Final Exam

Tues 12/10/13 FRNY G124 7:00 - 9:00 PM

6 Questions

1. Short Answers *
2. String
3. Sound Fields
4. Reflection
5. Sources
6. Duct

(vii) When considering sound transmission through a limp barrier, doubling either the mass/unit area freq causes the transmission loss of the barrier to increase by 6 dB.

mass law

(viii) When can a reflecting surface be modeled as being "locally reacting"?

$z_n \neq f(\theta)$

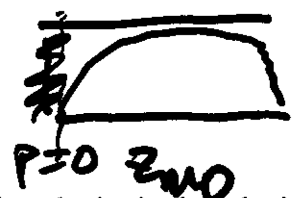
(ix) An unbaffled loudspeaker can be modeled as a dipole.

(x) When a point monopole is placed at the junction of three rigid, perpendicular surfaces, 7 image sources are required to satisfy the hard wall boundary conditions.

(xi) Consider a piston of area S and oscillatory velocity U in a rigid baffle. When the piston radius is very small compared to a wavelength, the piston source may be modeled as a monopole in free space having source strength $2US$.

(xii) In a public address system, why is it normal to use many high frequency drivers, and a relatively small number of low frequency drivers?

(xiii) The first plane wave resonance of a open-ended tube occurs when the tube is approximately what fraction of a wavelength long? $\lambda/2$



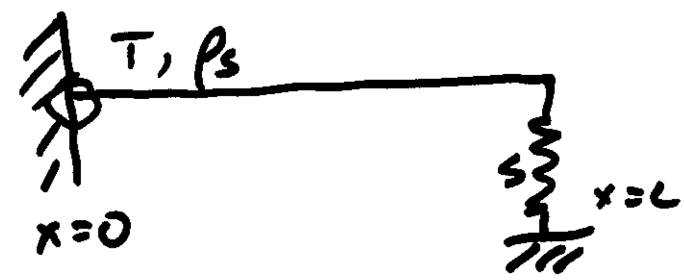
(xiv) A duct terminates in an unflanged opening. The acoustic particle velocity in the tube is approximately a maximum at the opening.

(xv) Acoustic loading of a loudspeaker usually causes the natural frequency of the loudspeaker to be reduced.

Problem 2.

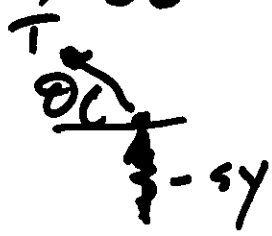
A uniformly tensioned string of finite length (tension T and mass per unit length ρ_s) is attached to a rigid support at $x = 0$, and at $x = L$ it is transversely constrained by a spring of stiffness s.

(i) Give the general complex harmonic form for the transverse displacement of the string.
(ii) Give the boundary conditions at both ends of the string.
(iii) By applying the boundary conditions, derive the transcendental characteristic equation (written in terms of kL, where k is the wave number for transverse wave motion on the string) that could be solved to give the natural frequencies of the system.
(iv) Sketch both sides of the characteristic equation as a function of kL, and show how the natural frequencies could be located graphically.



$$y = Ae^{-ikx}e^{i\omega t} + Be^{ikx}e^{i\omega t}$$

$y(0,t) = 0$



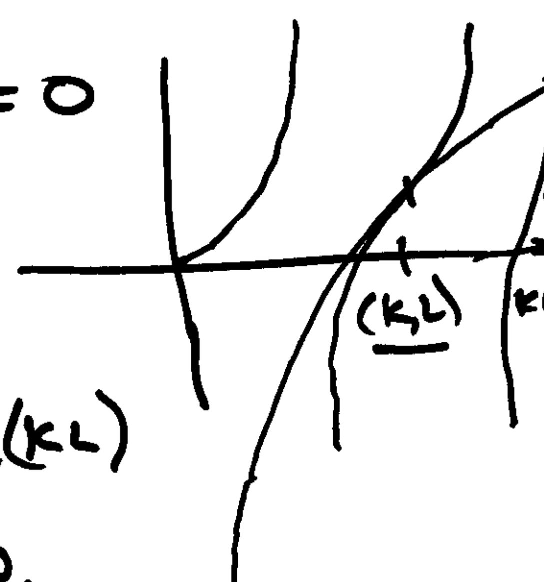
$\Sigma f_y = 0$

$$T\sin\theta|_{x=L} - sy|_{x=L} = 0$$

$$-T\frac{\partial y}{\partial x}\Big|_{x=L} - sy|_{x=L} = 0$$

$$\tan(kL) = \_\_\_(kL)$$

$k_1 \to \omega_1$



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Problem 5.

A circular rigid piston in a rigid baffle radiates into air at 100 Hz. The radius of the piston is 0.01 m.

$ka \ll 1$

(i) Calculate the *displacement* amplitude of the piston required to produce a sound pressure level of 90 dB re 20 μPa at a distance 2 m in front of the piston. Make use of whatever simplifying assumptions you feel appropriate under these conditions (but justify your assumptions). Comment on why such a large displacement amplitude is required in this case.

(ii) By using the appropriate form of the radiation impedance, calculate the sound power radiated by the piston.

$$p \sim j\rho_0 c \frac{kQ}{2\pi r} e^{j(\omega t - kr)}$$

$$L_p = 90\text{ dB re } 20\mu\text{Pa} \qquad L_p = 10\log\frac{p_{rms}^2}{p_{ref}^2}$$

$$p_{rms}^2 = p_{ref}^2\, 10^{L_p/10}$$

note mistake in class - should be $Q = U\pi(0.01)^2$ since 0.01 is the radius

$$p_{rms}^2 = \frac{pp^*}{2} = \sim\sim Q^2 \quad \text{solve for } Q$$

$$Q = U\pi\frac{(0.01)^2}{4}$$

$$u = j\omega\xi$$

$$|\xi| = \frac{|u|}{\omega}$$

$$\Pi = \frac{1}{2}U_0^2 R_r \qquad ka \ll 1$$

$$R_r = \frac{1}{2}\rho_0 c S (ka)^2$$

radius

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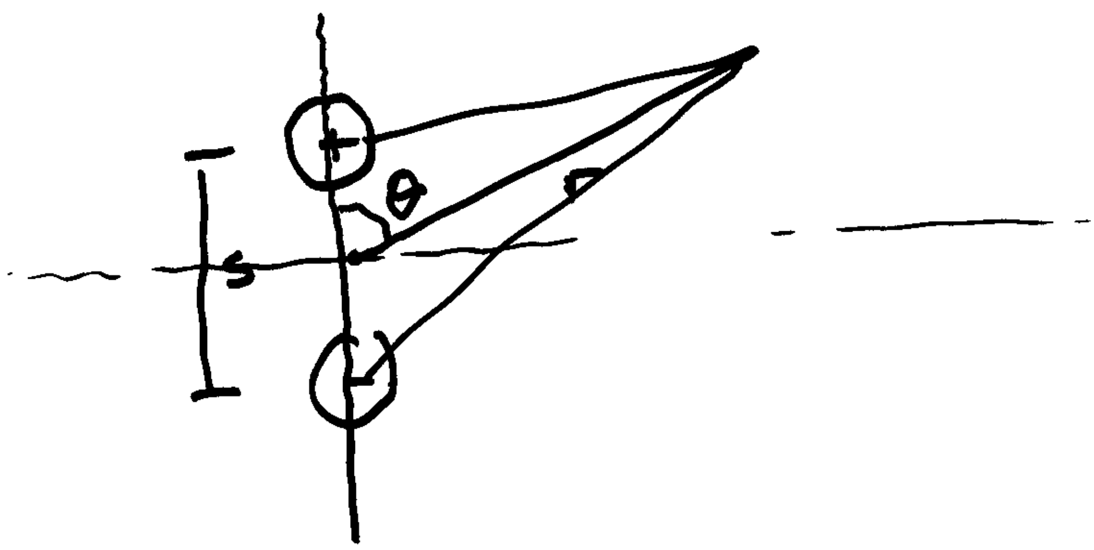
Problem 4.

A dipole can be considered to consist of two monopoles of equal strength operating 180 deg. out-of-phase with each other. The sound field radiated by the dipole is zero on the plane defined by $\theta = \pi/2$, where θ is the polar angle measured from the dipole axis. However, it may be desirable that the sound field be zero on some other plane.

So, imagine that the phase, φ, of the first of the two monopoles that make up the dipole is set to $\pi/4$: i.e., the sound field radiated by the first monopole is $(A/r_1)e^{-jkr_1}e^{j\pi/4}$.

By following an approach similar to that used to derive the farfield of a dipole, find an expression for the polar angle at which the radiated sound pressure is zero in this case.



$$\tilde{p}(r) = \frac{A_1}{r_1} e^{-jkr_1} e^{j\frac{\pi}{4}} + \frac{A_2}{r_2} e^{-jkr_2}$$

$$r \gg s \qquad A_2 = -A_1 = -A$$

$$\tilde{p}(r) = \frac{A}{r-\Delta} e^{-jk(r-\Delta)} e^{j\frac{\pi}{4}} - \frac{A}{r+\Delta} e^{-jk(r+\Delta)} \qquad \Delta = \frac{s}{2}\cos\theta$$

$$= A e^{j\frac{\pi}{8}} \left(\frac{e^{-jk(r-\Delta)}}{r-\Delta} e^{j\pi/8} - \frac{e^{-jk(r+\Delta)}}{r+\Delta} e^{-j\frac{\pi}{8}} \right)$$

$$\Delta^2 \ll r^2$$

$$\Delta \ll r$$

$$\tilde{p}(r) = \frac{A}{r} e^{-ikr} e^{j\frac{\pi}{8}} \underline{\left\{ e^{j(k\Delta + \frac{\pi}{8})} - e^{-j(k\Delta + \frac{\pi}{8})} \right\}}$$

$$= 2j \frac{A}{r} e^{-jkr} e^{j\pi/8} \underbrace{\sin\left(k\Delta + \frac{\pi}{8}\right)}_{\Delta = \frac{s}{2}\cos\theta}$$

$$\sin\left(k\Delta + \frac{\pi}{8}\right) = 0$$

$$k\Delta + \frac{\pi}{8} = n\pi$$

$$\theta = \cos^{-1}\left(\frac{2\pi}{ks}\left(n - \frac{1}{8}\right)\right)$$

Problem 5.

A loudspeaker is placed at one end of a duct of constant cross-sectional area S that is terminated at $x = L$ by a massless piston connected to a pure stiffness reactance as shown (note that you may assume that there is a vacuum outside the tube: i.e., there is no acoustic loading applied to the exterior of either the massless piston or the loudspeaker diaphragm). The loudspeaker diaphragm has a mass m, and its suspension has been designed to provide negligible stiffness and damping. The delivery of a voltage to the loudspeaker voice coil causes an oscillatory force to be applied to the diaphragm as shown below.

(i) Give an expression for the termination impedance, Z_{mL}, provided by the combination of the massless piston and the spring.

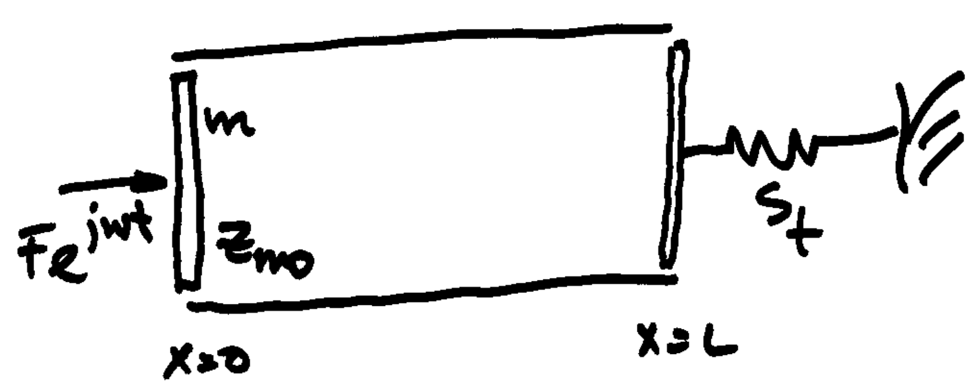
(ii) Give an expression for the acoustic loading, Z_{m0}, acting on the loudspeaker in this case. Give the result in terms of kL and the ratio of the termination stiffness to the stiffness of the air in the duct: i.e., the ratio $b = s_t/(S\rho_0 c^2/L)$.

(iii) Give a simplified expression for Z_{m0} that is valid when $kL << 1$.

(iv) Also under the condition $kL << 1$, give an expression for the total impedance experienced by the force acting on the loudspeaker diaphragm.

(v) Show that when $kL << 1$, the natural frequency of the acoustically-loaded loudspeaker diaphragm is:

$$\omega = \sqrt{\frac{s_t}{(1+b)}\frac{1}{m}}$$



(i) $-j\frac{s_t}{\omega} = Z_{mL}$

(ii) $$\underline{Z_{m0}} = \rho_0 c S \left[\frac{\frac{Z_{mL}}{\rho_0 c S} + j \tan kL}{1 + j \frac{Z_{mL}}{\rho_0 c S} \tan kL} \right] \qquad b = \frac{s_f}{\left(\frac{s \rho_0 c^2}{L}\right)}$$

$$= \rho_0 c S \left[\frac{-j\frac{b}{kL} + j \tan kL}{1 + \frac{b}{kL} \tan kL} \right]$$

(iii) $kL \ll 1$

$$Z_{m0} \simeq \rho_0 c S \left[\frac{-j\frac{b}{kL} + j kL}{1 + b} \right]$$

(iv) $$z_{m_{total}} = j\omega m + \rho_0 c S \left[\qquad \right]$$

(v) ~~From~~ finding the frequency that makes

$$Im\{z_{m_{total}}\} = 0$$

$$z_{m_{total}} = \frac{j\omega m(1+b) - j\rho_0 c \frac{Sb}{kL} + j\rho_0 c S kL}{1+b}$$

$$\frac{c}{L}\,\omega m \frac{L}{c}(1+b) - \rho_0 c \frac{Sb}{kL}\Big|_{\omega/c} + \rho_0 c S kL\Big|_{\omega/c} = 0$$