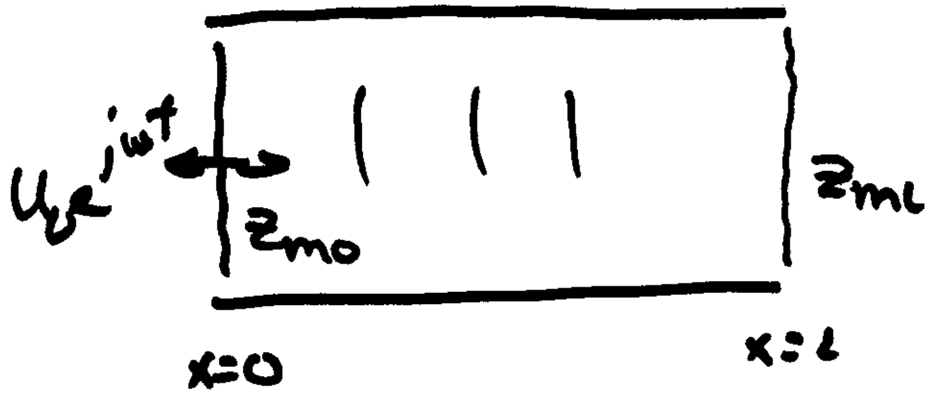
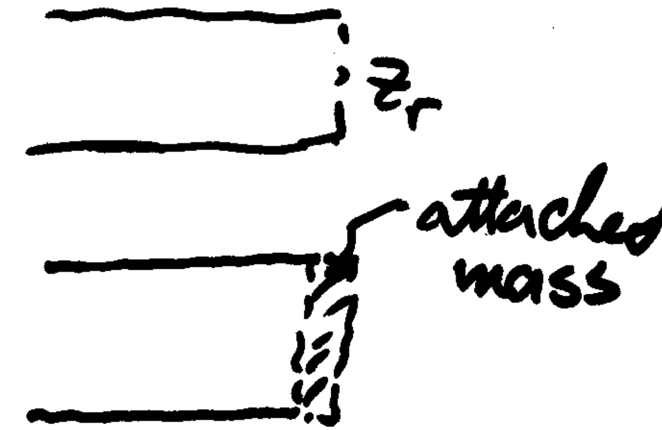
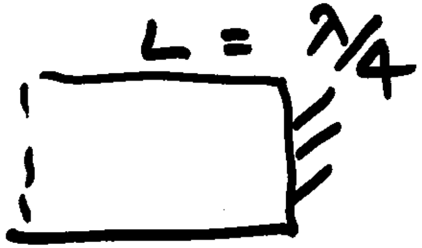
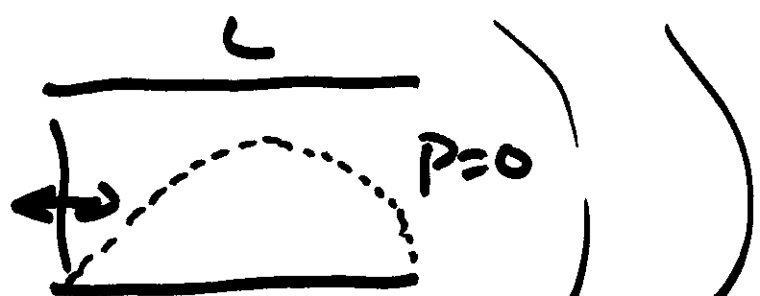




$$L_e = L + \frac{8}{3\pi} a$$



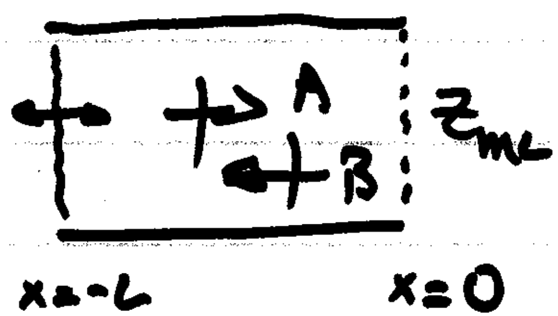
1/4 wave resonator



resonance when $L = \lambda/2$

6.2 Sound radiation from a pipe

quantify the sound power radiated by an open duct



$$R = \frac{B}{A}$$

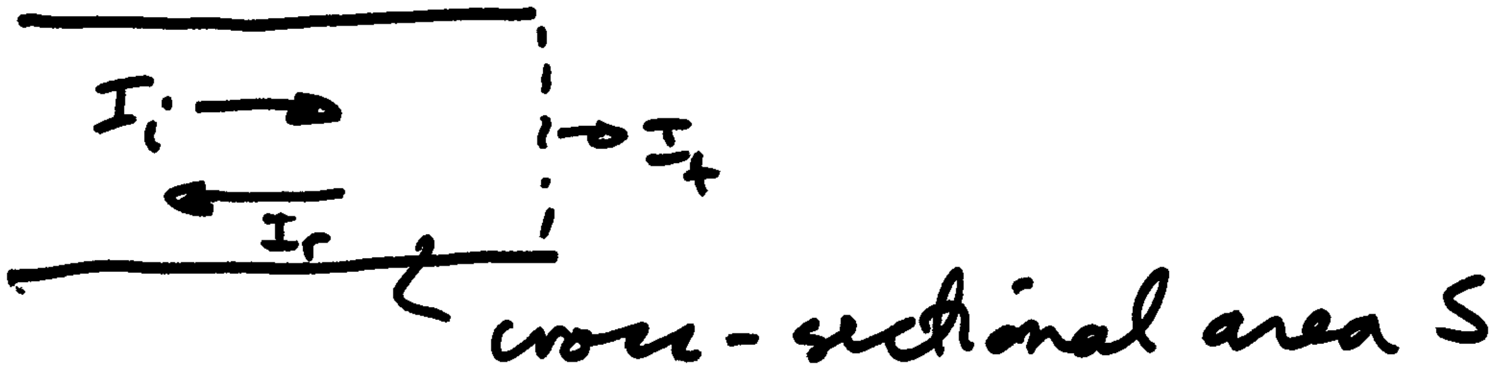
$$\tilde{p}(x) = A e^{-jkx} + B e^{+jkx}$$

$$\tilde{u}(x) = \frac{A}{\rho c} e^{-jkx} - \frac{B}{\rho c} e^{+jkx}$$

$$Z_{mL} = \left.\frac{S\tilde{p}}{\tilde{u}}\right|_{x=0} = \rho_0 c S \,\frac{A+B}{A-B}$$

$$= \rho_0 c S \,\frac{1+R}{1-R}$$

$$R = \frac{\dfrac{Z_{mL}}{\rho_0 c S} - 1}{\dfrac{Z_{mL}}{\rho_0 c S} + 1}$$



$$I_i = \frac{1}{2} \mathrm{Re}\{ \tilde{p}_i \tilde{u}_i^* \} = \frac{1}{2\rho_0 c} |A|^2$$

$$I_r = \frac{1}{2} \mathrm{Re}\{ \tilde{p}_r \tilde{u}_r^* \} = \frac{1}{2\rho_0 c} |B|^2$$

we know $I_i - I_r = I_t$ Energy conservation

$$W_t = (I_i - I_r)S = I_t S$$

$$W_t = I_i\left(1 - \underbrace{\frac{I_r}{I_i}}_{|R|^2}\right)S$$

$$\frac{I_t}{I_i} = T_\pi \approx \text{power transmission coefficient}$$

$$T_\pi = 1 - \frac{I_r}{I_i} = 1 - |R|^2$$

$$R = \frac{\frac{Z_{ML}}{\rho c S} - 1}{\frac{Z_{ML}}{\rho c S} + 1}$$

$$W_t = I_i (1 - |R|^2) S$$

↑ calculate knowing Z_{mL}

flanged opening → Z_{mL} ← unflanged

Flanged case

$$\frac{Z_{mL}}{\rho_0 c S} = \frac{1}{2}(ka)^2 + j \frac{8}{3\pi} a$$

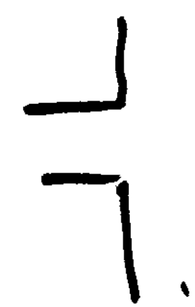
$ka \ll 1$
small pipe
low freq

$$T_\pi = \frac{2(ka)^2}{\left[1 + \frac{1}{2}(ka)^2\right]^2 + \left(\frac{8}{3\pi}\right)^2 (ka)^2}$$

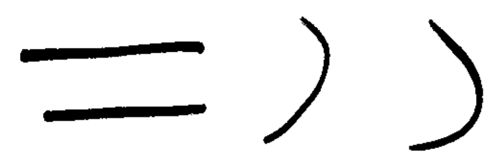
if $ka \ll 1$

so

$$T_\pi \simeq 2(ka)^2 \ll 1$$

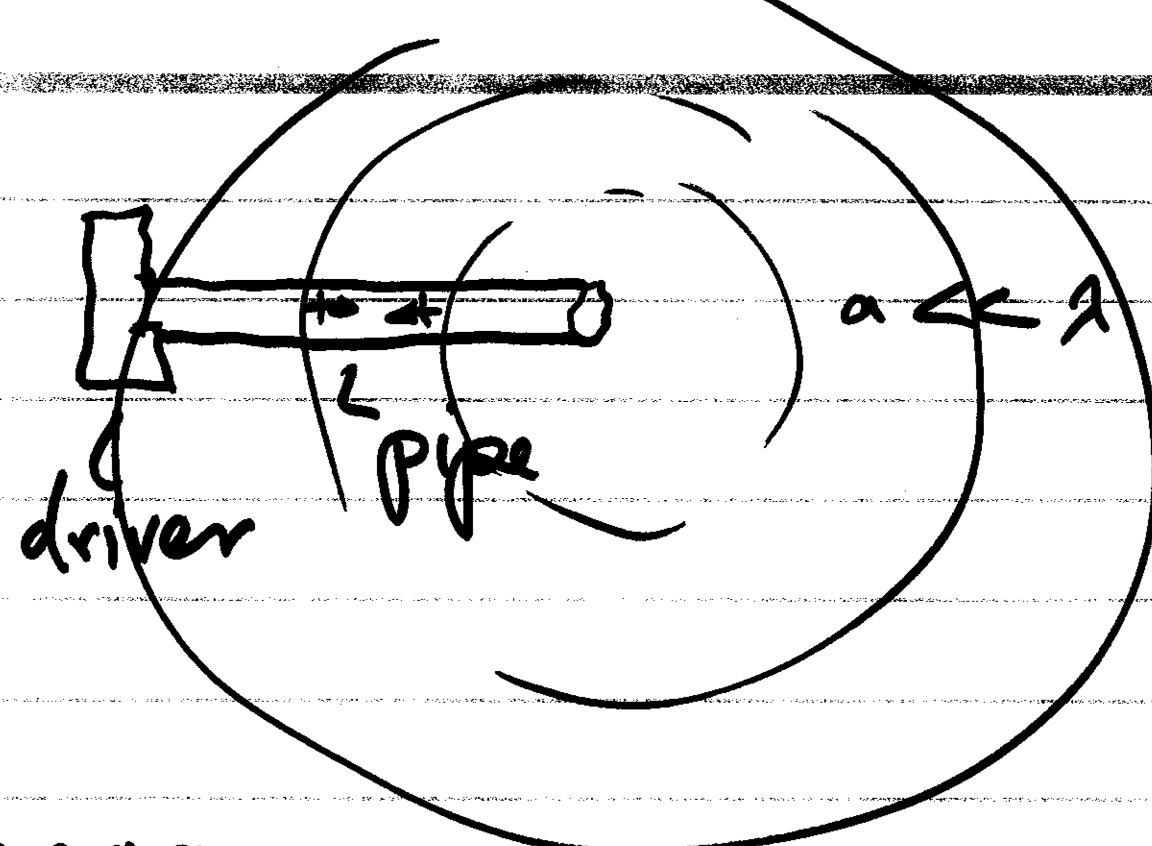


Unflanged opening



$$T_\pi \simeq (ka)^2$$

unflanged opening is a less effective radiator than a flanged pipe.



Main message

- we can calculate the sound power radiated from a duct without explicitly solving for the exterior sound field (if the termination impedance is known)

Homework Hints

7.1.3

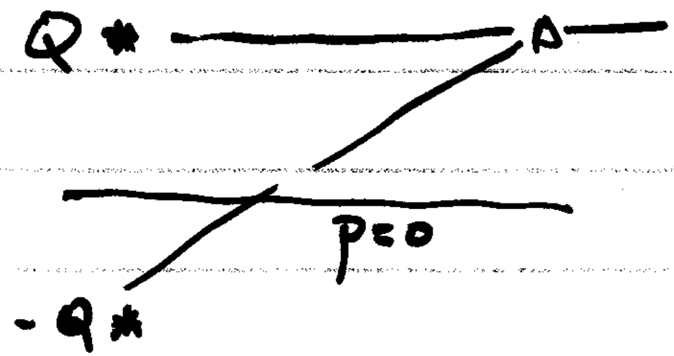
(a) - Eq. 7.1.2

- compare 7.1.6 with 7.1.8

(b) 7.1.8

$$|\tilde{a}| = \omega \hat{\tilde{u}}$$

6.8.3



follow the dipole derivation that we did in class

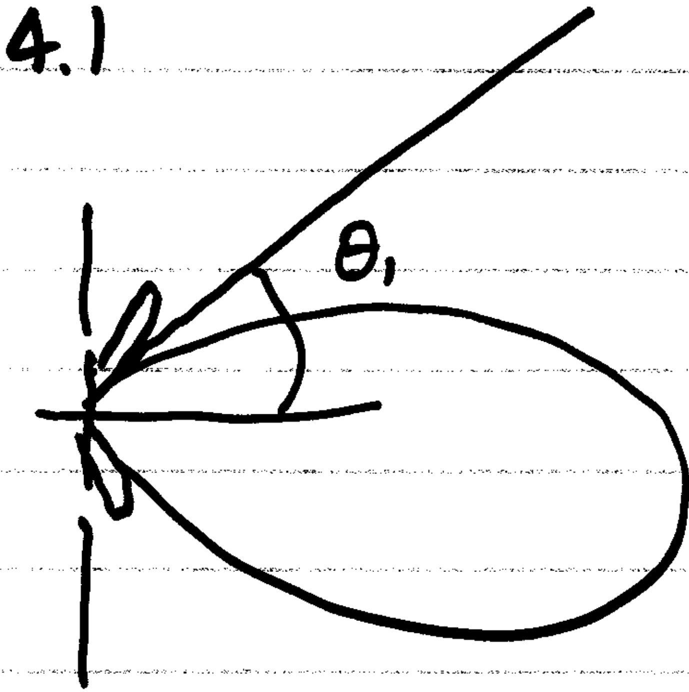
7.2.3

$kd \ll 1$

start with eqn 6.8.6

$kd \neq 0$

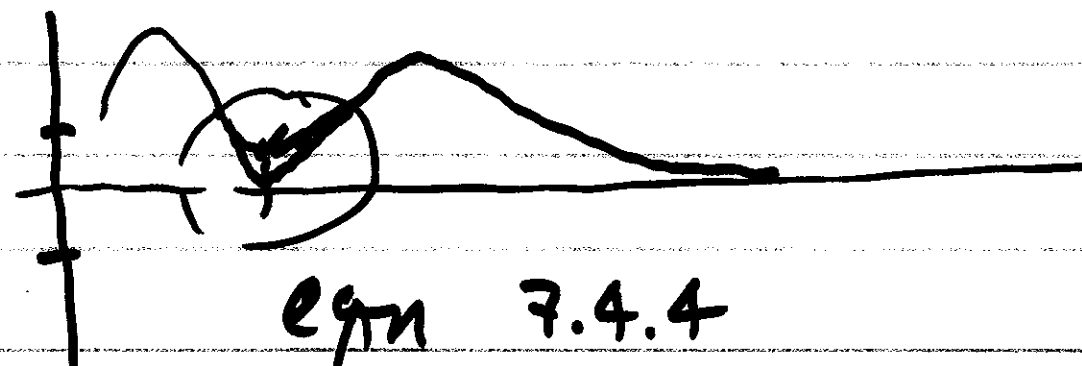
7.4.1



(a) θ_1 as a function of ka

j_{11}

(b)



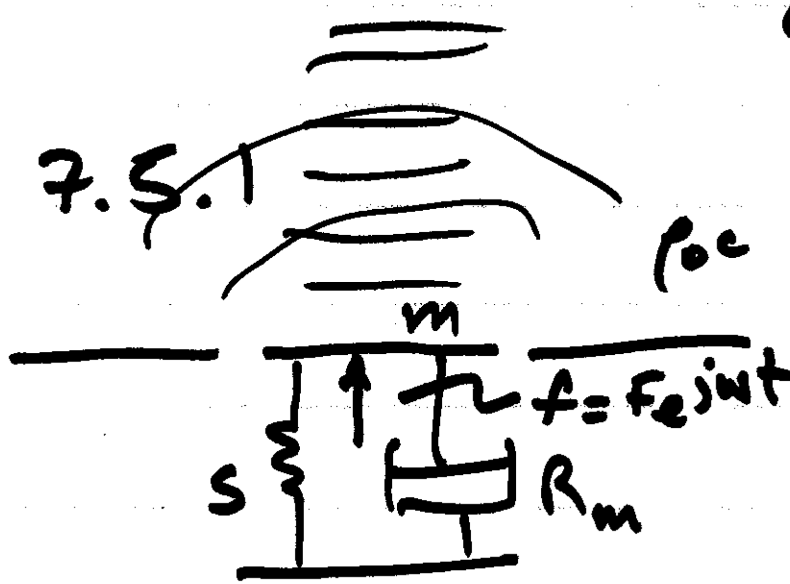
eqn 7.4.4

7.4.6c

Eqn 7.4.5

asymptotic soln 7.4.7

7.5.1

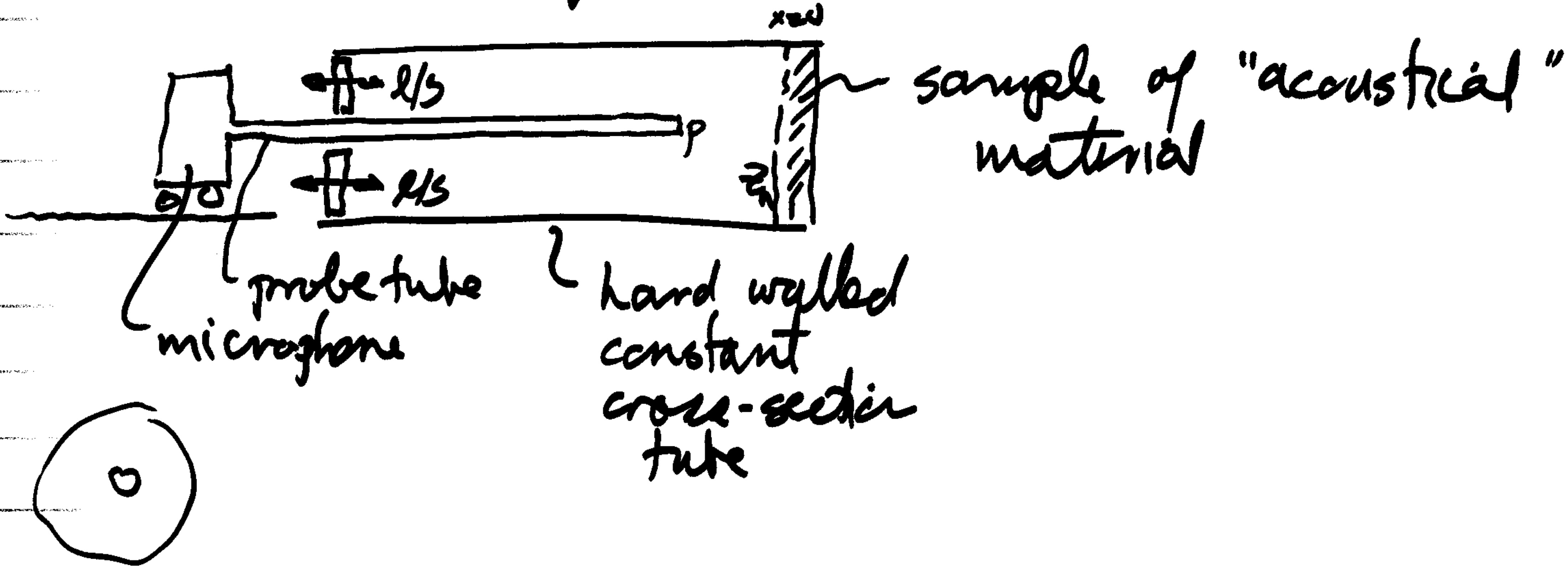


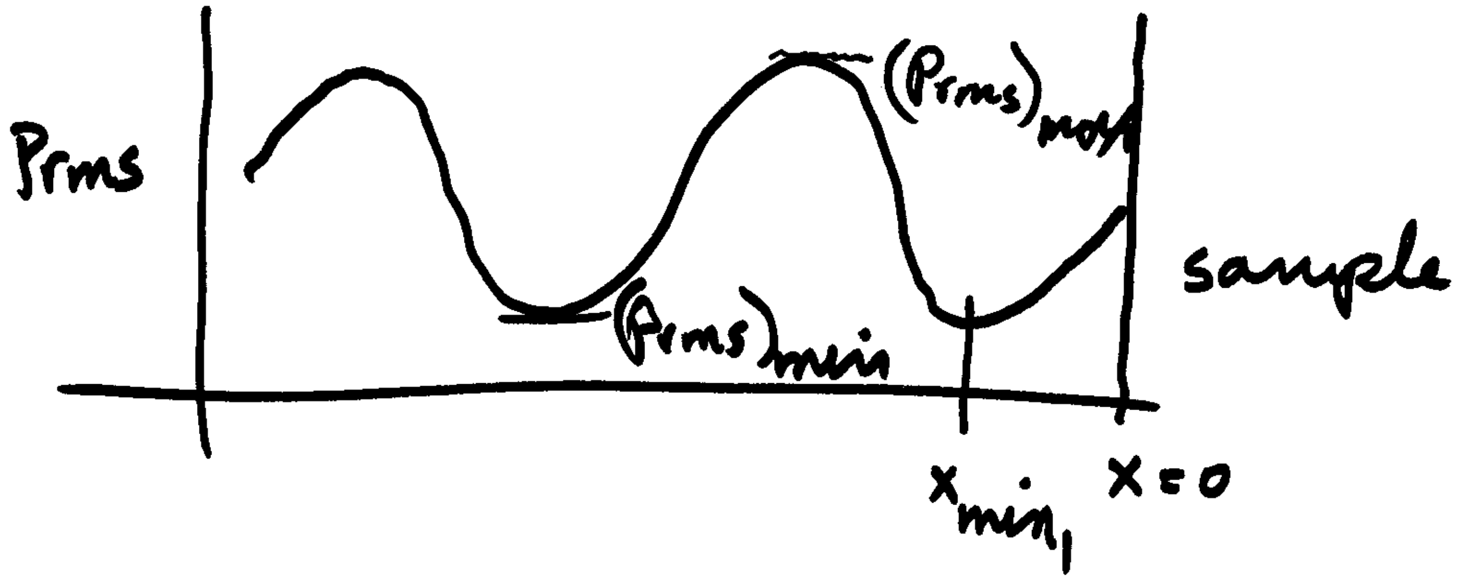
assume $ka \gg 2$

radiation is similar to plane wave radiator

6.3 Standing Waves in ducts: measurement of termination impedance

6.3.1 Single frequency approach





Standing Wave Ratio (SWR)

$$SWR = \frac{(P_{rms})_{max}}{(P_{rms})_{min}}$$