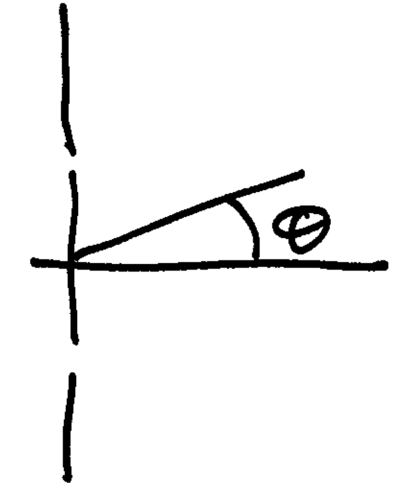
Extended sources.

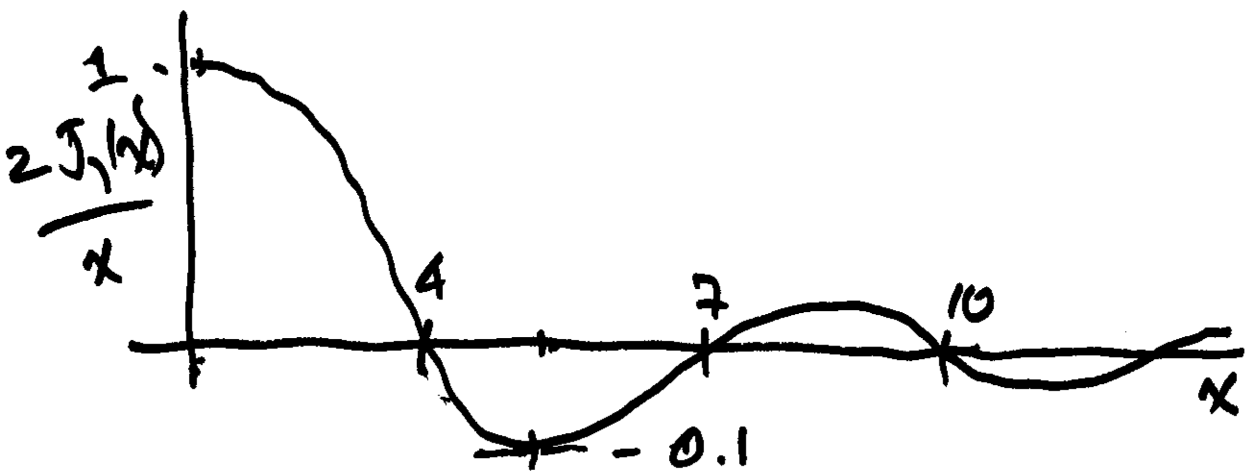
$ka \ll 1 \rightarrow$ monopole.

far field

$$\tilde{p}(r,\theta) = \underline{j\frac{\rho_0 c}{2} u_0 \left(\frac{a}{r}\right)} e^{-jkr}\left[2\,\frac{J_1(ka\sin\theta)}{ka\sin\theta}\right]$$



df

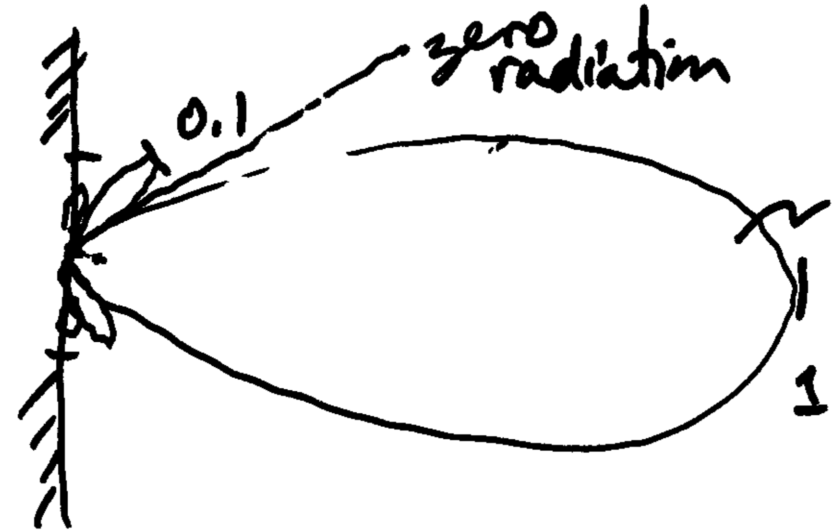


$$x = \underbrace{ka}\,\underbrace{\sin\theta}$$

$ka \ll 1 \quad [\quad] \approx 1$ for all angles

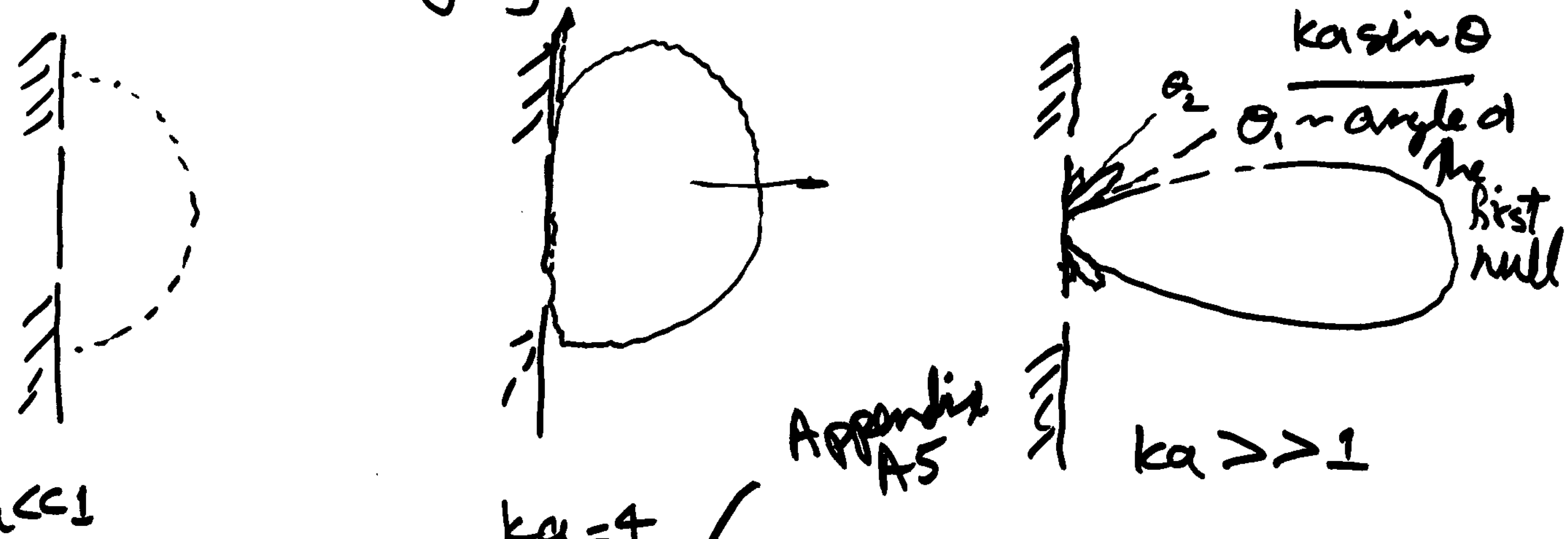
monopole-like radiation

$ka > 1$ we have the possibility of sidelobes & ~~...~~ nulls.



main lobe becomes narrower as the frequency increases

$ka \gg 1$ highly directional radiation



First minimum happens when $ka \sin\theta_1 = j_{11}$ First zero of the Bessel func

$J_1(j_{11}) = 0 \quad J_1(j_{12}) = 0 \quad J_1(j_{13}) = 0$

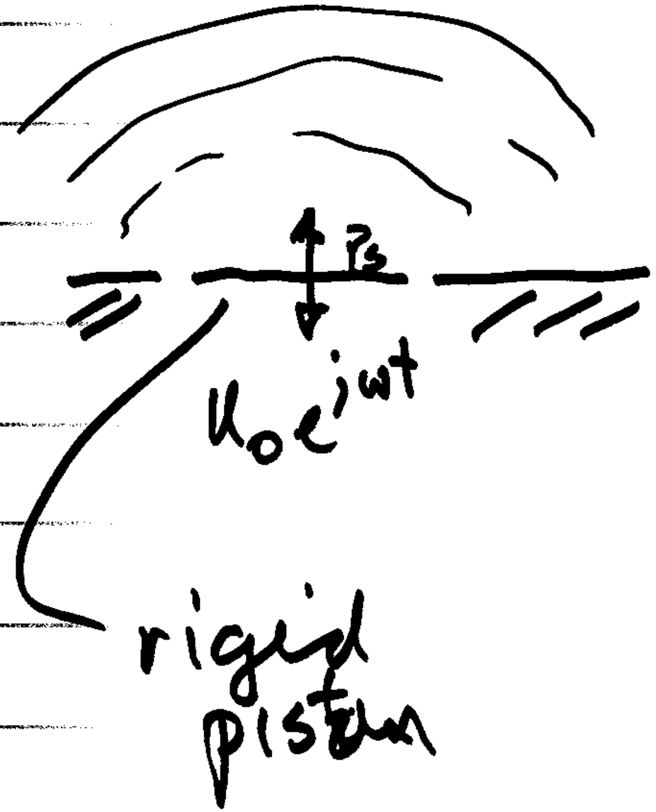
at high frequencies ($ka \gg 1$) j_{11} is reached at progressively smaller angles

"beam width" becomes narrower

Public Address Systems

- many h.f. drivers to achieve uniform dispersion
- smaller number of low frequency drivers.

5.4.1.2 Radiation Impedance



$$Z_r = \frac{\text{force exerted on the radiator by the sound field}}{\text{source velocity}}$$

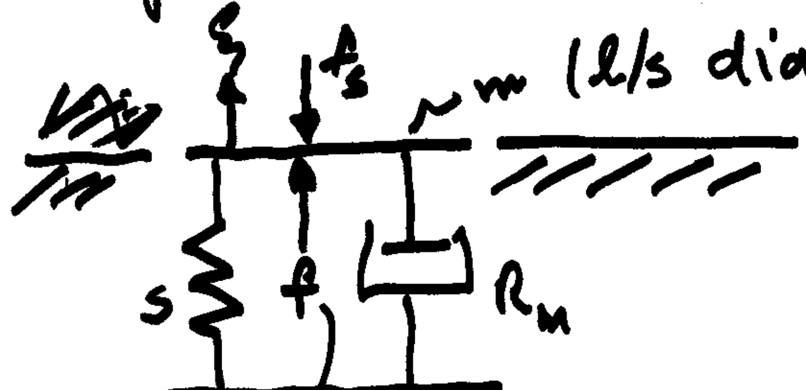
mechanical impedance

$$Z_r = \frac{\int_S p_{surface}\, dS}{u_0 Z}$$

~ total force acting on piston

uniform rigid piston

Loudspeaker - finite internal mechanical impedance



force generated by the voice coil

Equation of Motion

$$f - f_s = m\frac{d^2\xi}{dt^2} + R_m\frac{d\xi}{dt} + s\xi$$

assume harmonic motion $e^{j\omega t}$

$$u = j\omega\xi \qquad \text{diaphragm velocity}$$

$$f - f_s = j\omega m u + R_m u + \frac{s}{j\omega} u$$

$$= \Big[\underbrace{\overset{Z_m}{R_m + j\left(\omega m - \frac{s}{\omega}\right)}}_{\text{in vacuo mechanical impedance of the l/s}}\Big] u$$

$$f = f_s + z_m u$$

$$= (z_r + z_m) u \qquad z_r = \frac{f_s}{u}$$

radiation impedance

mechanical impedance

$$u = \frac{f}{z_r + z_m}$$

loudspeaker response is determined by both mechanical & radiation impedance

$$Z_r = \underset{\uparrow \atop \text{resistance}}{R_r} + j\underset{\uparrow \atop \text{reactance}}{X_r}$$

Power Delivered by the piston to the sound field

$$\Pi = \frac{1}{T}\int_0^T \mathrm{Re}\{f_s\}\,\mathrm{Re}\{u\}\,dt$$

harmonic case $e^{j\omega t}$

$$Z_r = \frac{f_s}{u}$$

$$f_s = Z_r u$$

$$\Pi = \frac{1}{2}\mathrm{Re}\{f_s u^*\}$$

$$= \frac{1}{2}\mathrm{Re}\{Z_r u u^*\}$$

$$\overline{\Pi} = \frac{|u|^2}{2} \underbrace{\mathrm{Re}\{Z_r\}}_{R_r}$$

$$= \frac{|u|^2}{2} R_r$$

m.s. velocity of the piston

Circular Piston

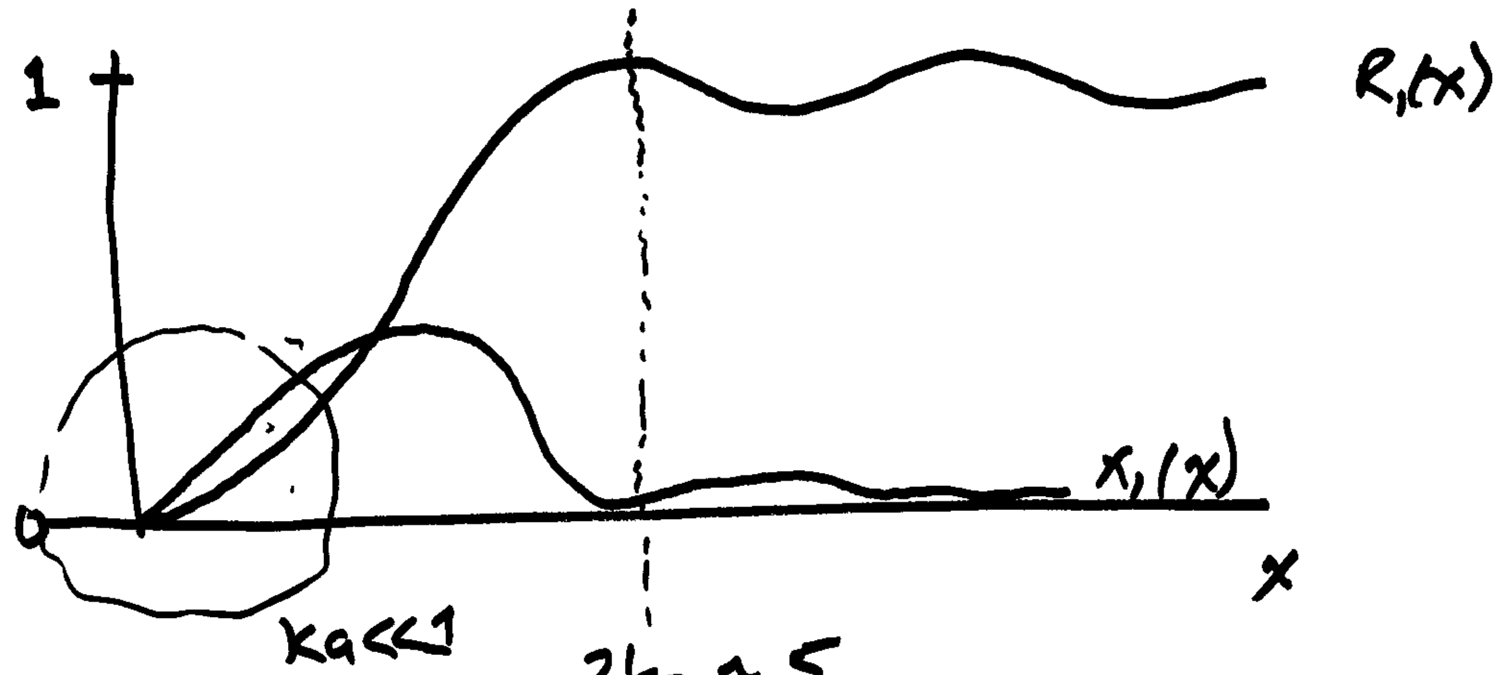
$$R_r = \pi a^2 \rho_0 c \, R_1(2ka)$$
$$X_r = \pi a^2 \rho_0 c \, X_1(2ka)$$

circular rigid piston in an infinite baffle.

piston radius of a

$$R_1(x) = 1 - \frac{2 J_1(x)}{x}$$

$$X_1(x) = \frac{4}{\pi}\left[\frac{x}{3} - \frac{x^3}{3^2 \cdot 5} + \frac{x^5}{3^2 \cdot 5^2 \cdot 7} - \cdots\right]$$



small piston case

$R_1(x)$
$X_1(x)$

$2ka$

$$R_r \simeq \frac{\pi a^2}{2} \rho_0 c \, (ka)^2$$

$$X_r \simeq \frac{\pi a^2}{2} \rho_0 \frac{8}{3\pi} (ka) \longrightarrow \text{mass-like impedance}$$