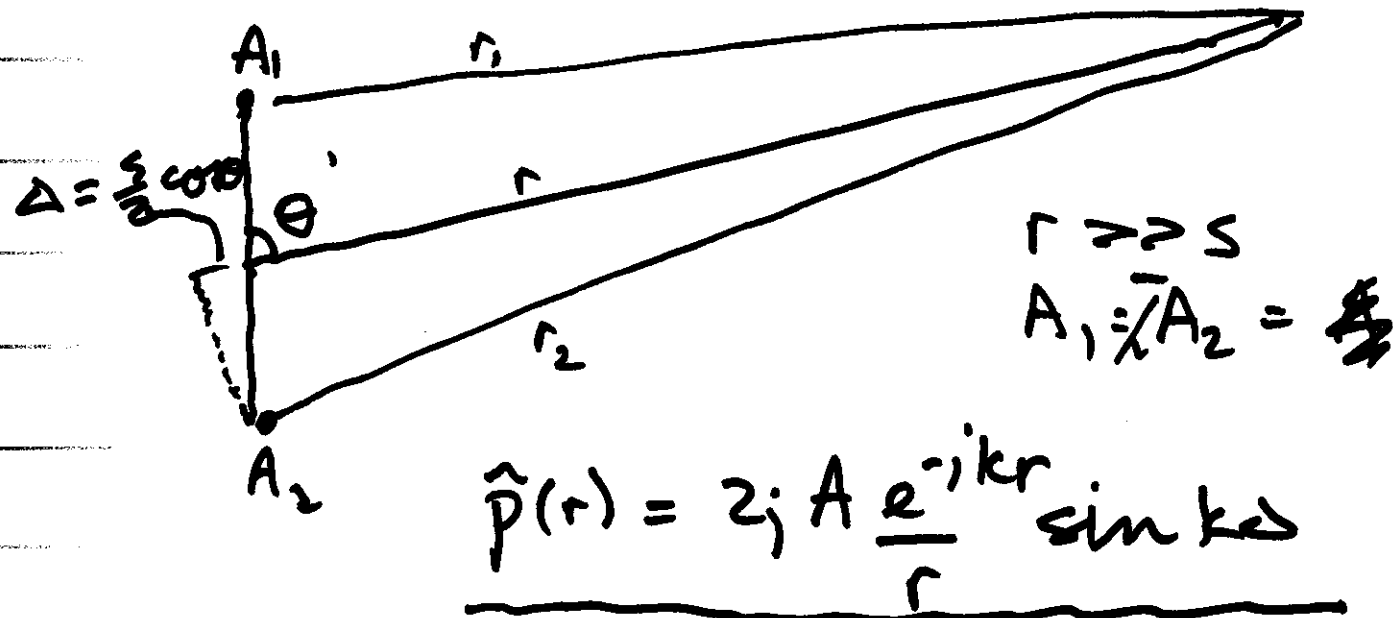


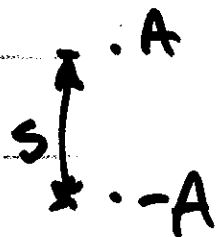
-dipole



Force same



Compact source assumption



$$ks \ll 1$$

$s$  is small compared to a wavelength.

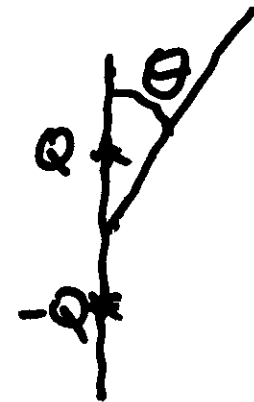
$$\sin kd$$

$$= \sin \left( \underbrace{\frac{ks}{2} \cos \theta}_{\text{very small}} \right)$$

$$\approx \frac{ks}{2} \cos \theta$$

$$\sin \theta \approx \theta \quad \theta \ll 1$$

$$\tilde{p}(r) = 2j A \frac{e^{-jkr}}{r} \frac{ks}{2} \cos\theta$$



$$A = j \rho_0 c \frac{kQ}{4\pi} \Big] \text{monopole derivation}$$

$Q = \text{volume source strength.}$

$$\tilde{p}(r) = - \rho_0 c k^2 \underbrace{(QS)}_D \frac{e^{-jkr}}{4\pi r} \cos\theta$$

$\underbrace{\hspace{1.5cm}}_D$  Dipole source strength

$$D = QS$$

$\rightarrow = \text{Dipole moment}$

sound field radiated by a  
point dipole

Assumptions:

- compact source  $ks \ll 1$
- receiver be in the farfield

$$\Delta \ll r \quad s \ll r$$

For a monopole

$$\tilde{p}(r) = i \rho_0 c \frac{kQ}{4\pi r} e^{-ikr}$$

For a dipole

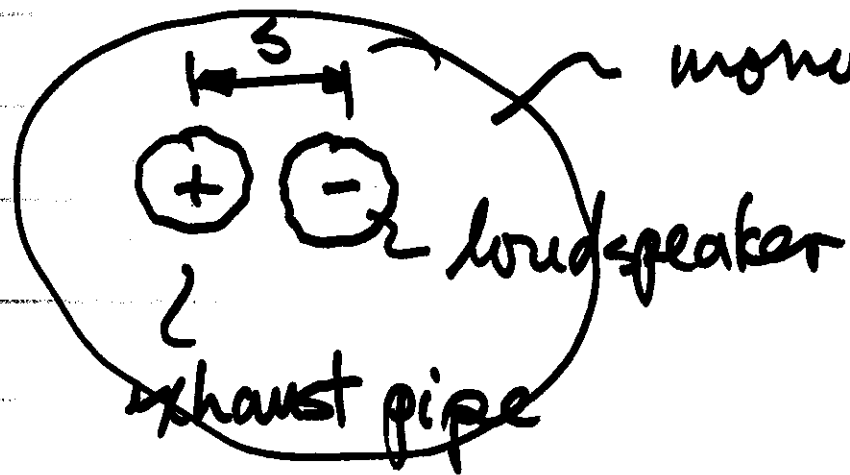
$$\tilde{p}(r) = -\rho_0 c k^2 (Qs) \frac{e^{-ikr}}{4\pi r} \cos\theta$$

$$\frac{|\tilde{p}(r)|_{\text{dipole}}}{|\tilde{p}(r)|_{\text{monopole}}} = ks \cos\theta \quad ks \ll 1$$

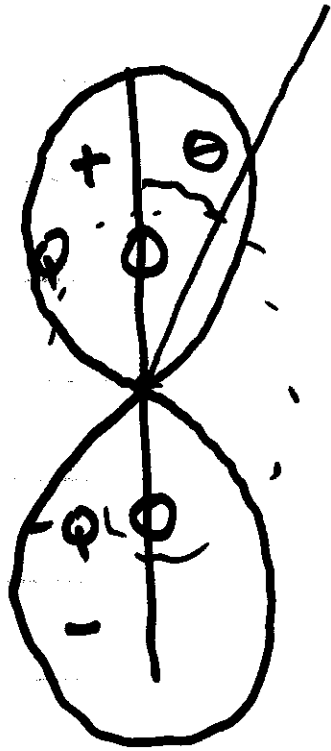


for a given monopole source strength

$$|\tilde{P}_{\text{dipole}}| \ll |\tilde{P}_{\text{monopole}}|$$



- reduced radiation  
because radiation  
efficiency is  
reduced.



$$\tilde{p}(r) = -\rho_0 c k^2 Q_s \frac{e^{-jkr}}{4\pi r} \cos \theta$$

pressure is a maximum when

$$\theta = 0 \text{ or } \underline{\underline{\pi}}$$

pressure is a minimum when

$$\theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

$$\tilde{p}(\theta=0) = -\tilde{p}(\theta=\pi)$$

sound radiation is axisymmetric

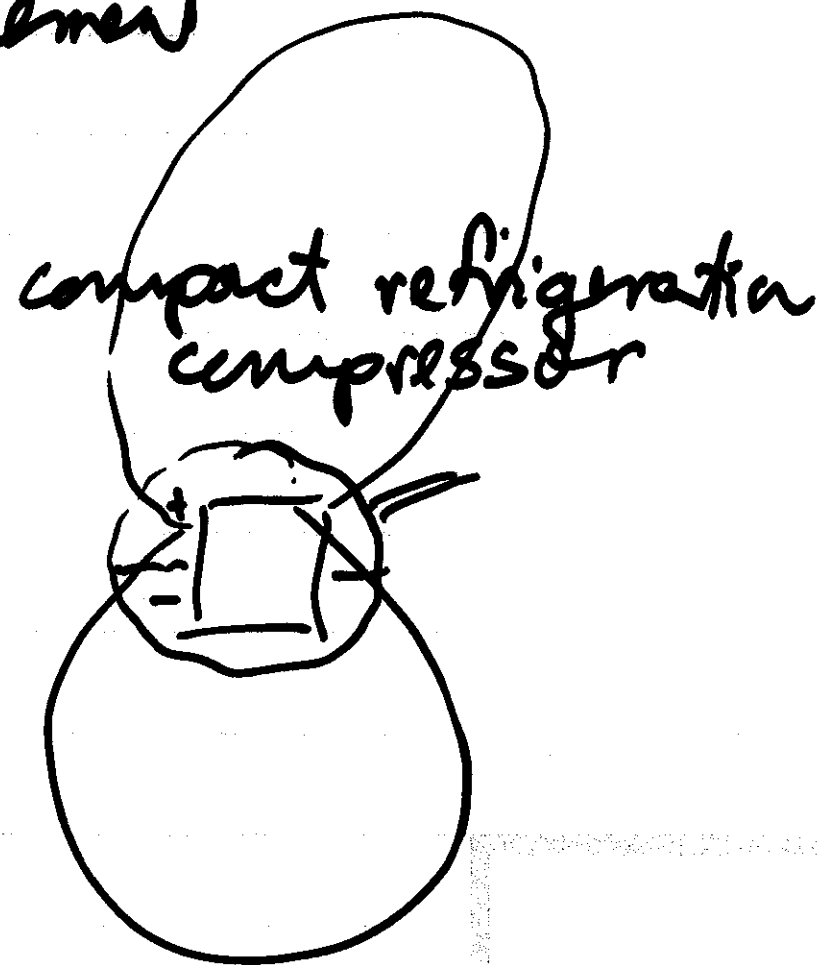
Dipole - no net volume displacement

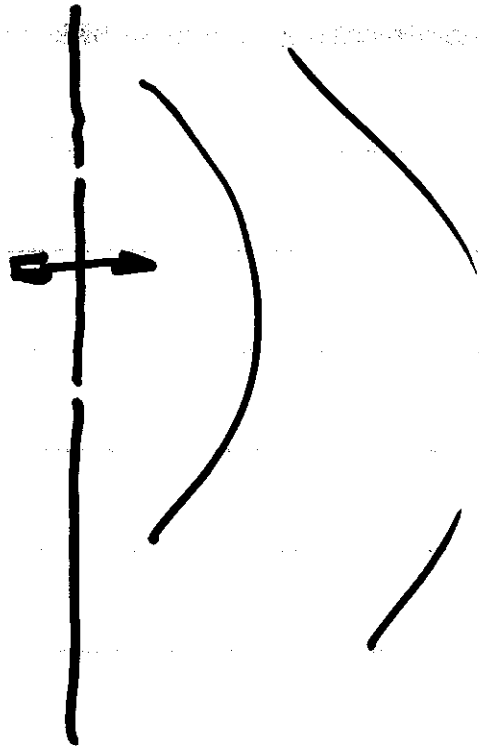
7

unaffixed



8  
axial  
fan





increase the radiation  
efficiency of a loudspeaker  
by baffling it

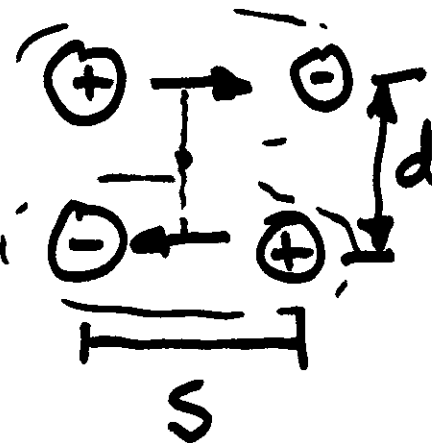


### 5.3.6 Quadrupole, etc.

9

- Array of two dipole

lateral  
quadrupole

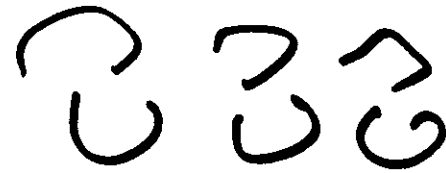


Compact  
 $kS \ll 1$   
 $kd \ll 1$

quadrupole is used to represent a compact source that induces oscillatory rotation

- represents a point moment

# Turbulent flows



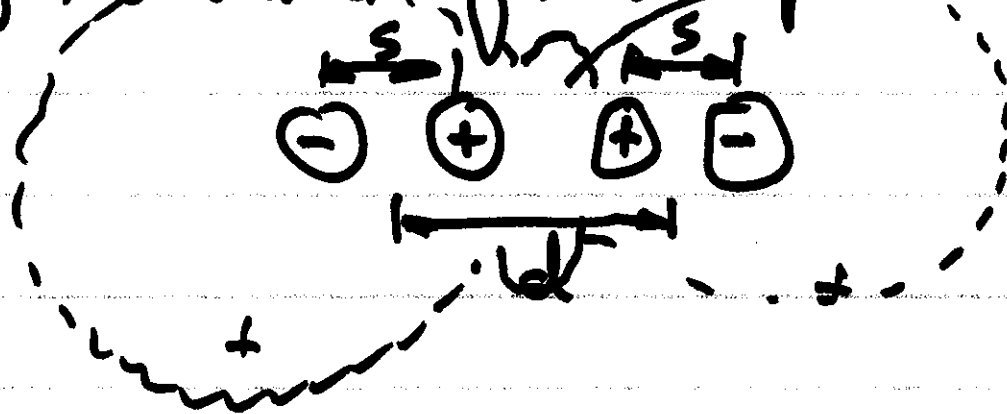
- many vortices  
interacting &  
changing rotation rates

lateral quadrupoles are used to  
represent sound radiation from  
homogeneous turbulence (no boundaries)

M. J. Lighthill

- 11
- turbulent interaction with a hard boundary
  - dipole in character

longitudinal quadrupole

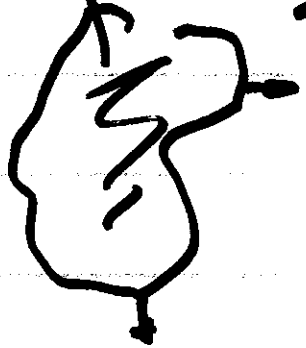


$$k_s \ll 1$$

$$kd \ll 1$$

# Multipole Decomposition

arbitrary  
source (regardless of size)



$$\vec{p}(r) = \sum \text{monopole}$$

+  $\sum$  dipoles

+  $\sum$  quadrupoles

+  $\sum$  octupoles

+

⋮

ESM

Equivalent

source

method.

per unit source strength

$|P_{monopole}| \gg |P_{dipole}| \gg |P_{quadrupole}|$   
 $\rightarrow$  increasing internal cancellation