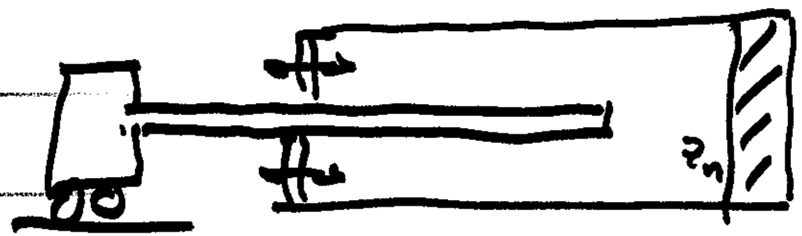
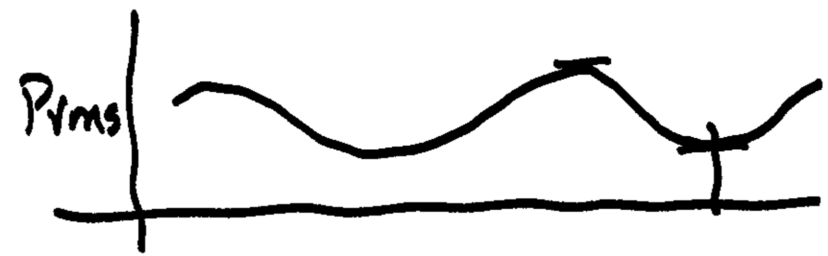
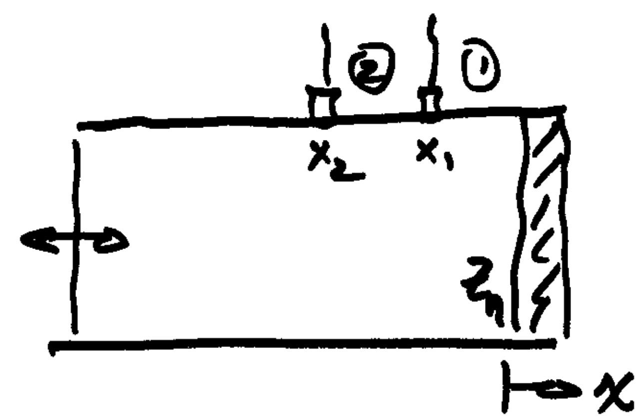




$$R \rightarrow \alpha_0$$
$$\rightarrow z_n$$

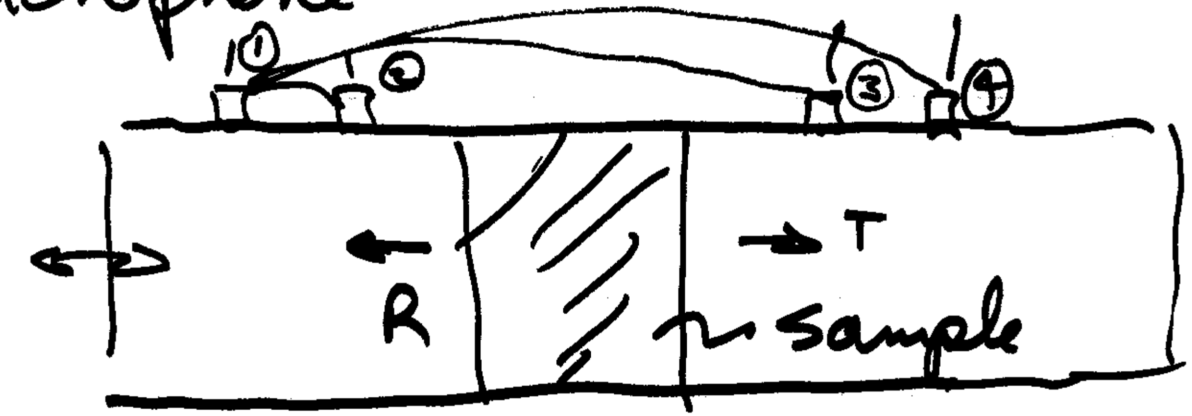


ASTM E1050

$$H_{12} = \frac{\tilde{p}(x_1)}{\tilde{p}(x_2)}$$

$$R = \frac{H_{12} e^{-jkd_2} - e^{+jkd_1}}{e^{-jkd_1} - H_{12} e^{-jkd_2}} \rightarrow z_n$$

four microphone

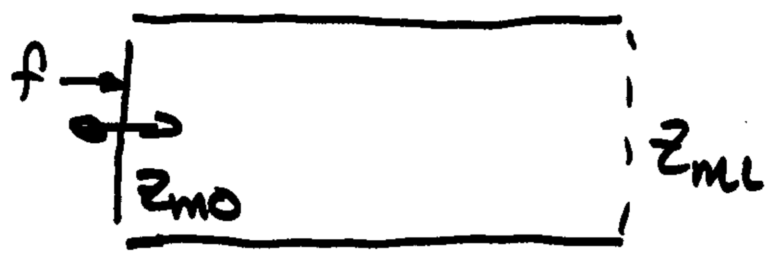


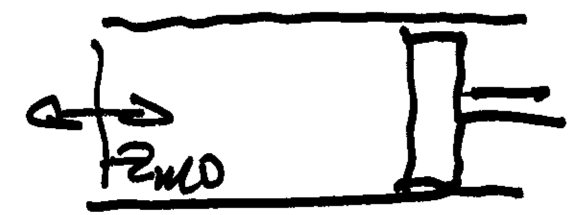
arbitrary termination

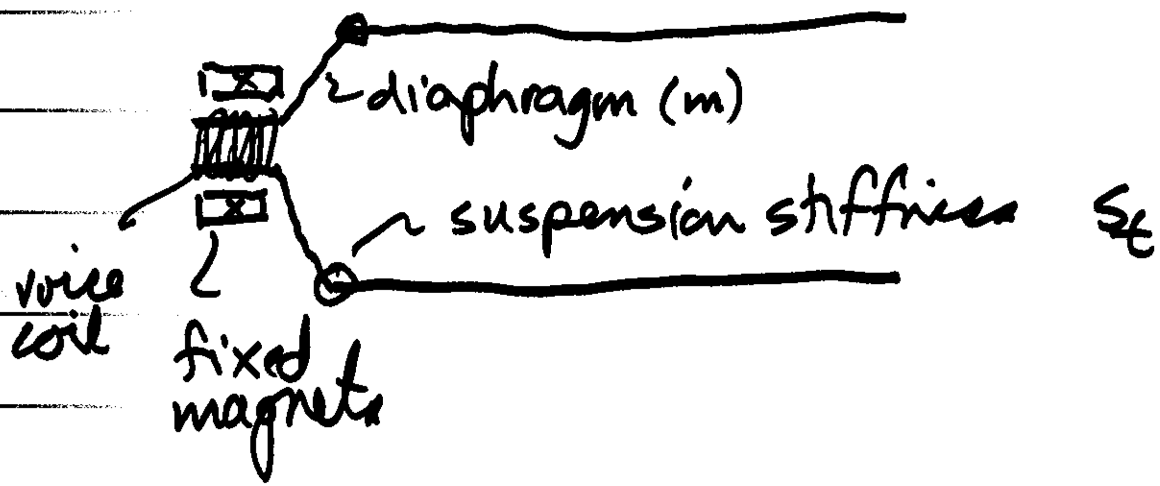
α_d energy dissipated within the material

ASTM E 2611-09

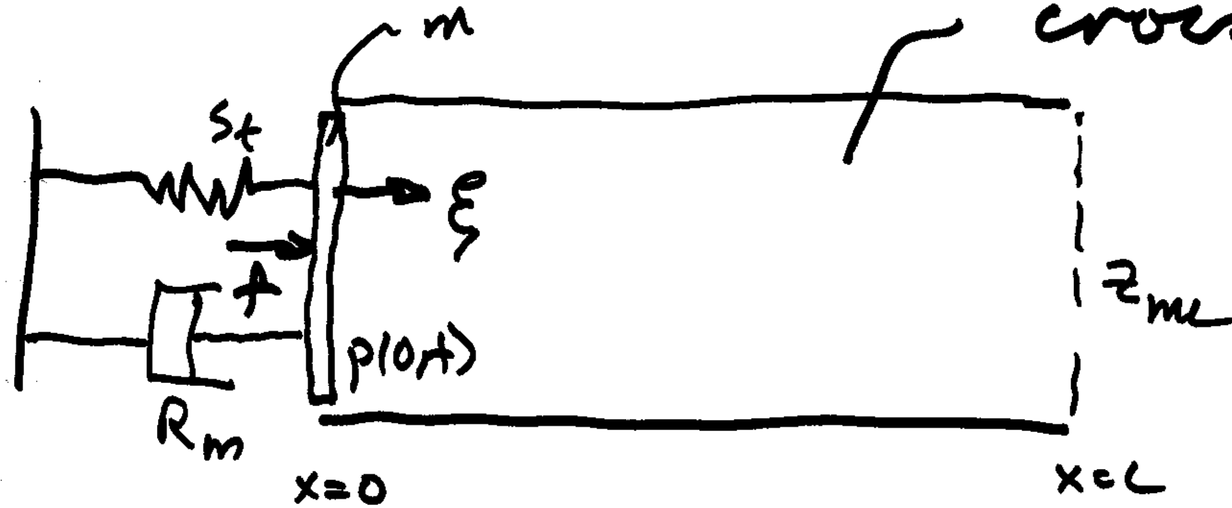
6.4 Combined Driver-Pipe System







l/s is a resonant system
- modified by the sound field



$$f = F e^{j\omega t}$$
$$\xi = \hat{\xi} e^{j\omega t}$$

EOM of the l/s

$$m \frac{d^2\xi}{dt^2} = -s_t \xi - R_m \frac{d\xi}{dt} - S p(0,t) + f$$

$$\underbrace{u(0,t)}_{\text{particle velocity in the sound field}} = \frac{d\xi}{dt} = j\omega\hat{\xi}e^{j\omega t} = \hat{u}e^{j\omega t}$$

$$p(0,t) = \hat{p}(0)e^{j\omega t}$$

EOM becomes

$$j\omega m\hat{u} = -R_m\hat{u} - \frac{s_t}{\omega}\hat{u} - S\hat{p}(0) + F$$

$$F = \Big[\underbrace{R_m + j\left(\omega m - \frac{s_t}{\omega}\right)}_{Z_{md}} + \underbrace{\frac{S\hat{p}(0)}{\hat{u}}}_{Z_{mo}}\Big]\hat{u}$$

In vacuo mechanical impedance of the l/s

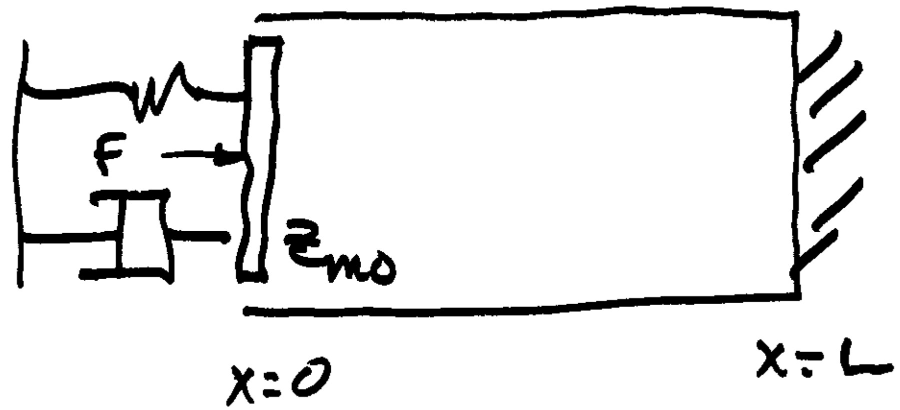
loading mechanical impedance created by the sound field

$$F = (Z_{md} + Z_{mo})\,\tilde{u}$$

Total mechanical impedance

$$\frac{F}{\tilde{u}} = Z_{md} + Z_{mo} = Z_m$$

In general, resonance occurs when $\mathrm{Im}\{Z_m\} \to 0$



example:
rigid termination

$$z_{m0} = -j\rho_0 c S \cot kL$$

$$z_{md} = R_m + j\left(\omega m - \frac{s_t}{\omega}\right)$$

Resonance $\quad \mathrm{Im}\{z_{m0} + z_{md}\} \to 0$

$$\omega m - \frac{s_f}{\omega} - \rho_0 c S \cot kL = 0$$

solutions of this equation
define the natural frequencies

Express all terms in terms of kL

$$\cot kL = \frac{\omega m}{\rho_0 c S}\frac{L}{L} - \frac{s_f}{\omega \rho_0 c S}\frac{L}{L} \qquad k = \frac{\omega}{c} \qquad \omega = kc$$

$$= (kL)\underbrace{\left(\frac{m}{\rho_0 S L}\right)}_{a} - \frac{1}{(kL)}\underbrace{\left(\frac{s_f}{\left(\frac{\rho_0 c^2 S}{L}\right)}\right)}_{b}$$

$$\cot kL = a(kL) - \frac{b}{(kL)}$$

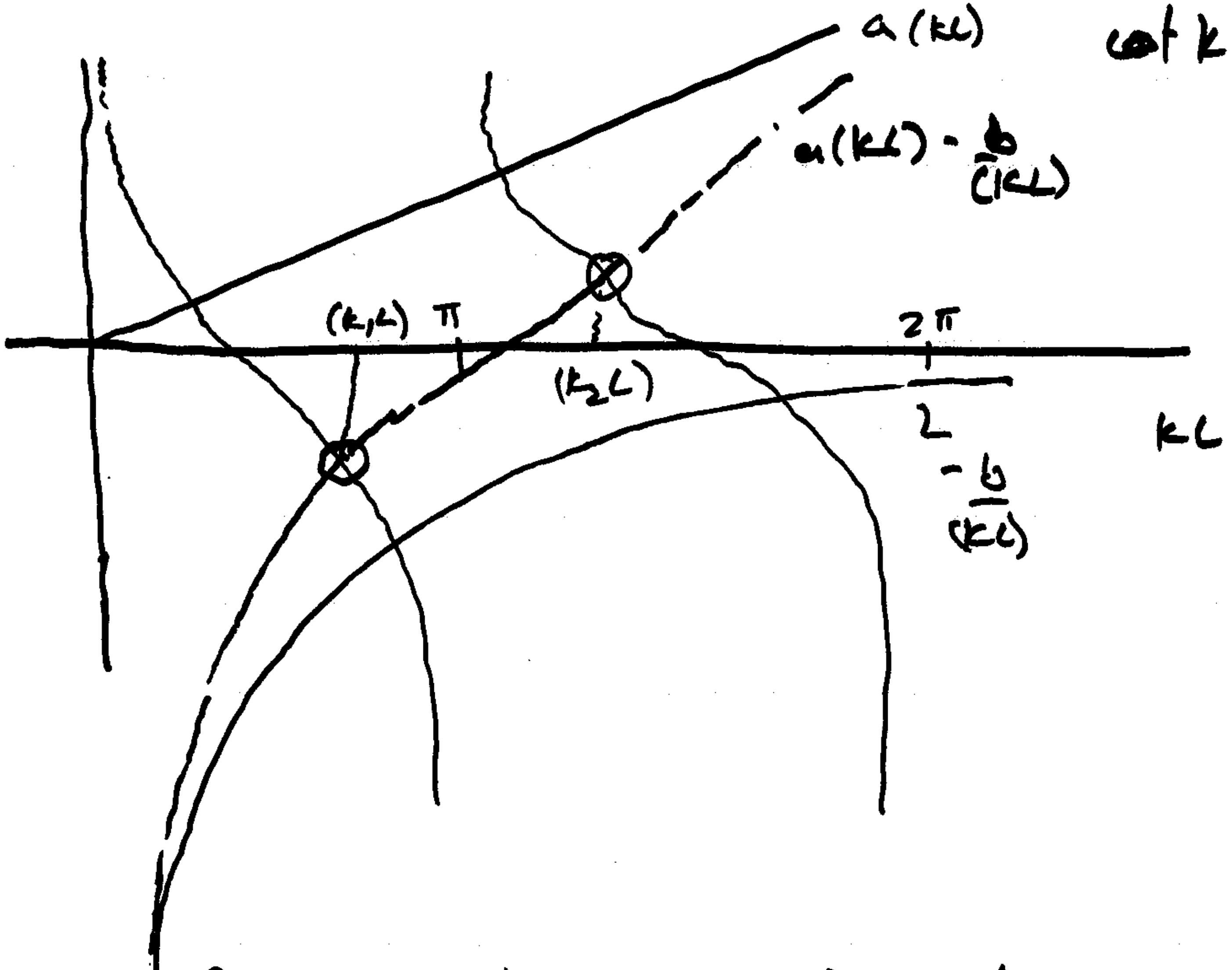
a = mass ratio

$= \dfrac{m}{\rho_0 SL}$ ~ mass of diaphragm / ~ mass of air in duct

b = stiffness ratio

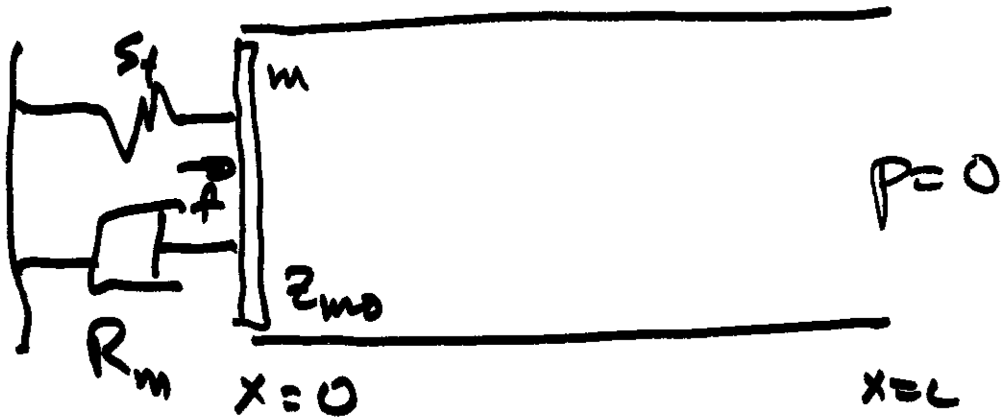
$= \dfrac{s_t}{\left(\frac{\rho_0 c^2 S}{L}\right)}$ ~ diaphragm / ~ static stiffness of the rigidly terminated air column

$$\cot kL = a(kL) - \frac{b}{(kL)}$$



solve for $(k_1 L)$ $(k_2 L)$ etc

$$k_1 L = \sim$$
$$k_1 = \sim / L$$
$$\left(\frac{\omega_1}{c}\right) = k_1$$

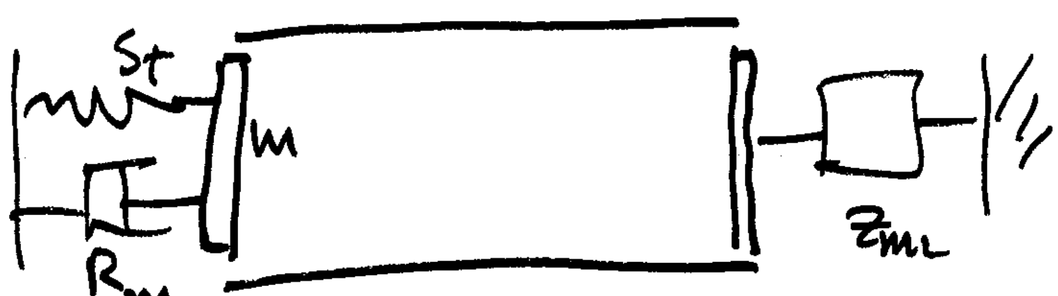


pressure release

~~Bad~~ Loading impedance has a significant impact on the dynamics of the driver system

$$z_{mo} = j\rho_0 c S \tan kL$$

$$z_{md} + z_{mo} = \underbrace{z_m}$$

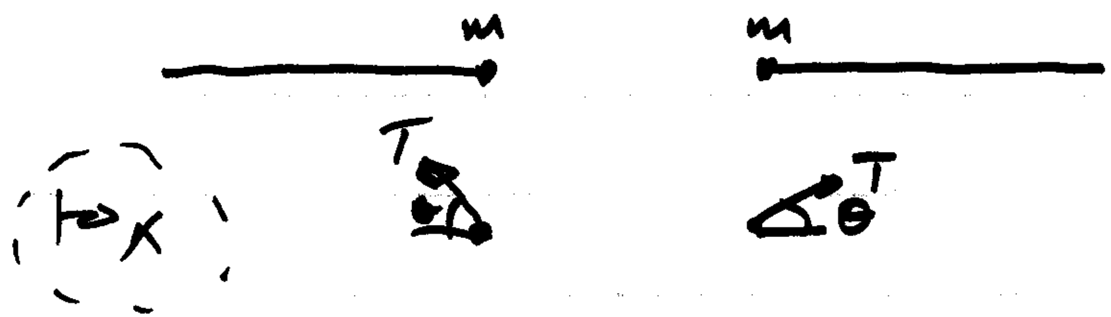


Review

1. SDOF – impedances $\frac{F}{u}$
   - mass-like
   - stiffness-like

2. string – governing eqn
   - wave equation
   - solutions of the wave equation
   - application of b.c.'s.

<u>wavenumber</u> $e^{-ikx}e^{i\omega t}$



$T\sin\theta$ $\quad$ $T\sin\theta$

$-\dfrac{\partial y}{\partial x}T$ $\quad$ $\dfrac{\partial y}{\partial x}T$

$\dfrac{\partial}{\partial x}$

$\dfrac{\partial}{\partial t}$

$\Sigma f_y = ma$ $\quad$ $\dfrac{\partial^2 y}{\partial t^2}$