– Compact Sources

$D \ll \lambda$

special types of compact
simple

Monopole - volume
Dipole - force
Quadrupole - moment

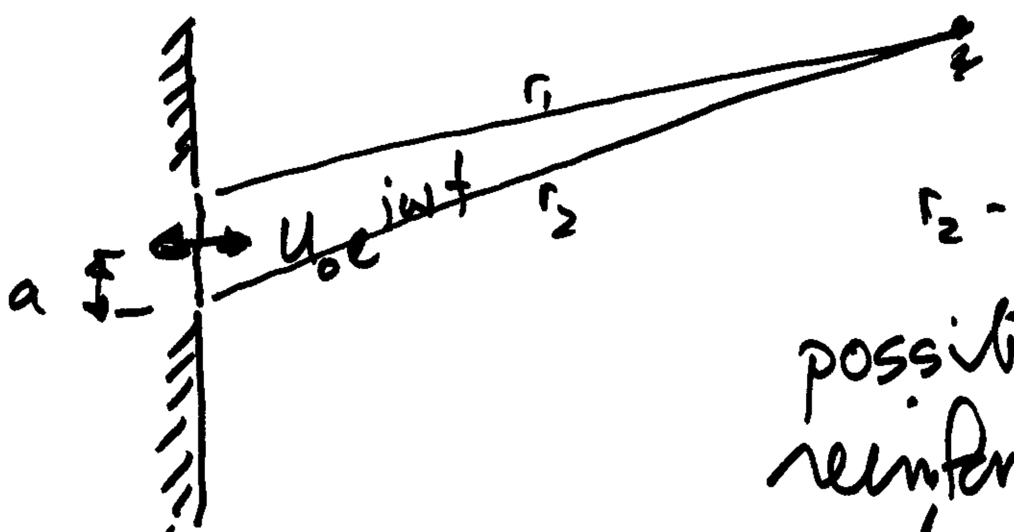
$\downarrow$ radiation efficiency drops

5.4 Sound radiation from extended sources

- finite extent

$D = O(\lambda)$

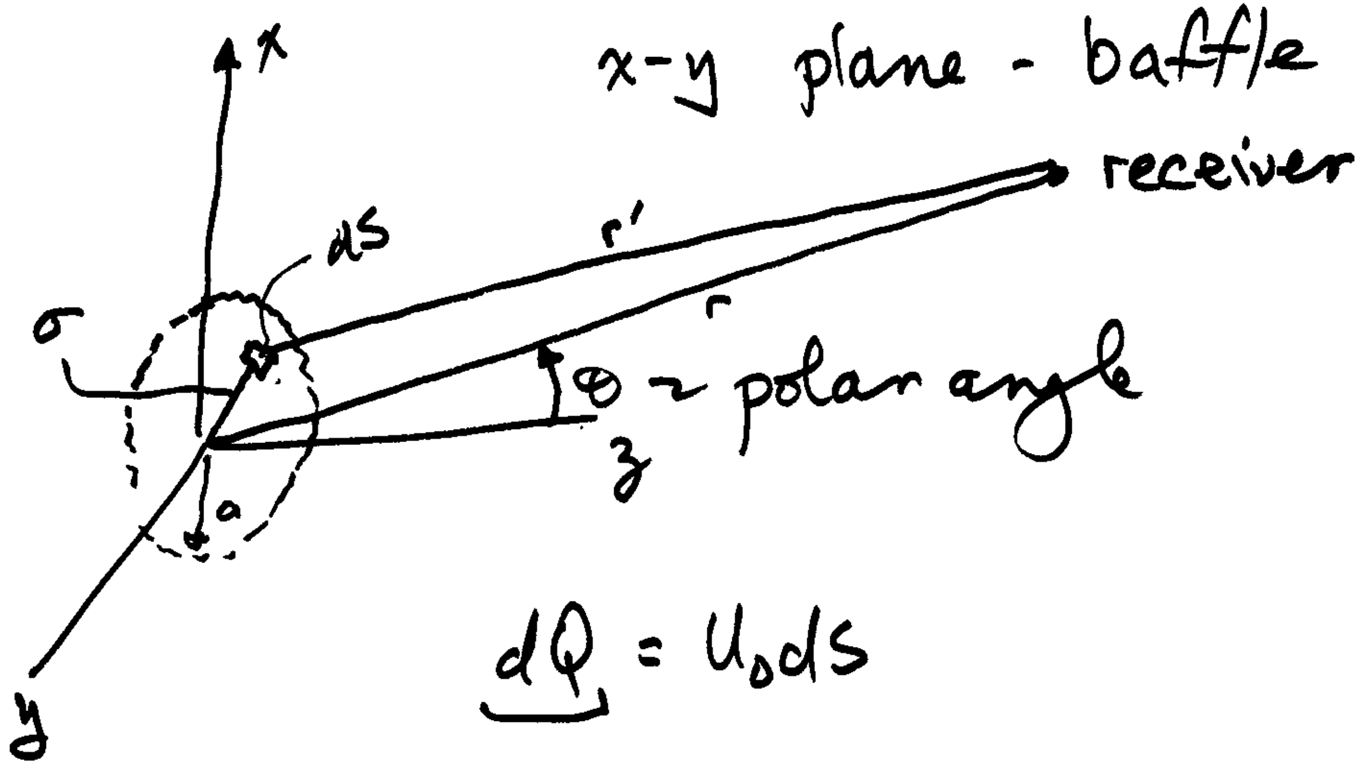
5.4.1 Piston in a baffle



$a = O(\lambda)$

$r_2 - r_1 = O(\lambda)$

possibility of reinforcement and cancellation



$$\underbrace{dQ} = U_0\, dS$$

axisymmetric

Contribution from the incremental source dS

$$d\tilde{p} = j\rho_0 c \, \frac{k\,(dQ)}{2\pi r'} \, e^{-jkr'}$$

incremental volume source strength

monopole on a hard surface.

$$dQ = U_0 dS$$

$$d\hat{p}(\bar{r}) = j\rho_0 c\, k\, U_0\, dS \, \frac{e^{-jkr'}}{2\pi r'}$$

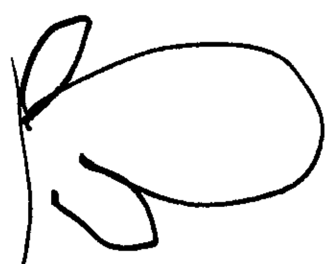
Integrate over the surface of the source

$$\tilde{p}(\bar{r},\theta) = j\rho_0 c \frac{u_0}{2\pi} k \int_{S_2} \frac{e^{-jkr'}}{r'} dS$$

piston area

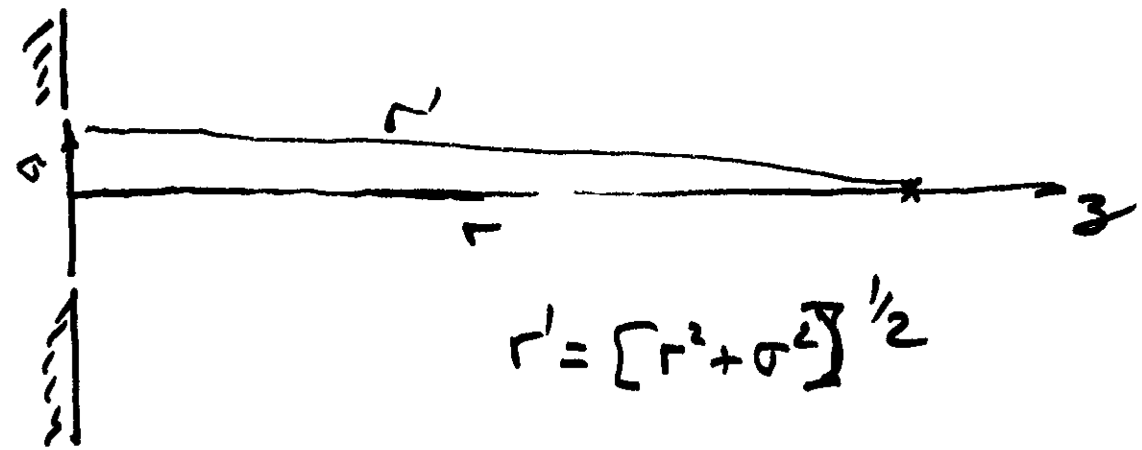
(i) on-axis

(ii) farfield

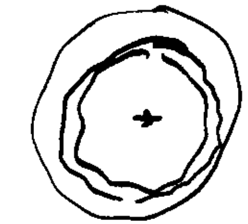


Evaluate the pressure on-axis

$\hat{p}(r,0)$

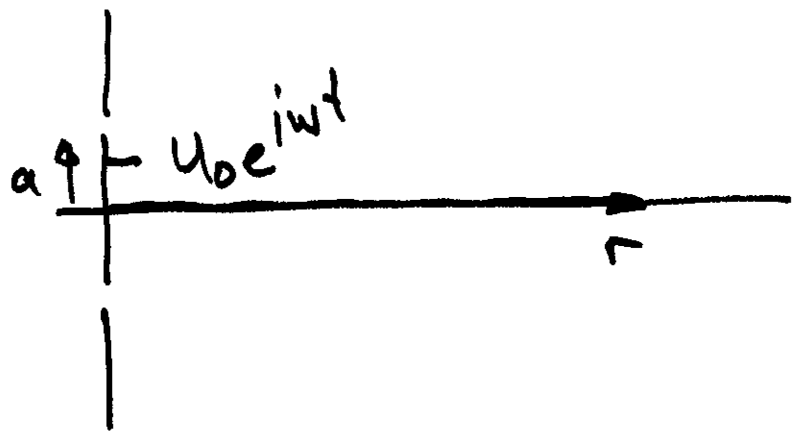


$$r' = [r^2+\sigma^2]^{1/2}$$



$$\hat{p}(r,0) = j\rho_0 c \frac{U_0 k}{2\pi} \int_0^a \frac{e^{-jk(r^2+\sigma^2)^{1/2}}}{(r^2+\sigma^2)^{1/2}} \underbrace{2\pi\sigma\, d\sigma}_{dS}$$

$$\hat{p}(r,0) = j\rho_0 c\, U_0 e^{-jkr} \underbrace{e^{-j\frac{kr}{2}\left[\sqrt{1+\frac{a^2}{r^2}}-1\right]}}_{\text{phase}}\; 2j \underbrace{\sin\left[\frac{kr}{2}\left[\sqrt{1+\frac{a^2}{r^2}}-1\right]\right]}_{\text{oscillatory function of position and also dictates the spatial variation}}$$



special case

$ka \ll 1$ (compact source)

farfield $\frac{a}{r} \ll 1$

should reduce to monopole on hard surface farfield

$$\sqrt{1+\frac{a^2}{r^2}}$$

$$\frac{a^2}{r^2} << 1$$

$$(1+x)^{1/2}$$

$$\simeq 1 + \frac{1}{2}\frac{a^2}{r^2}$$

$$= 1 + \frac{x}{2} + \frac{x^2}{2\cdots}$$

$$\sin\frac{kr}{2}\left[\cancel{1} + \frac{1}{2}\frac{a^2}{r^2} - \cancel{1}\right]$$

$$\sin\left(\frac{ka}{4}\left(\frac{a}{r}\right)\right)$$

$ka << 1$ compact

$\frac{a}{r} << 1$ farfield

$$\simeq \frac{ka}{4}\left(\frac{a}{r}\right)$$

substitute into the complete expression + take the magnitude

$$|\tilde{p}(r,0)| \simeq \rho_0 c \frac{U_0}{2}(ka)\frac{a}{r} \qquad Q = \pi a^2 U_0$$

$$= \rho_0 c \frac{kQ}{2\pi r}$$

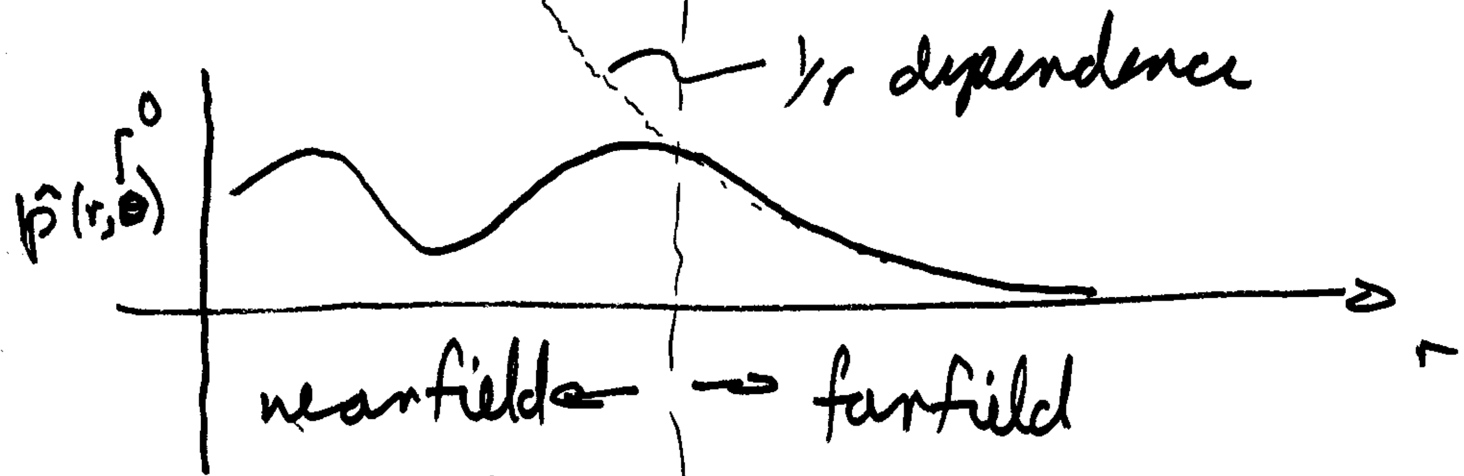
exactly the same as a monopole on a hard surface.

Piston in a baffle

loudspeaker flush mounted with a wall
if $ka \ll 1$ - model the l/s as a monopole on a hard surface.

on-axis – complete solution

– solution for the case $ka \not\ll 1$

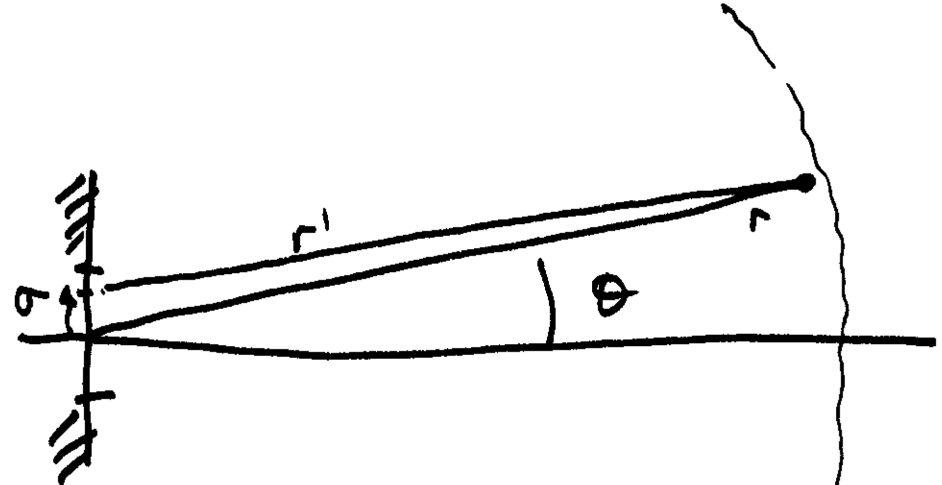


oscillatory sound field

both increase & decrease in the nearfield

2 relatively large pld's in the nearfield

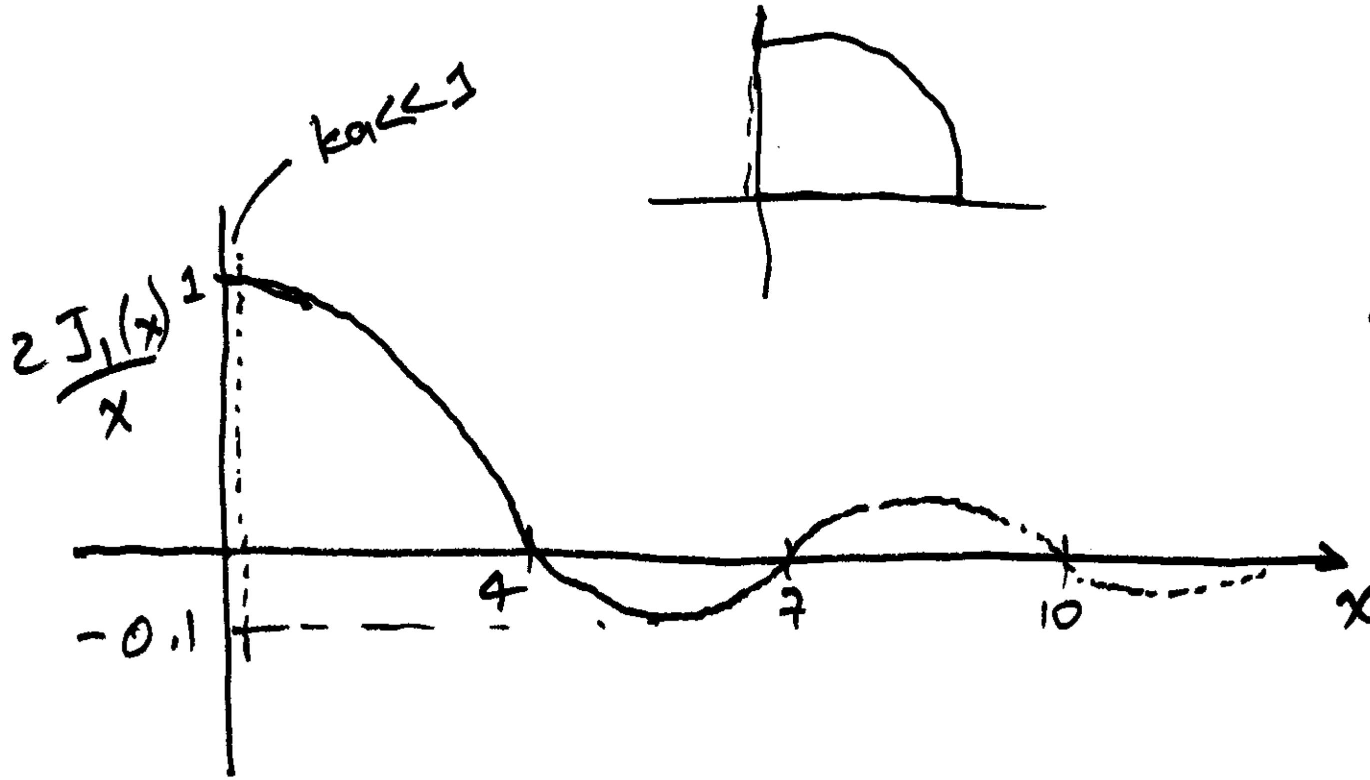
farfield



$$\frac{a}{r} \ll 1$$

- replace r' by r in the denominator
- replace r' by $r + \sigma$ in the phase term

$$\tilde{p}(r,\theta) = \underbrace{j \frac{\rho_0 c}{2} U_0 \left(\frac{a}{r}\right) e^{-jkr}}_{\text{monopole}} \underbrace{\left[2 \frac{J_1(ka\sin\theta)}{ka\sin\theta} \right]}_{\text{directivity factor}} \checkmark$$



$$\left[2\,\frac{J_1(ka\sin\theta)}{ka\sin\theta} \right]$$

$ka\sin\theta = x$

$k = \frac{\omega}{c}$

a - piston radius

θ - polar angle.

if $ka \ll 1$ $[df] \simeq 1$ for all angles