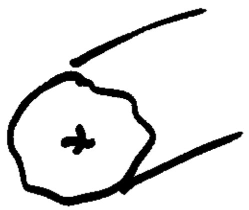
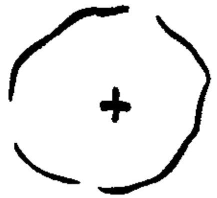
3. Wave Equation & simple solutions

wave equation $c = \sqrt{\frac{\gamma P_0}{\rho_0}}$

- plane
- cylindrical
- spherical



$p \propto \frac{1}{r^{1/2}}$



$p \propto \frac{1}{r}$

linearized momentum eqn

$u_x = -\frac{1}{j\omega\rho_0}\frac{\partial p}{\partial x}$ relates p & $\{\bar{u}\}$

vector $\bar{u} = u_x\bar{i} + u_y\bar{j} + u_z\bar{k}$

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \qquad k = \beta - j\alpha$$

$$e^{-jkx} \qquad e^{-j\beta x} e^{-\alpha x}$$

$$u \propto \frac{\partial p}{dx} \qquad \boxed{-jke^{-jkx}}^{*} \qquad +jk^{*}e^{+jk^{*}x}$$

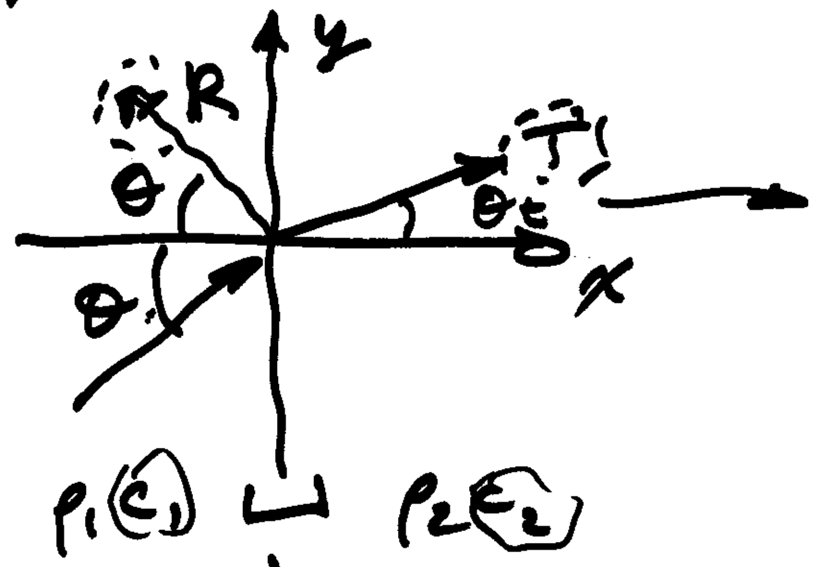
$$k^{*} = \beta + j\alpha$$

$$p \propto \underbrace{A}_{\text{complex}} e^{-jkx} \qquad I_x = \frac{1}{2}\mathrm{Re}\{p u_x^{*}\}$$

$$\frac{pp^{*}}{2} \qquad \frac{AA^{*}}{2} = |A|^2 \neq A^2$$

$$\bar{I} = I_x \bar{i} + I_y \bar{j} + I_z \bar{k}$$

4. Reflection & Transmission



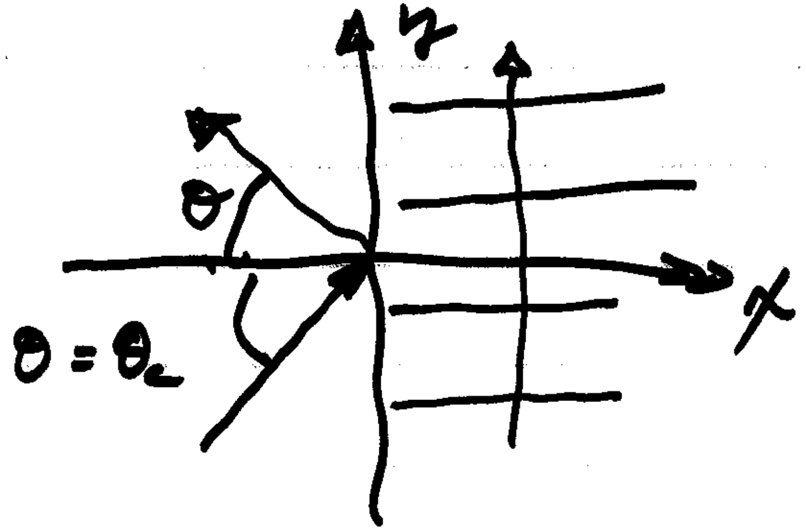
Snell's Law

2 b.c.'s

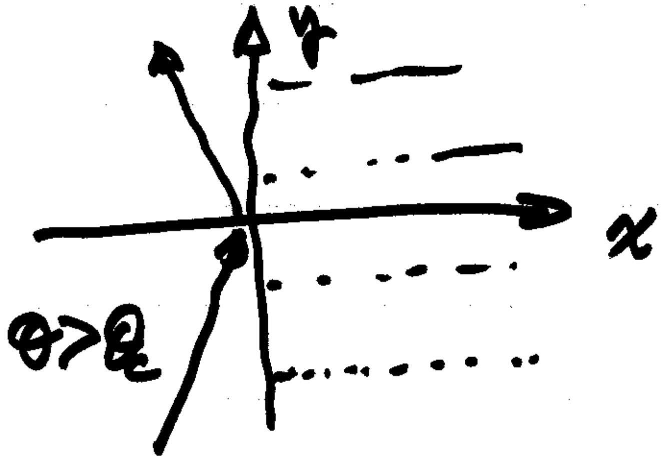
$p_1 = p_2$

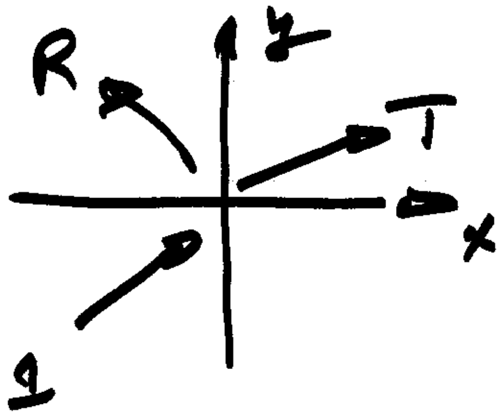
$u_{1x} = u_{2x}$

$c_2 > c_1$



θ_c = critical angle

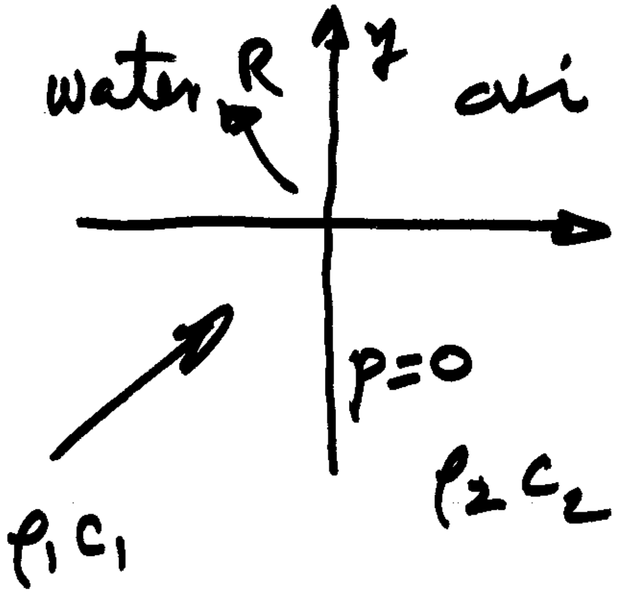




$$R + T \neq 1$$

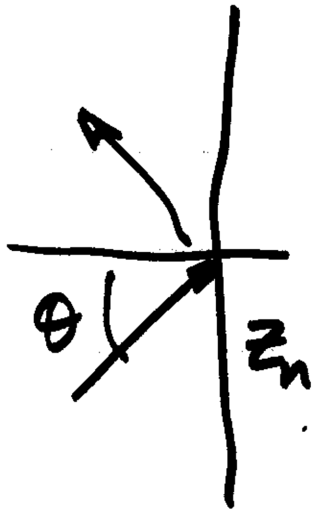
$$R_\pi + T_\pi = 1$$

Pressure Release B.C.



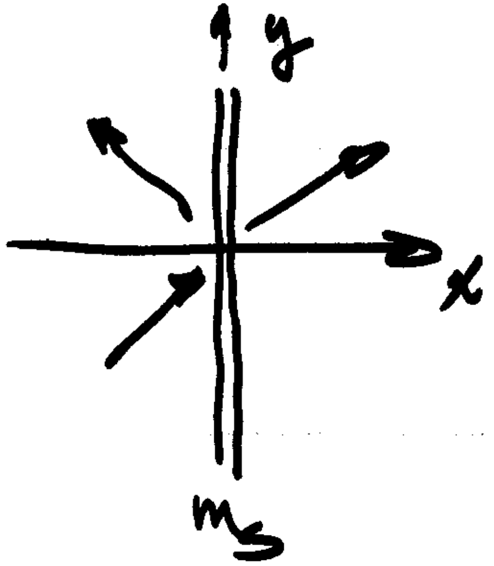
$$R \to -1$$

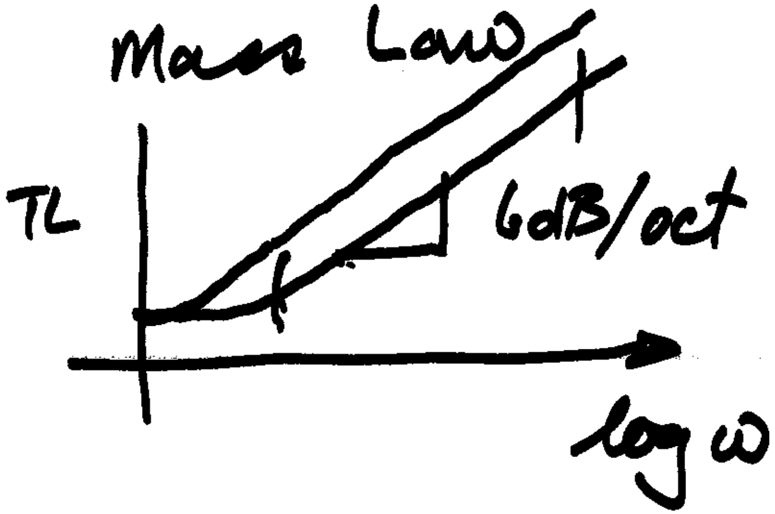
$$\rho_2 c_2 << \rho_1 c_1$$



$\rightarrow R$

$Z_n \neq f(\theta)$ surface of local reaction





$\omega\, m_s$

5. Sources

Compact sources $D \ll \lambda$ $kD \ll 1$

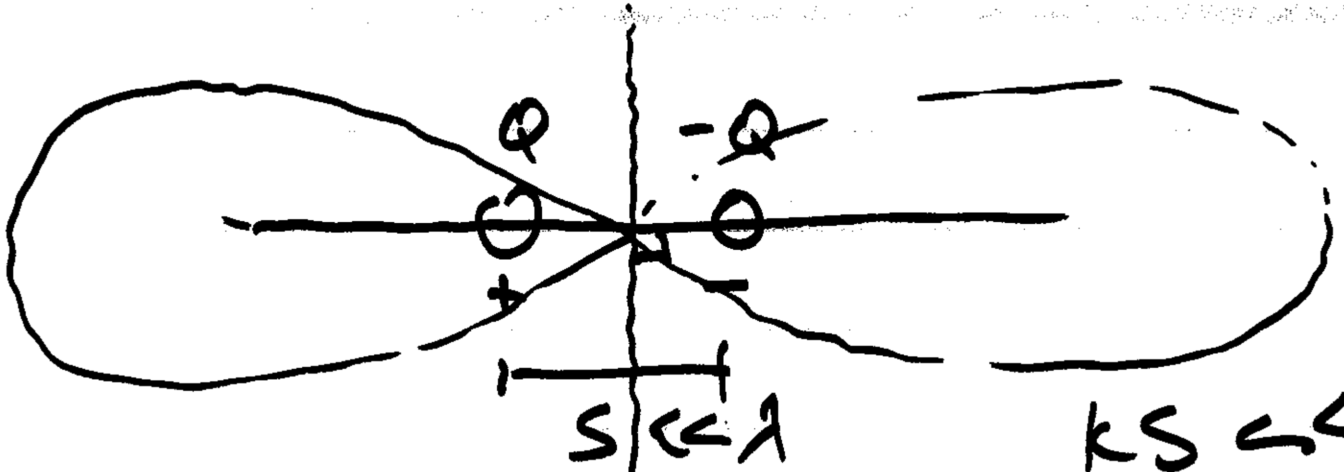
Simple Sources

- monopole - volume change, mass addition
- dipole - force - unbaffled l/s
- quadrupole - moment - turbulence

Q volume source strength $D \ll \lambda$

$$p = j\rho_0 c \frac{kQ}{4\pi} e^{-jkr}$$

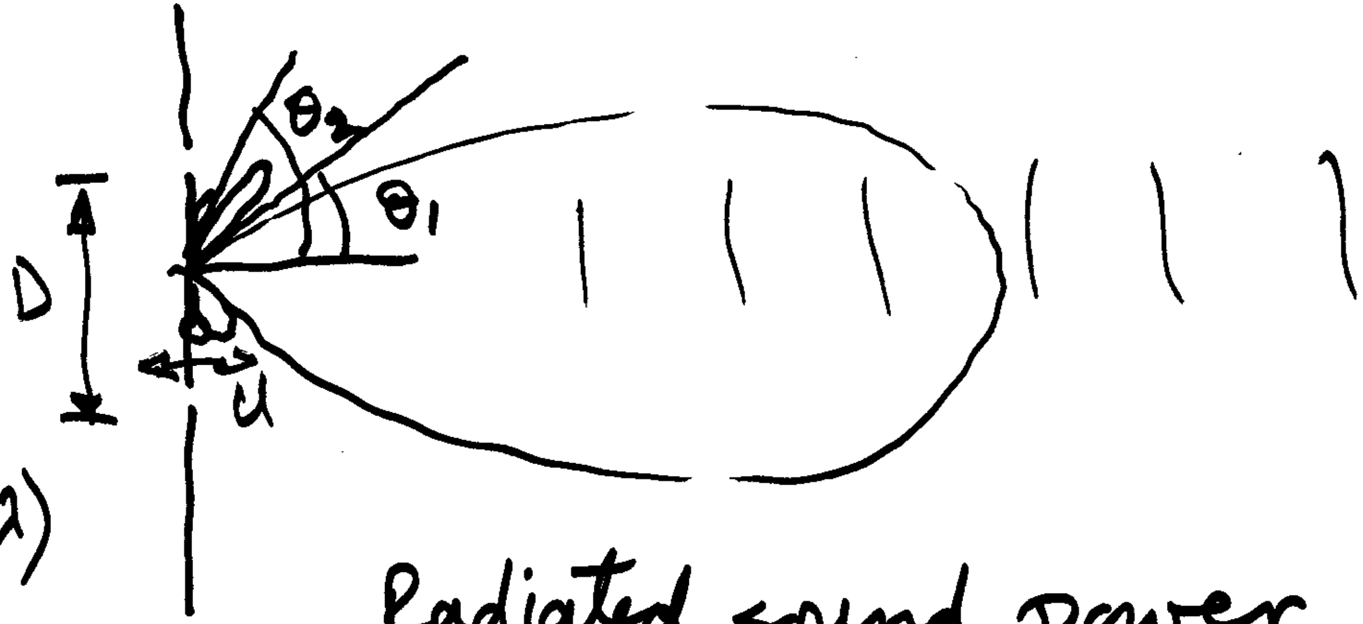
$$Q = \int u_n dS$$



$kS << 1$

Dipole strength (moment) = QS

$$\frac{|P_{dipole}|}{|P_{monopole}|} << 1$$

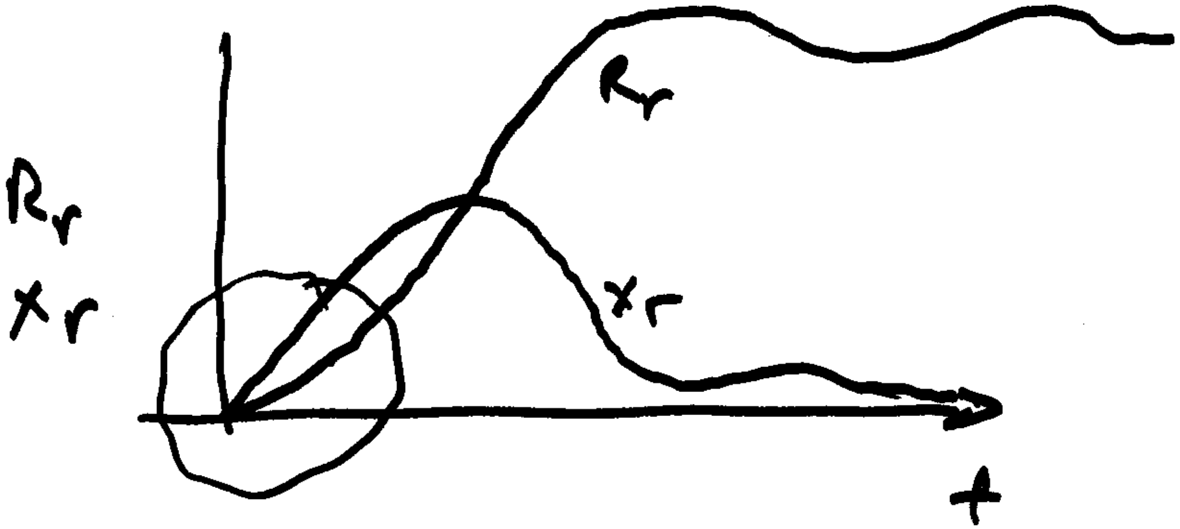


$D \sim O(\lambda)$

Radiated sound power

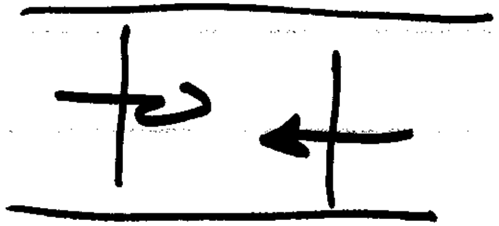
→ Radiation Impedance

$$\underset{\underset{\text{power}}{\uparrow}}{R_r} + j\underbrace{X_r}_{\text{mass loading}}$$

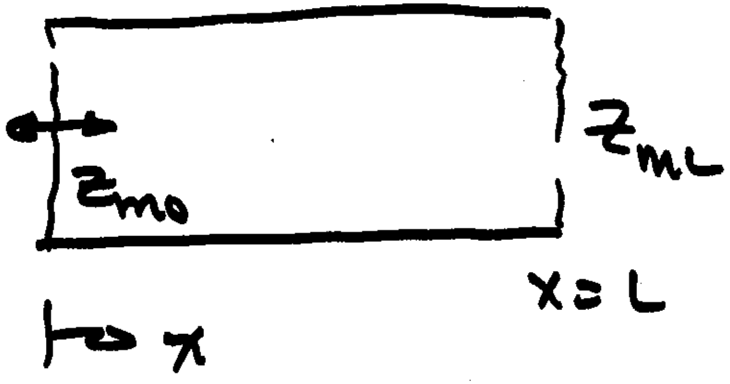


small sources are inefficient radiators

Duct - plane waves



low freq.

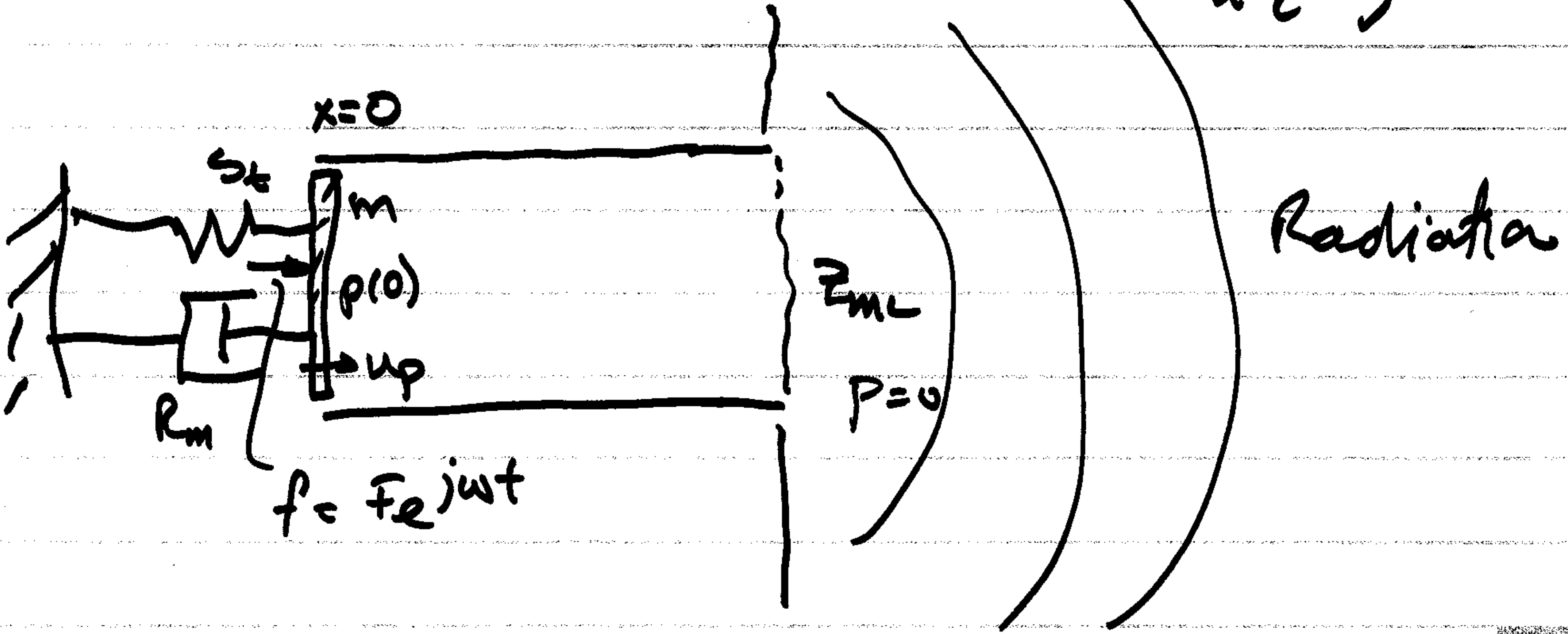


$$\underset{\sim}{Z_{mo}} = \rho_0 c S \; \frac{\frac{Z_{mL}}{\rho_0 c S} + j \tan kL}{\underline{1} + j\left(\frac{Z_{mL}}{\rho_0 c S}\right) \tan kL}$$

impedance transfer

Acoustic Resonances

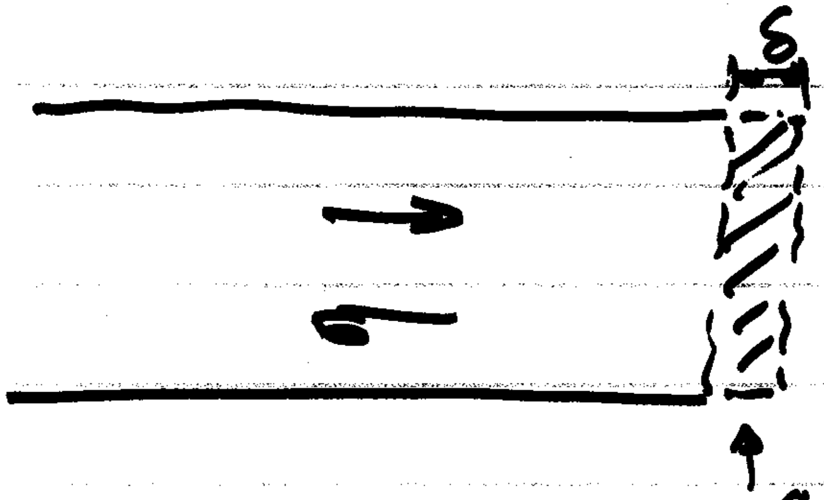
$\mathrm{Im}\{Z\} \rightarrow 0$



$$Z_{total} = \frac{F}{u_p} = Z_{ml} + Z_{m0}$$

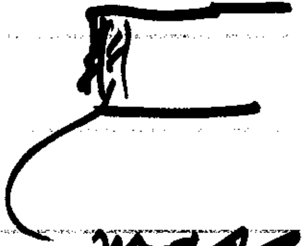
$$\mathrm{Im}\{Z_{total}\} = 0$$

End Correction

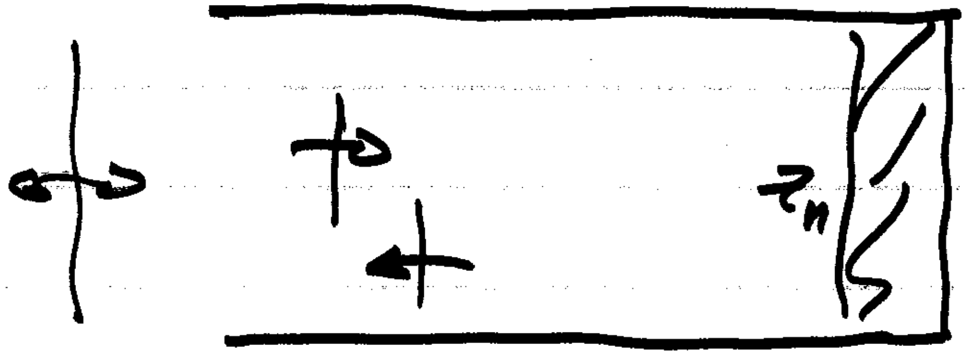


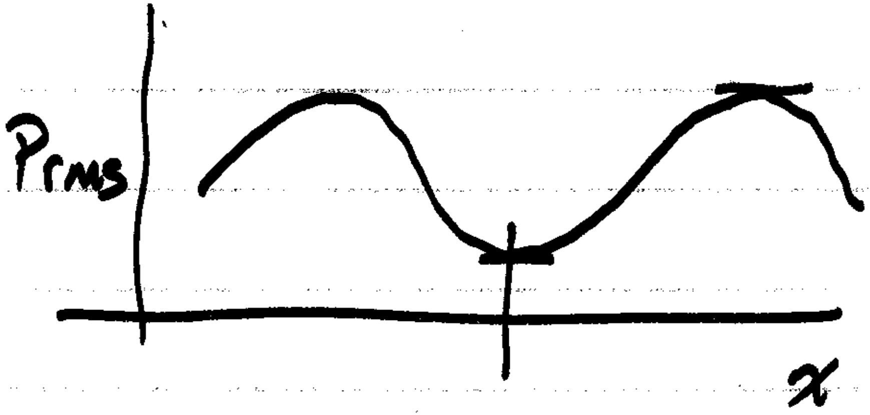
$$L' = L + \delta$$

natural frequencies are decreased



mass loading → decrease in natural freq.





$R \rightarrow z_n$