \sqrt{\frac{s}{m}}$

$x = A_1 \cos \omega_0 t$ $\quad$ $x = A_2 \sin \omega_0 t$

$\gamma = \omega_0$ $\quad$ ω_0 [rad/s]

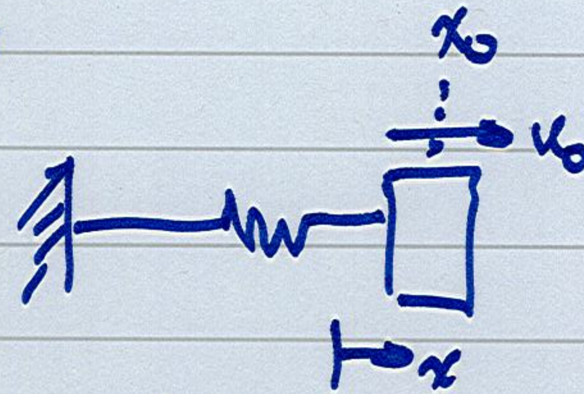
Complete $\quad$ $f_0 = \frac{\omega_0}{2\pi}$ [Hz]

$x = \hat{A}_1 \cos \omega_0 t + \hat{A}_2 \sin \omega_0 t$

1.1.3 Initial conditions (boundary conditions in time)

2 constants A_1 & A_2

2 independent conditions



e.g. at $t=0$

$x = x_0$ displacement

$\frac{dx}{dt} = \dot{x} = u_0$ velocity

at $t=0$

since $\cancel{A}$, $x = A_1 \cos \omega_0 t + A_2 \sin \omega_0 t$

at $t=0$ $\quad x = A_1$

$A_1 = x_0$

Velocity $u = \dot{x} = \frac{dx}{dt} = -\omega_0 A_1 \sin \omega_0 t + \underline{\omega_0 A_2} \cos \omega_0 t$

at $t = 0$ $\dot{x} = \omega_0 A_2 = u_0$

$$A_2 = \frac{u_0}{\omega_0}$$

Complete Solution

$$x = x_0 \cos \omega_0 t + \frac{u_0}{\omega_0} \sin \omega_0 t \quad (4)$$

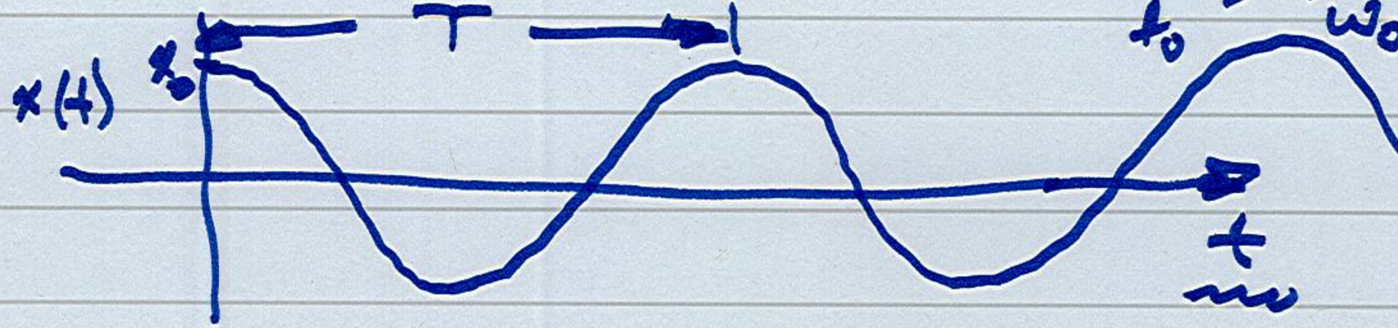
$$\omega_0 = \sqrt{\frac{s}{m}}$$

if initial velocity $= 0$

T = period

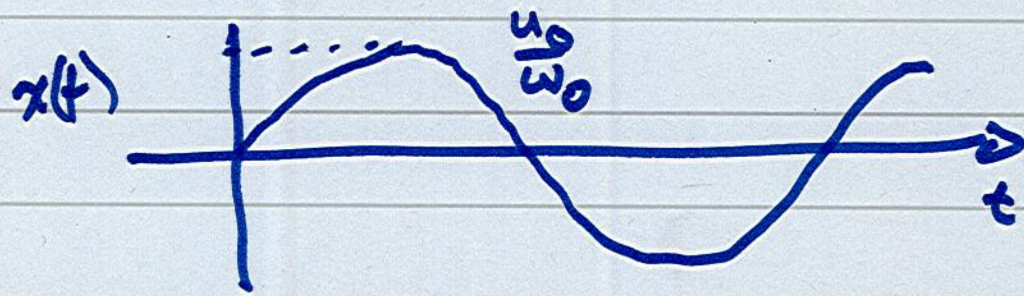
$x = x_0 \cos \omega_0 t$

$= \frac{1}{f_0} = \frac{2\pi}{\omega_0}$



if initial displacement $= 0$

$x = \frac{u_0}{\omega_0} \sin \omega_0 t$

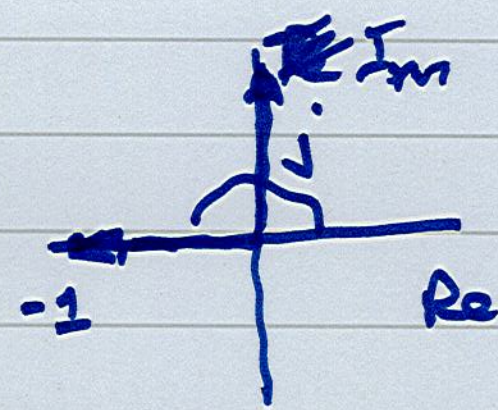


complete response - sum of the two.

1.1.4 complex solution

$$\boxed{\tilde{x} = \tilde{A} e^{\gamma t}}$$

Assumed solution



$\hat{x} = x_r + j x_i \qquad j = \sqrt{-1}$

$\tilde{A} = a + j b$

$Re\{\tilde{x}\} \rightarrow$ physical soln

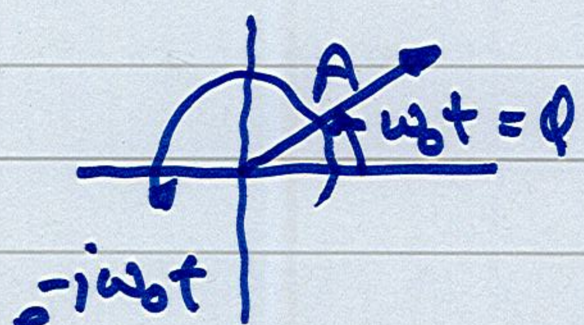
(3) $\frac{d^2\tilde{x}}{dt^2} + \omega_0^2 \tilde{x} = 0$ $\quad \tilde{x} = \tilde{A} e^{\gamma t}$

$\gamma^2 \cancel{\tilde{A} e^{\gamma t}} + \omega_0^2 \cancel{\tilde{A} e^{\gamma t}} = 0$

$\gamma^2 = -\omega_0^2$ true if

$\gamma = \sqrt{-\omega_0^2} = \sqrt{-1}\,\sqrt{\omega_0^2}$

$= \pm j\omega_0$

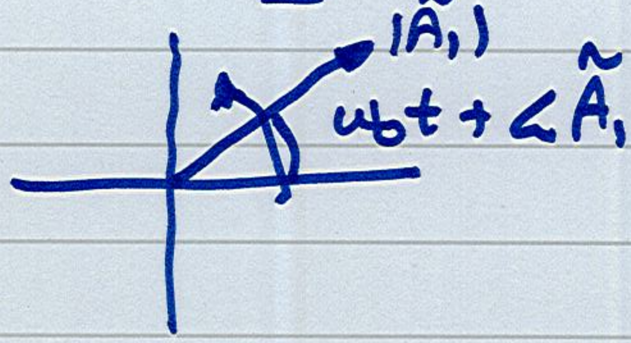


$A e^{j\omega_0 t}$

$e^{j\phi}$

complete solution

$$x = \underline{\tilde{A}_1} e^{j\omega_0 t} + \underline{\tilde{A}_2} e^{-j\omega_0 t} \qquad \tilde{A}_1 \text{ \& } \tilde{A}_2$$



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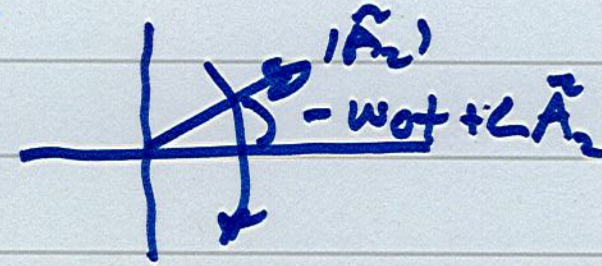

Determine $\hat{\tilde{A}}_1$ & $\hat{\tilde{A}}_2$ by applying initial conditions

at $t=0$ $\quad \tilde{x} = x_0 = \hat{A}_1 + \hat{A}_2$ $\qquad e^{\pm j\omega_0 t}$

$\dot{\tilde{x}} = u_0 = j\omega_0 \hat{\tilde{A}}_1 - j\omega_0 \hat{\tilde{A}}_2$

2 eqns in 2 unknowns

$\hat{A}_1 = \frac{1}{2}\left(x_0 - j\frac{u_0}{\omega_0}\right)$

$\hat{A}_2 = \frac{1}{2}\left(x_0 + j\frac{u_0}{\omega_0}\right)$

$\hat{\tilde{A}}_2 = \tilde{\hat{A}}_1^{*}$ conjugate so that

$$\left.\begin{aligned}\tilde{A}_1 &= a + jb \\ \tilde{A}_2 &= a - jb\end{aligned}\right]$$ just 2 real constants

Complete solution

$$\tilde{x} = \frac{1}{2}\left(x_0 - j\frac{u_0}{\omega_0}\right)\overbrace{e^{j\omega_0 t}}^{\cos\omega_0 t + j\sin\omega_0 t} + \frac{1}{2}\left(x_0 + j\frac{u_0}{\omega_0}\right)\overbrace{e^{-j\omega_0 t}}^{\cos\omega_0 t - j\sin\omega_0 t}$$

$$\tilde{x} = x_0\cos\omega_0 t + \frac{u_0}{\omega_0}\sin\omega_0 t$$

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same soln as before

Real part of soln → physical

$$x(t) = \mathrm{Re}\{\tilde{x}(t)\}$$

$$\tilde{x} = \underline{\tilde{A} e^{j\omega_0 t}} \quad \text{displacement}$$

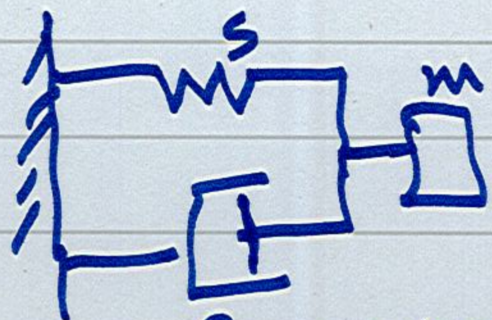
$$\frac{d\tilde{x}}{dt} = \dot{\tilde{x}} = \tilde{u} = j\omega_0 \tilde{A} e^{j\omega_0 t} = j\omega_0 \tilde{x}$$

$$\frac{d^2\tilde{x}}{dt^2} = \ddot{\tilde{x}} = \tilde{a} = -\omega_0^2 \tilde{A} e^{j\omega_0 t} = -\omega_0^2 \tilde{x}$$

$$e^{j\underbrace{\omega_0 t}} \quad e^{j\phi} \quad \phi = \omega_0 t$$

$$\frac{d\phi}{dt} = \omega_0 = \text{time rate of change of the phase}$$

1.2 Damped Oscillations



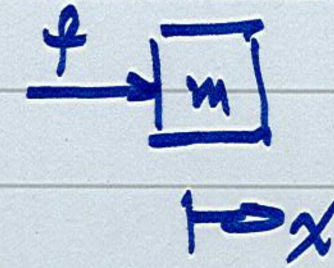
R_m ~ viscous damper

force ~ velocity

most systems dissipate energy as they vibrate

1.2.1 Governing Eqns

(i) Restoring force



$$f = -sx - R_m \frac{dx}{dt} \quad (1)$$

(ii) EOM

$f = ma$

$$f = m \frac{d^2x}{dt^2} \quad (2)$$

(1) → (2)

$$m\frac{d^2x}{dt^2} + R_m\frac{dx}{dt} + sx = 0$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{R_m}{m}\frac{dx}{dt} + \omega_0^2 x = 0} \qquad \omega_0 = \sqrt{\frac{s}{m}}$$

1.7.2 Soln

$$\tilde{x} = \tilde{A}e^{\gamma t}$$

result

$$\gamma^2 \cancel{\tilde{A} e^{\gamma t}} + \gamma\left(\frac{R_m}{m}\right)\cancel{\tilde{A} e^{\gamma t}} + \omega_0^2 \cancel{\tilde{A} e^{\gamma t}} = 0$$

$$\gamma^2 + \left(\frac{R_m}{m}\right)\gamma + \omega_0^2 = 0$$

$$\gamma = \frac{-\left(\frac{R_m}{m}\right) \pm \sqrt{\left(\frac{R_m}{m}\right)^2 - 4\omega_0^2}}{2}$$

let $\frac{R_m}{2m} = \beta$

$$\gamma = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

$\beta = 0 \qquad \gamma = \pm j\omega_0$ same as before