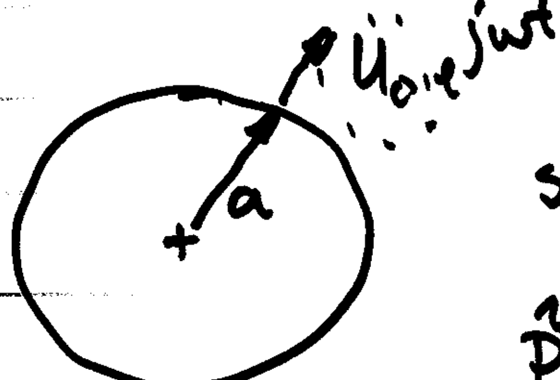
Forney G124 12/10/13 Tuesday
7 - 9 pm

Compact source - small compared to a wavelength

simple source - special compact source - each represents a particular physical source mechanism



spherical symmetry

$$\hat{p} = \frac{\hat{A}}{r} e^{-jkr}$$

apply b.c. on velocity at r = a

$$\underline{\tilde{u}_r(r)} = -\frac{1}{j\omega\rho_0}\frac{d\hat{p}}{dr} = \frac{\tilde{A}}{\rho_0 c}\frac{e^{-jkr}}{r}\left(1 - \frac{j}{kr}\right)$$

$$\hat{p}(r) = \frac{\hat{A}}{r}e^{-jkr}$$

nearfield

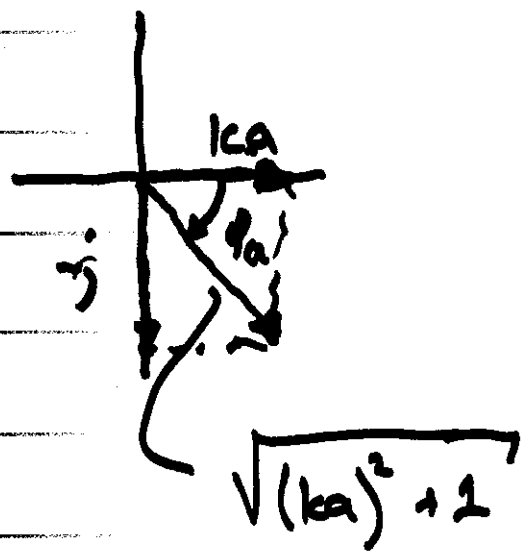
$U_0 e^{j\omega t}$ at $r = a$

$$\tilde{u}_r(a) = U_0 \qquad \text{b.c.}$$

solve for $\tilde{A}$

$$\hat{A} = \rho_0 c\, a\, U_0\, e^{jka} \frac{1}{1 - \frac{j}{ka}}$$

$$\tilde{A} = \rho_0 c a U_0 e^{jka} \frac{ka}{ka - j} \qquad ka - j = \sqrt{(ka)^2+1}\, e^{-j\phi_a}$$



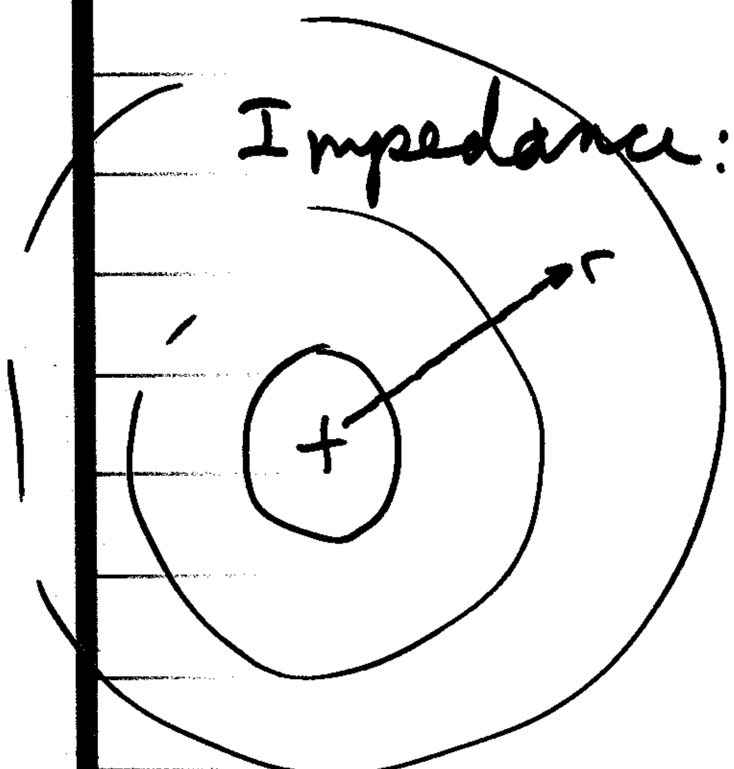
$$= \rho_0 c a U_0 e^{jka} \underbrace{\frac{ka}{\sqrt{(ka)^2+1}}}_{\cos\phi_a} e^{-j\phi_a}$$

$$= \tilde{A} = \rho_0 c a U_0 e^{jka} \cos\phi_a \, e^{+j\phi_a}$$

$$\tilde{p}(r) = \left(\rho_0 c a U_0 e^{jka} e^{j\phi_a} \cos\phi_a\right) \frac{e^{-jkr}}{r}$$

sound field generated by a pulsating sphere of radius a

5.2.3 Impedance, Intensity & Sound Power



Impedance: $$\frac{\tilde{p}(r)}{\tilde{u}_r(r)} = \tilde{z}(r) = \rho_0 c \, \frac{1}{1 - \frac{j}{kr}}$$

$$= \rho_0 c \, \frac{kr}{kr - j}$$

$$= \rho_0 c \cos\phi_r \, e^{j\phi_r}$$

$$\cos\phi_r = \frac{kr}{\sqrt{(kr)^2 + 1}}$$

farfield $kr \gg 1$ $\quad \tilde{z}(r) \rightarrow \rho_0 c$

plane wave

$\cos\phi_r \rightarrow 1$

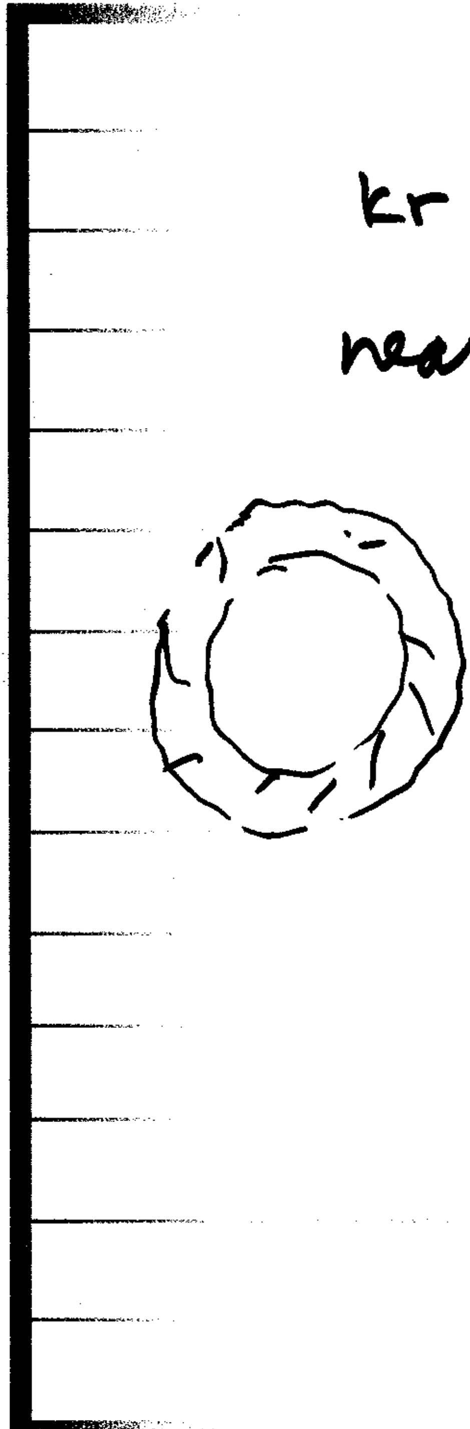
$kr \ll 1$

nearfield

$\hat{z} \simeq +j\rho_0 c\,(kr)$

mass-like
impedance

$k = \frac{\omega}{c}$



$$I_r = \frac{1}{2} \mathrm{Re}\left\{ \tilde{p}(r)\, \tilde{u}_r^{*}(r) \right\}$$

$$= \frac{1}{2} \left(\frac{a}{r}\right)^2 \rho_0 c\, U_0^2 \cos^2 \phi_a$$

$I_r \propto \dfrac{1}{r^2}$ Inverse Square Law

Sound Power

$$W = \int_S I_r \, dS = I_r \int_S dS = 4\pi r^2 I_r$$

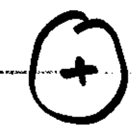
$$\underline{W = 2\pi a^2 \rho_0 c\, U_0^2 \cos^2 \phi_a}$$ independent of r

5.3 simple sources

5.3.1 Point Monopole

- compact source that changes volume
- volume source

exhaust



$$\tilde{p} = \hat{A}\frac{e^{-jkr}}{r}$$ free field

$$\hat{A} = j\rho_0 c\, a\, U_0 e^{jka}\frac{ka}{ka-j}$$ sphere

$4\pi a^2 u_0$ = Volume displaced by the sphere / s
$[m^3/s]$
= Q = monopole source strength.

$$\tilde{A} = j\omega\rho_0 (ka) \frac{Q}{4\pi (ka)^2} e^{jka} \frac{ka}{jka + 1}$$

$ka = 2\pi\left(\frac{a}{\lambda}\right)$ non-dim radius of the source

$$\lim_{(ka)\to 0} \tilde{A} = j\omega\rho_0 \frac{Q}{4\pi}$$

] for a source that is very small compared to a wavelength

"point source"

point monopole

$$\hat{p}(r) = j\omega\rho_0 \frac{Q}{4\pi r} e^{-jkr}$$

$$\boxed{\hat{p}(r) = j\rho_0 c \frac{kQ}{4\pi r} e^{-jkr}}$$

– omni-directional source

Radial Intensity

I_r for $ka \ll 1$

$$\cos\varphi_a = \frac{ka}{\sqrt{(ka)^2+1}}$$

$\cos\varphi_a \to (ka)$ for small ka

$$\lim_{(ka)\to 0} I_r = \frac{\rho_0 c}{2} k^2 \frac{Q^2}{(4\pi r)^2}$$

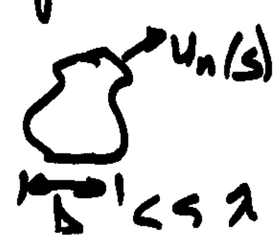
$$I_r \propto a^4$$

small sources are very ineffective radiators

Use a point source to represent any compact source that exhibits a periodic volume change and which displaces ϕ volume of fluid/s.

If a source is small compared to a wavelength in <u>all</u> dimensions

- <u>The sound radiation does not depend on the spatial distribution of the velocity</u>



$$\phi = \int_S u_n(s)\,dS$$

represent as point monopole.