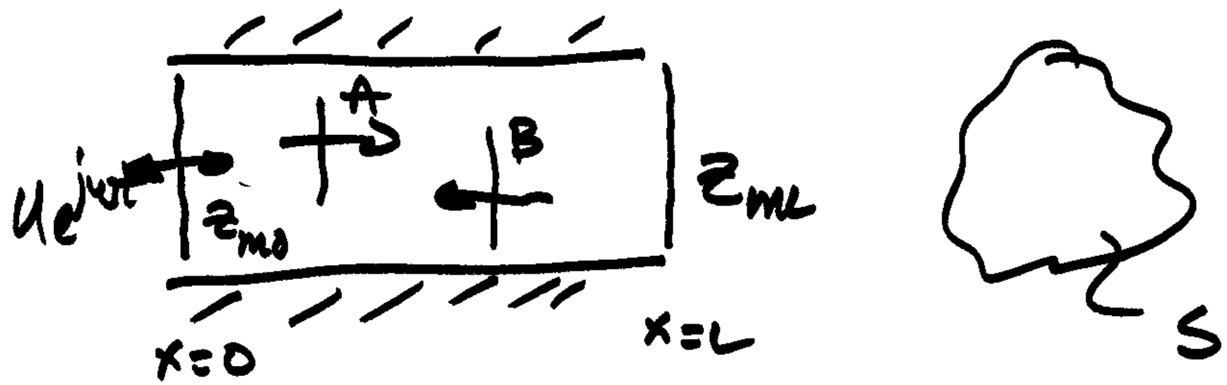
Pipes, Cavities and Waveguides



Applied B.C.'s

$$z_{mL} = \left.\frac{S\hat{p}}{\hat{u}}\right|_{x=L} = S\rho_0 c\,\frac{Ae^{-jkL} + Be^{+jkL}}{Ae^{-jkL} - Be^{+jkL}} \qquad (1)$$

$$z_{m0} = \left.\frac{S\hat{p}}{\hat{u}}\right|_{x=0} = S\rho_0 c\,\frac{A + B}{A - B} \qquad (2)$$

Rewrite (1) $\qquad R = \frac{B}{A}$

$$Z_{mL} = S\rho_0 c \frac{e^{-jkL} + R e^{+jkL}}{e^{-jkL} - R e^{+jkL}}$$

solve for R in terms of Z_{mL}

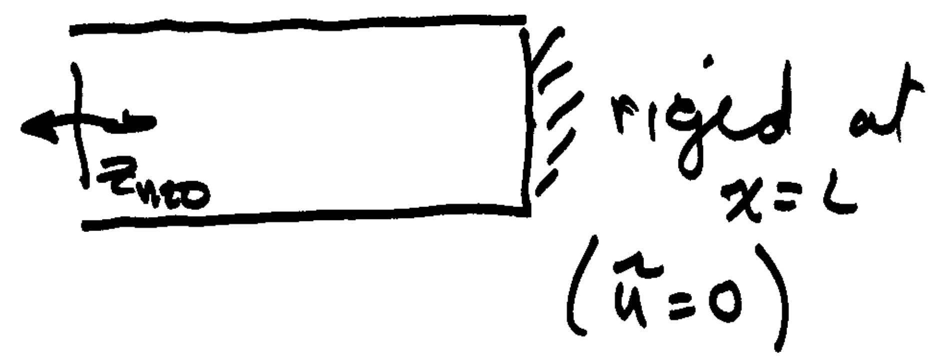
then substitute result into eqn (2)

$$\frac{Z_{m0}}{\rho_0 c S} = \frac{\frac{Z_{mL}}{\rho_0 c S} + j\tan(kL)}{1 + j\frac{Z_{mL}}{\rho_0 c S}\tan(kL)}$$

$$kL = 2\pi\left(\frac{L}{\lambda}\right)$$

non-dimensional duct length in terms of wavelength

Rigid Termination



$$\frac{Z_{mL}}{\rho_0 c S} \to \infty$$

$$\frac{Z_{m0}}{\rho_0 c S} = -j \cot kL$$

rigidly terminated air column

Resonance condition

$$\mathrm{Im}\{Z_{mo}\} = 0$$

$$\cot kL \to 0 \qquad \underbrace{\cos(kL) \to 0}$$

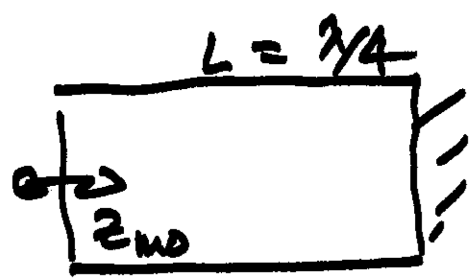
$$k_n L = (2n-1)\frac{\pi}{2} \qquad n = 1, 2, 3 \ldots\ldots$$

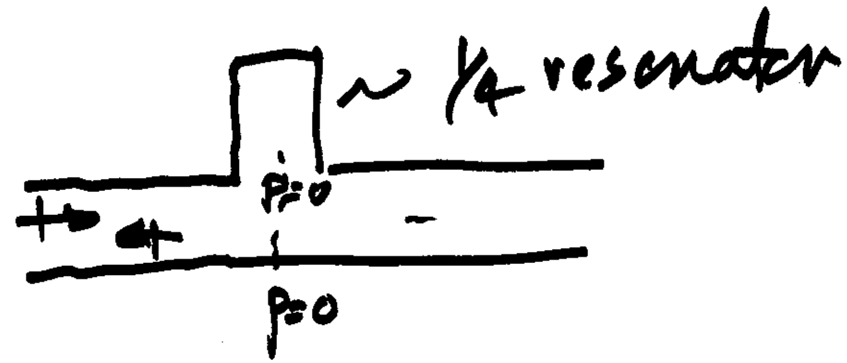
$$k_n = \frac{\omega_n}{c} = \frac{2\pi f_n}{c} \qquad \frac{2\pi}{\lambda_n}$$

$$f_n = \frac{2n-1}{4}\frac{c}{L}$$

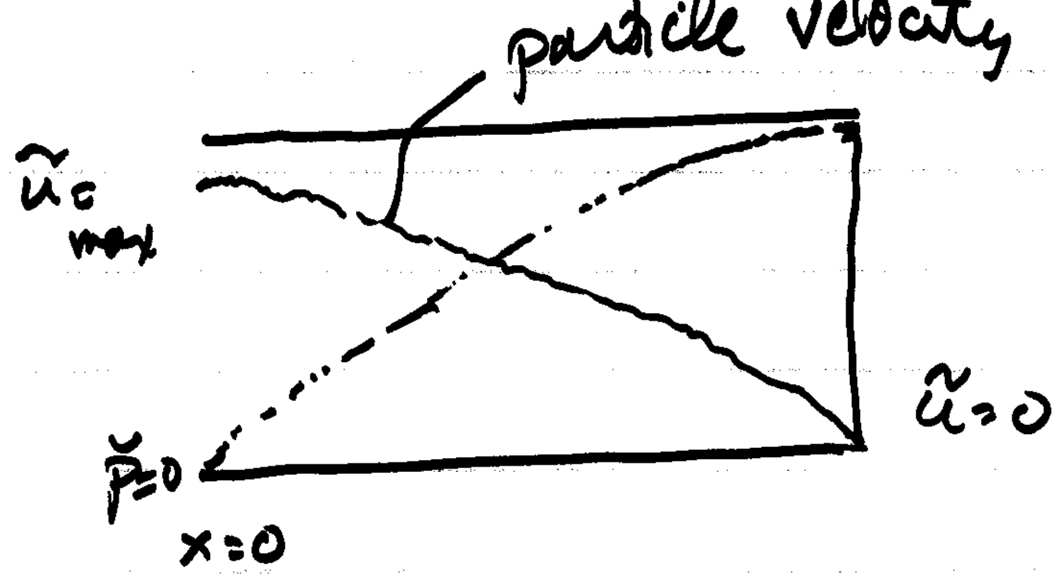
$$\left(\frac{L}{\lambda_n}\right) = \frac{2n-1}{4} \qquad n = 1, 2, 3 \ldots$$

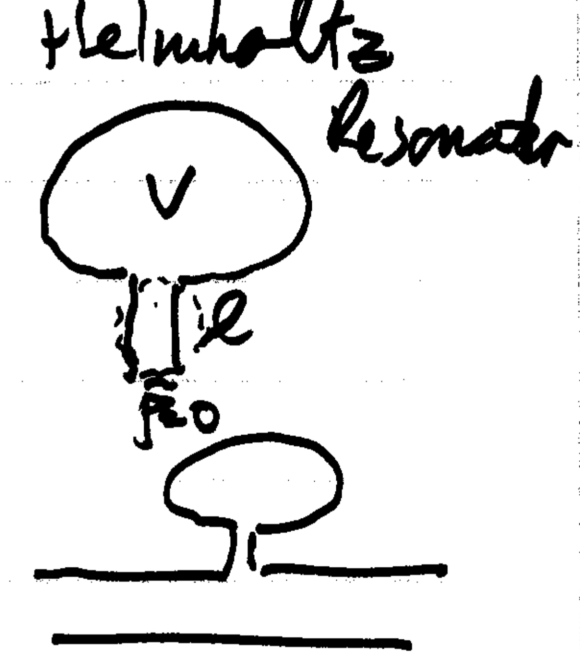
$n = 1 \qquad \lambda_1 = 4L \qquad$ or $\qquad L = \dfrac{\lambda_1}{4}$



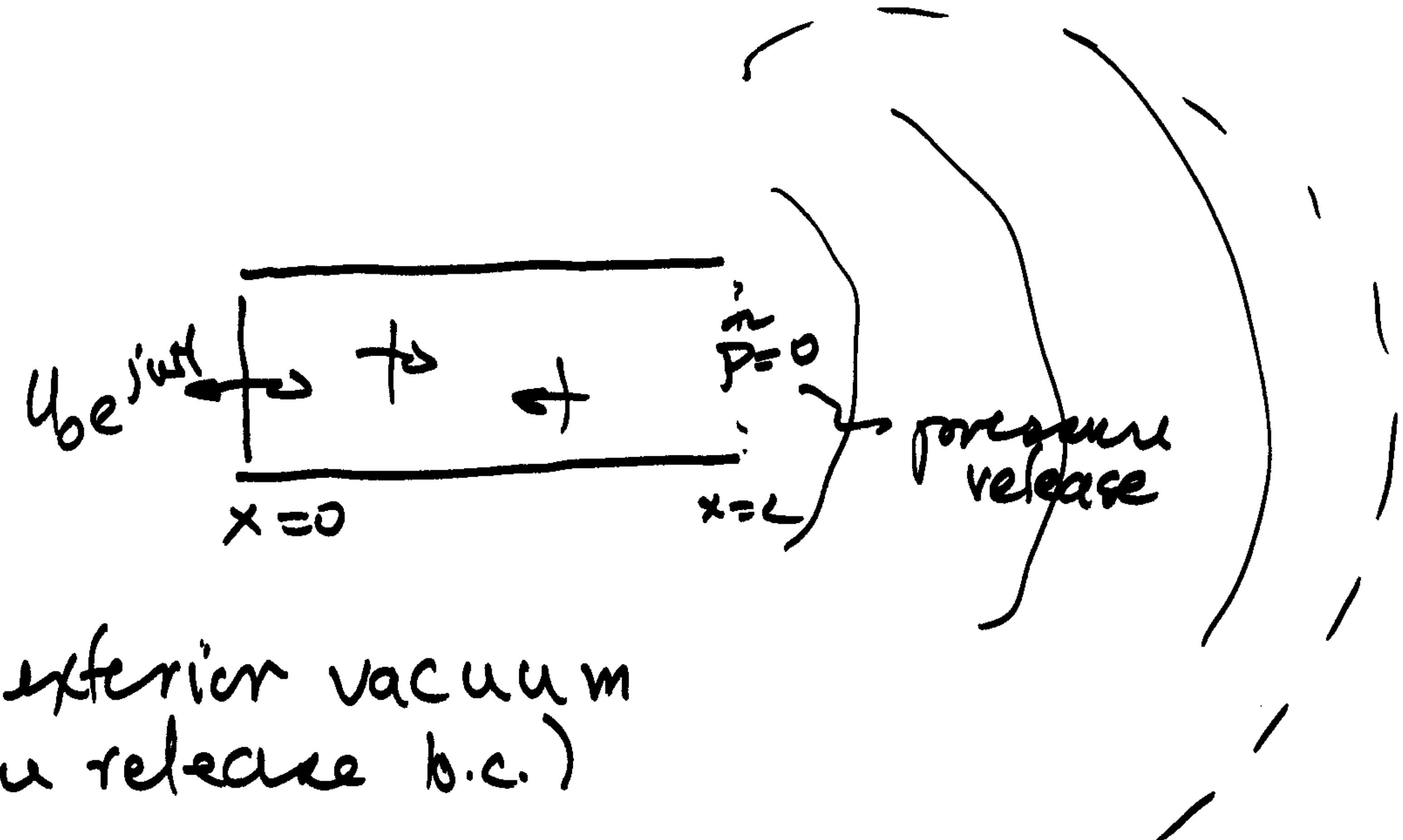


quarter wave resonator





$Z_{m0} = 0 = \left.\frac{S\tilde{p}}{\tilde{u}}\right|_{x=0}$ $\tilde{p} = 0$ at $x=0$ pressure release open pipe



(i) Assume an exterior vacuum
(pressure release b.c.)

good approximation when
tube diameter $\ll \lambda$

$ka \ll 1$

a pipe radius

$$Z_{mL} = \frac{S\tilde{p}}{\tilde{u}}\bigg|_{x=L} = 0$$

$$\frac{Z_{m0}}{\rho_0 c S} = j \tan kL$$

$$\mathrm{Im}\{Z_{m0}\} \rightarrow 0 \quad \text{when} \quad \tan kL \rightarrow 0 \qquad \sin kL$$

$$k_n L = n\pi \qquad n = 1, 2, 3 \ldots$$

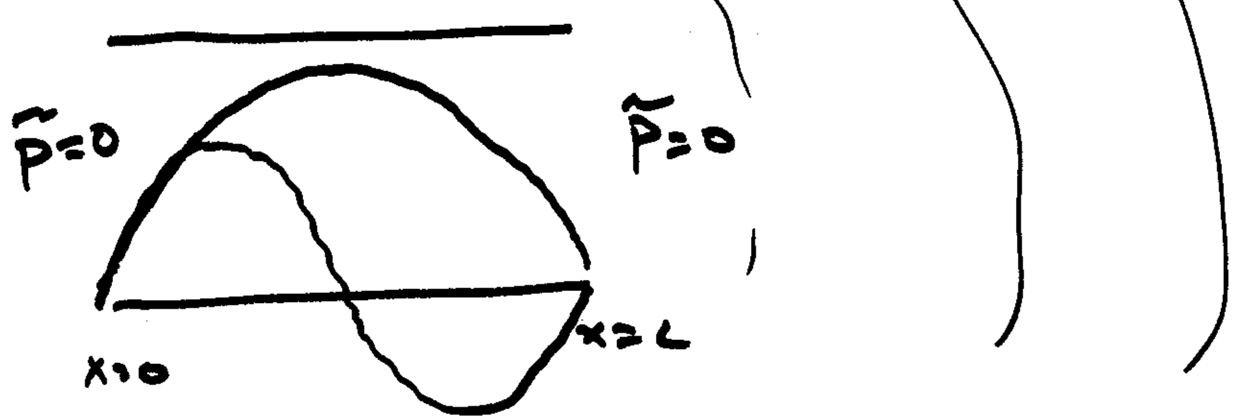
$$f_n = \frac{n}{2}\frac{c}{L}$$

$$\frac{L}{\lambda_n} = \frac{n}{2} \qquad n = 1, 2, 3 \ldots\ldots$$

at a resonance

$$\lambda_1 = 2L \qquad L = \frac{\lambda_1}{2}$$

first resonance



(ii) when z_{mL} depends on the radiated sound field

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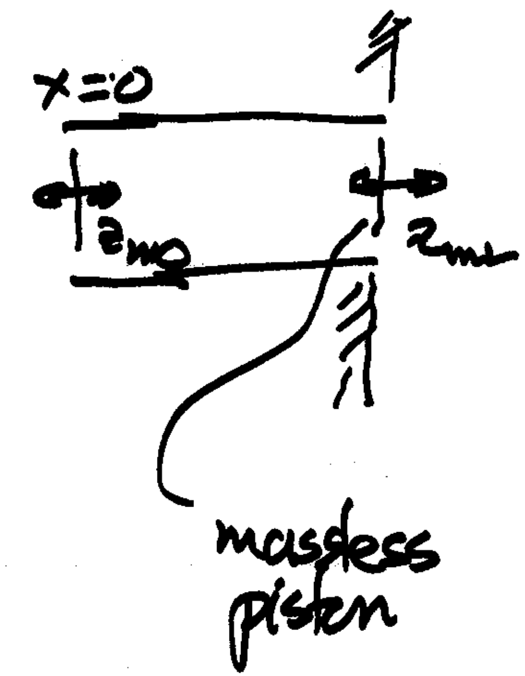
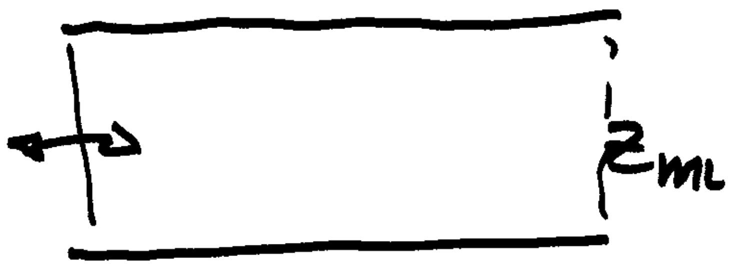


z_{mL} = baffled piston

$= z_r$ radiation impedance

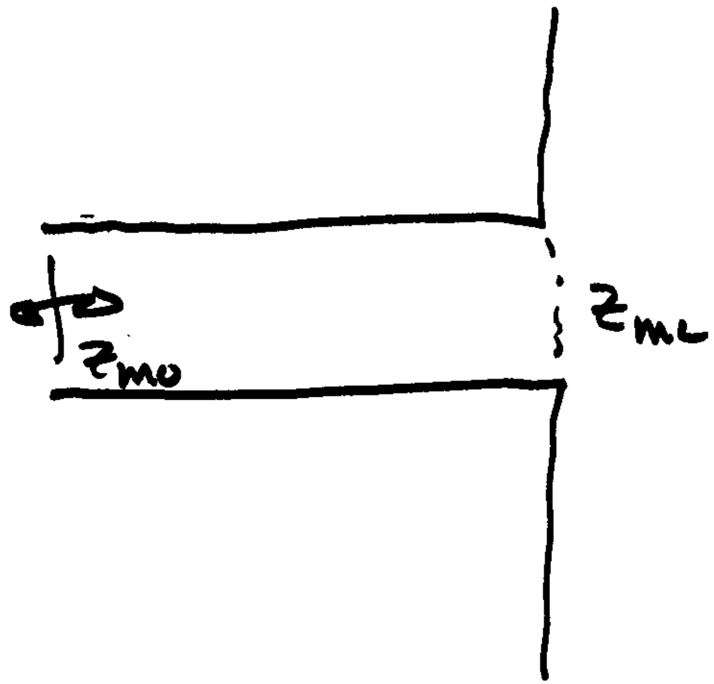
$ka \ll 1$



unbaffled case (unflanged)

$$\frac{Z_{mL}}{\rho_0 c S} \approx \frac{1}{4}(ka)^2 + j\,0.6\,(ka)$$

a = pipe radius



baffled case (flanged)

$$\frac{Z_{mL}}{\rho c} = \frac{1}{2}(ka)^2 + j\,\frac{8}{3\pi}(ka)$$

$$\frac{Z_{m0}}{\rho_0 c S} = \frac{\frac{Z_{mL}}{\rho_0 c S} + j \tan kL}{1 + j \frac{Z_{mL}}{\rho_0 c S} \tan kL}$$

At resonance

$$\mathrm{Im}\{Z_{m0}\} \to 0$$

termination impedance

$$\left(\frac{Z_{mL}}{\rho_0 c S}\right) = r + ix$$

$$\tan kL = -\mathrm{Im}\left\{\frac{Z_{mL}}{\rho_0 c S}\right\}$$

$$= -x$$

$$\tan kL = -\chi$$

~~since~~

$$\tan(n\pi - k_n L) = \chi$$

since $ka \ll 1$

$$n\pi - k_n L \simeq \chi = \frac{8}{3\pi}(ka)$$

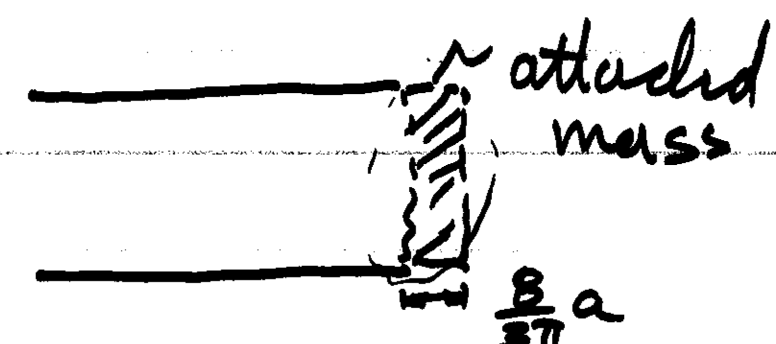
~~the~~ flanged case

$$\underbrace{k_n\left(L + \frac{8}{3\pi}a\right)}_{\text{effective duct length}} = n\pi$$

n : 1, 2, 3 resonance condition

effective duct length

$L_e > L$



pressure release b.c. $f_n = \frac{n}{2}\frac{c}{L}$

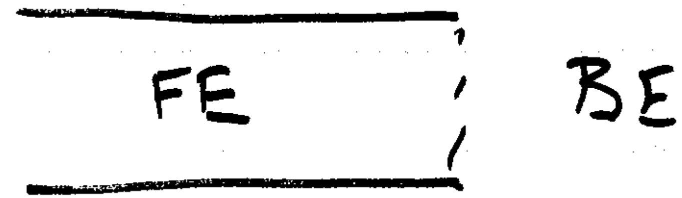
actual case ($ka \ll 1$) - accounting for the radiation loading

$f_n = \frac{n}{2}\frac{c}{L_e}$ L_e = effective length

$= L + \frac{8}{3\pi}a$ for a flanged opening

$L_e > L$

resonance frequencies are slightly reduced compared to the simple pressure release case



6.2 Sound Radiation from The pipe