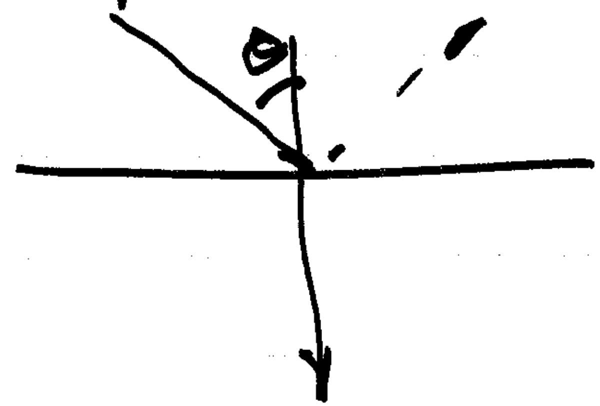
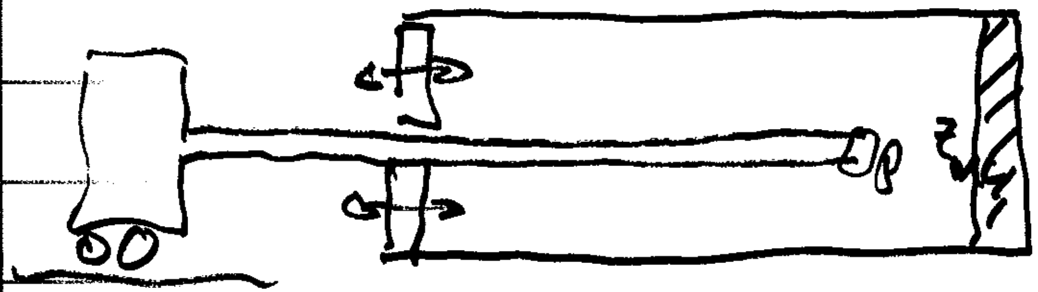
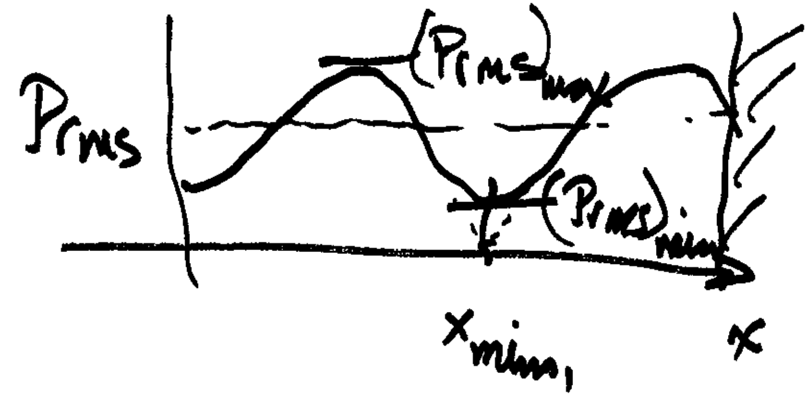
free space



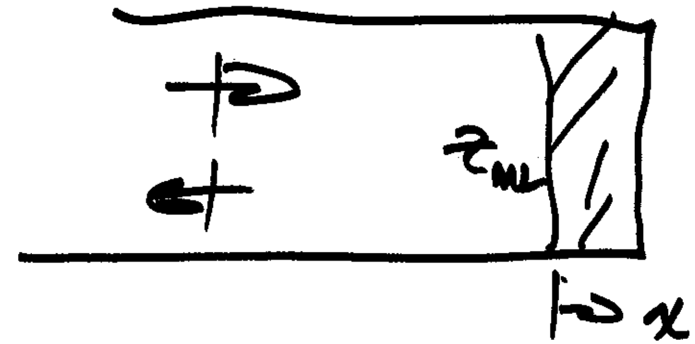




$$\underset{\sim}{SWR} = \frac{(P_{rms})_{max}}{(P_{rms})_{min}}$$

$SWR_{rigid} \to \infty$

$SWR_{perfect\ absorber} = 1$



what is z_{ml}

$$\hat{p} = \bar{A}e^{-jkx} + \bar{B}e^{jkx}$$

$$A = |A|e^{j\phi_a} \qquad B = |B|e^{j\phi_B}$$

plane wave reflection coefficient

$$R = \cancel{\frac{|B|}{|A|}} \; \frac{B}{A} = \frac{|B|}{|A|}e^{j(\phi_B - \phi_A)} = \frac{|B|}{|A|}e^{-j(\phi_A - \phi_B)}$$

$$|R| = \frac{|B|}{|A|}$$

$$p_{rms}^2 = \frac{\tilde{p}\,\tilde{p}^*}{2} = \frac{1}{2}\left[\,|A|^2 + |B|^2 + 2|A||B|\cos\left[(\phi_a - \phi_b) - \underline{\underline{2}}\,kx\right]\right]$$

$$e^{-jkx}$$

$$c = f\lambda$$

$$\left(p^2_{rms}\right)_{max} = \frac{1}{2}\left[|A|^2 + |B|^2 + 2|A|\,|B|\right]$$

$$= \frac{1}{2}\left[|A| + |B|\right]^2$$

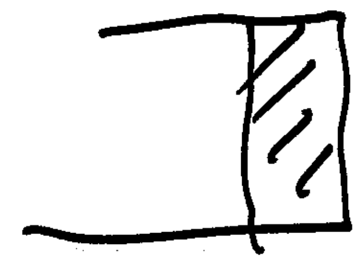
$$\left(p^2_{rms}\right)_{min} = \frac{1}{2}\left[|A|^2 + |B|^2 - 2|A|\,|B|\right]$$

$$= \frac{1}{2}\left[|A| - |B|\right]^2$$

$$SWR = \frac{\left(P_{rms}\right)_{max}}{\left(P_{rms}\right)_{min}} = \frac{|A| + |B|}{|A| - |B|}$$

$$SWR = \frac{1 + |R|}{1 - |R|} \qquad |R| = \frac{|B|}{|A|}$$

$$|R| = \frac{SWR - 1}{SWR + 1}$$



Z_{ML}

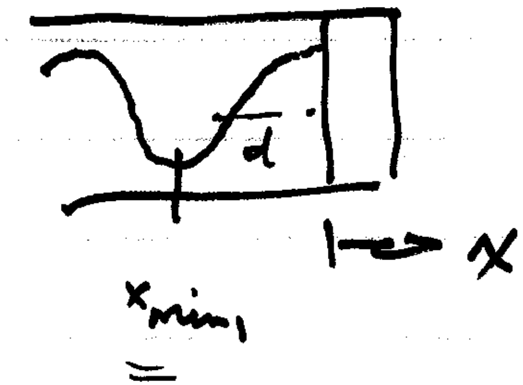
$$\alpha = 1 - |R|^2$$

location of the first minimum in the standing wave pattern

$$\cos\left[(\varphi_A - \varphi_B) - 2kx_{min,1}\right] = -1$$

location of 1st minimum

$$(\varphi_A - \varphi_B) - 2kx_{min,1} = \pi$$



$$\phi_A - \phi_B = \pi + 2kx_{min,1}$$

$$= \pi - 2k|x_{min,1}|$$

$$= \pi - 2\left(\frac{2\pi}{\lambda}\right)|x_{min,1}|$$

$$= \pi\left(1 - \frac{|x_{min,1}|}{\lambda/4}\right)$$

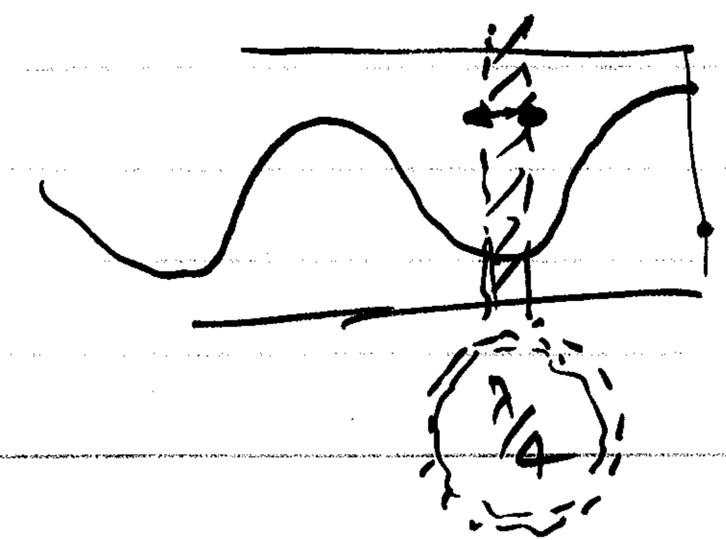
$$\underset{\checkmark}{R} = \underset{\substack{\uparrow \\ \text{SWR}}}{|R|}\, e^{-j\underbrace{(\phi_A - \phi_B)}_{x_{min,1}}}$$

phase of the reflection coefficient is proportional to the shift of the minimum location from $\lambda/4$

if $|x_{min,1}| = \frac{\lambda}{4}$ $\quad \underbrace{\phi_A - \phi_B}_{} = 0$

$|R| = R$ = purely real

R is real
- rigid termination
- purely resistive



In general $\phi_A - \phi_B \neq 0$

R is complex $\quad Z_{mc} \to$ complex

Measurement procedure

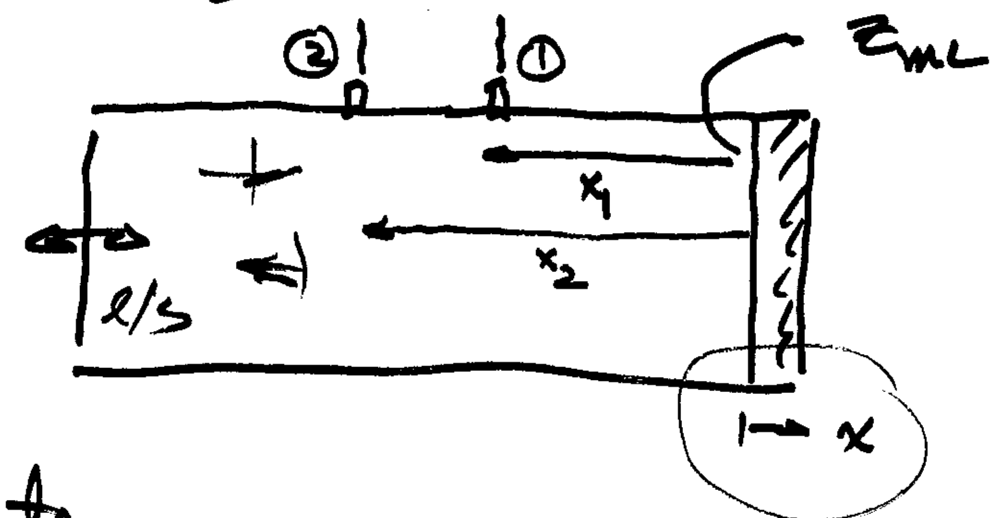
1. place the sample in the tube
2. Use the l/s to generate a single frequency standing wave pattern
3. Use the probe tube mic to map the mean square pressure → P_{rms}
4. Calculate SWR → $|R|$ → α
5. Find x_{min_1} → phase of R

6. Form complex

7. $$\frac{Z_{mL}}{\rho_0 c \underbrace{S}_{\text{cross-sectional area of the duct}}} = \frac{1+R}{1-R}$$

Repeat for every frequency of interest

6.3.2 Two-Microphone Method.



$x_1 = -d_1$

$x_2 = -d_2$

d_1 & d_2 are positive distances

generate white noise (across the plane wave region)

10 cm ID (0 → 1600 Hz)

2.9 cm ID (0 → 6400 Hz)

$$\hat{p}(x_3) = A e^{-jkx} + B e^{jkx}$$

$$\hat{p}(x_1) = A e^{+jkd_1} + B e^{-jkd_1}$$

$$\hat{p}(x_2) = A e^{+jkd_2} + B e^{-jkd_2}$$

Measure the transfer function between Mics 1 + 2

$$H_{12} = \frac{\hat{p}(x_1)}{\hat{p}(x_2)} = \frac{A e^{+jkd_1} + B e^{-jkd_1}}{A e^{+jkd_2} + B e^{-jkd_2}}$$

$\div$ above & below by A

$$H_{12} = \frac{e^{+jkd_1} + R\,e^{-jkd_1}}{e^{+jkd_2} + R\,e^{-jkd_2}}$$

$$R = \frac{H_{12}\,e^{+jkd_2} - e^{+jkd_1}}{e^{-jkd_1} - H_{12}\,e^{-jkd_2}}$$