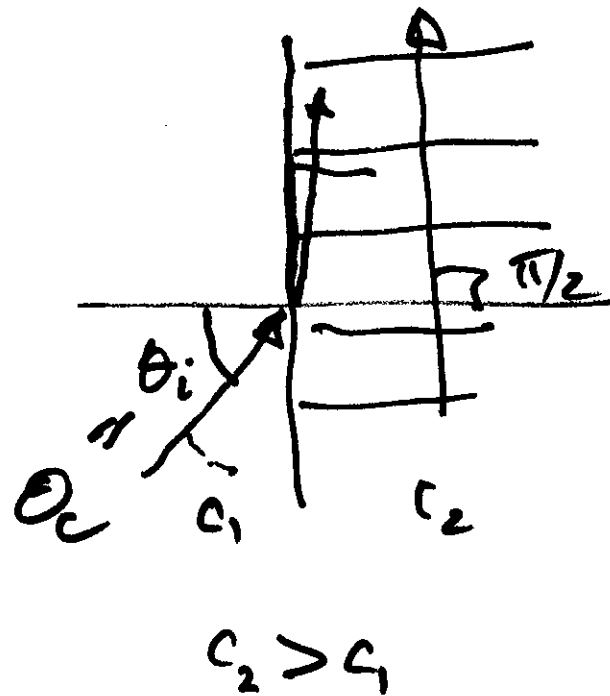


Snell's law

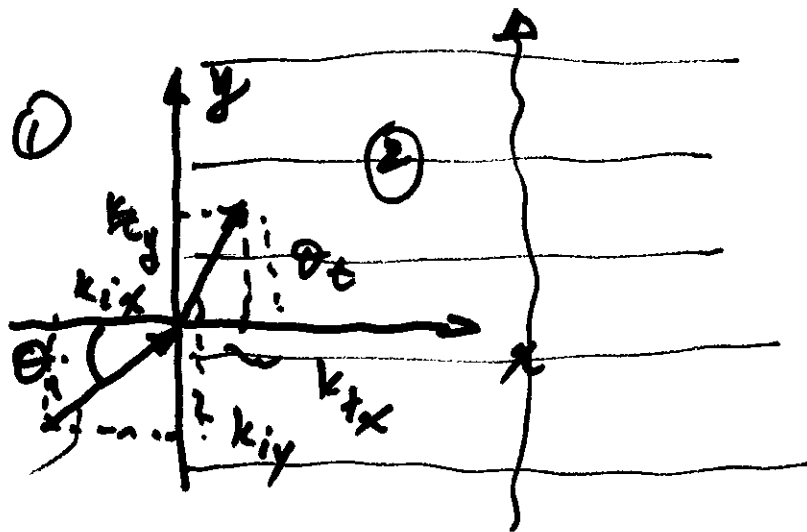
R, T $\frac{c_2}{c_1}$

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{c_2}{c_1}$$

Critical Angle



Transmitted Field



$$\tilde{P}_t = P_t e^{-j(k_{tx}x + k_{ty}y)}$$

$$k_{ty} = k_{iy} = k_1 \sin \theta_i$$

$$k_{tx} = k_2 \cos \theta_t$$

$$= \pm k_2 \sqrt{1 - \left(\frac{c_2}{c_1}\right)^2 \sin^2 \theta_i}$$

1 when $\theta_i = \theta_c$

when $\theta_i = \theta_c$ $k_{tx} = \pm k_2 \sqrt{1 - 1} = 0$

no prop in the
x-direction

when $\theta_i > \theta_c$

$$\sin \theta_t = \frac{c_2 \sin \theta_i}{c_1} > 1$$

when $\theta_i > \theta_c$

$$\sin \theta_t > 1$$

$$\sin \theta_t = \frac{e^{j\theta_t} - e^{-j\theta_t}}{2j}$$

$|\sin \theta_t|$ can be > 1 when θ_t is imaginary or complex

$$\theta_i > \theta_c$$

$$k_{tx} = \pm k_2 \sqrt{1 - \underbrace{\left(\frac{c_2}{c_1}\right)^2 \sin^2 \theta_i}_{\substack{\sin \theta_t \\ > 1}}}$$

negative

$$k_{tx} = \pm k_2 \sqrt{(-1) \left[\underbrace{\left(\frac{c_2}{c_1}\right)^2 \sin^2 \theta_i - 1}_{\text{positive}} \right]}$$

$$= \pm j \left(k_2 \sqrt{\left(\frac{c_2}{c_1}\right)^2 \sin^2 \theta_i - 1} \right) \quad \gamma \text{ real \& positive}$$

$$= \pm j\gamma$$

transmitted field

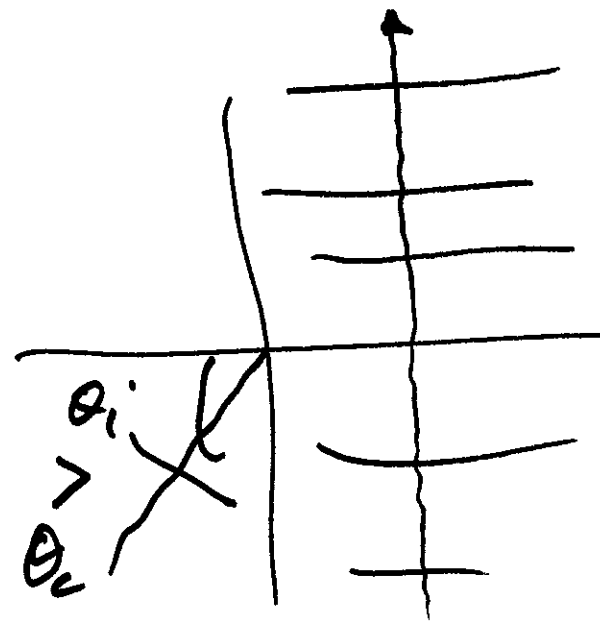
$$\begin{aligned} \hat{P}_t^z &= P_t e^{-j k_{tx} x} e^{-j k_{ty} y} \\ &= P_t e^{-j(\pm j\gamma)x} e^{-j k_{ty} y} \\ &= P_t e^{\pm \gamma x} e^{-j(k_1 \sin \theta_i) y} \end{aligned}$$

pure
exponential
growth or
decay.

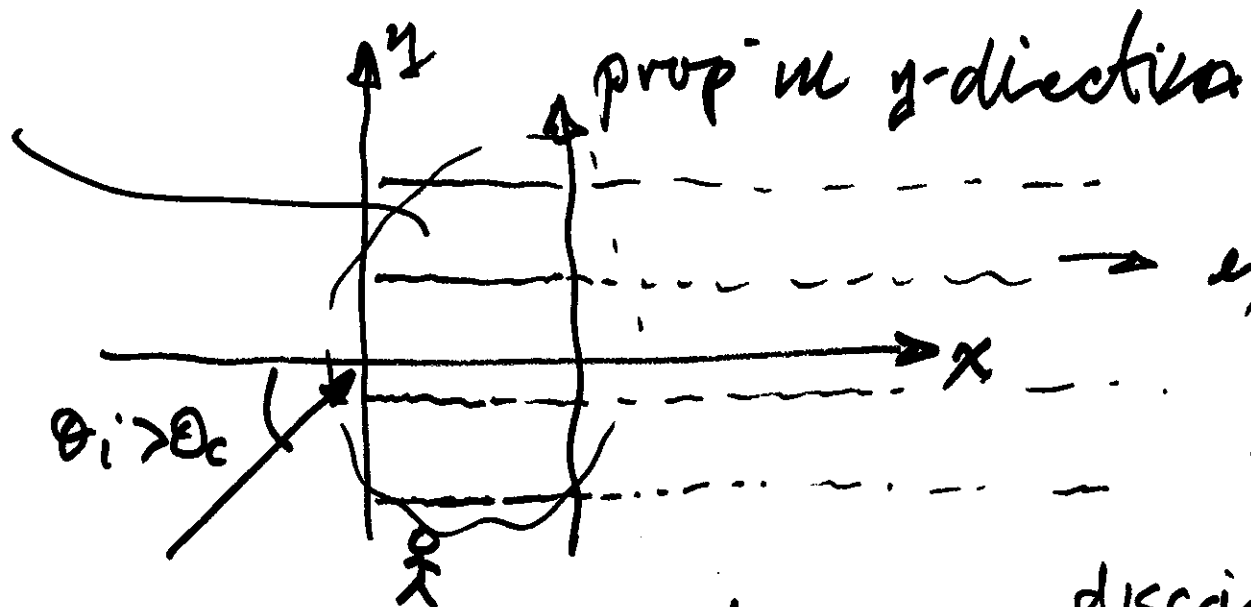
away from the interface

always real
(for real θ_i)

always represents
prop in the y-
direction



nearfield



$$|R| = 2$$

when $\theta_i > \theta_c$ no propagation is away from the boundary

discarded the exponentially growing solution on physical grounds

- sound field in the transmitted region - inhomogeneous plane waves
- prop in one direction & decay \perp to that direction

There is a sound field in region (2)

when $\theta_i > \theta_c$ - but it decays

exponentially away from the interface

when θ_i increases beyond θ_c

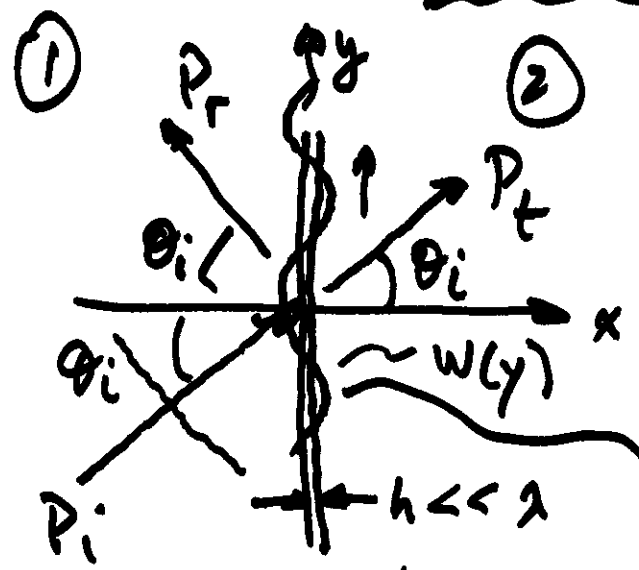
δ increases

rate of decay increases

& nearfield becomes smaller.



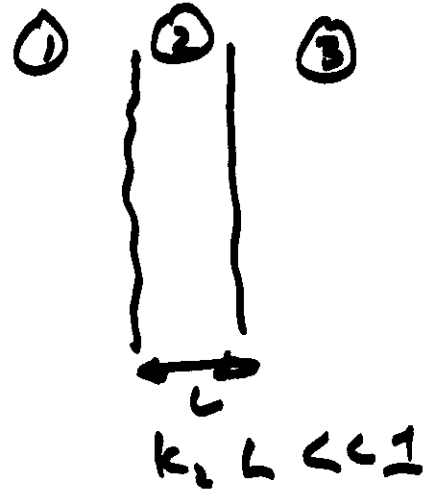
4.3.2 Reflection and Transmission at a limp, thin panel



$$\rho_1 c_1 = \rho_2 c_2 = \rho_0 c$$

no flexural stiffness (rubber sheet)

transverse displacement of the panel in response to the sound field



panel mass/unit area = m_s

one additional unknown $w(y)$

3 b.c.'s are necessary

①

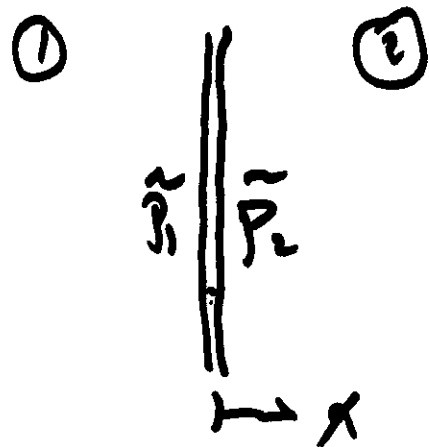
$$\hat{P}_i = P_i e^{-j(k_x x + k_y y)} + P_r e^{+j(k_x x - k_y y)}$$

$$\hat{u}_{ix} = \frac{P_i}{\rho_0 c} \cos \theta_i e^{-j(k_x x + k_y y)} - \frac{P_r}{\rho_0 c} \cos 2\theta_i e^{+j(k_x x - k_y y)}$$

$$k_x = k \cos \theta_i$$

$$k_y = k \sin \theta_i$$

$$k = \frac{\omega}{c}$$



panel is accelerated
in the x -direction
by the Δ pressure difference
across it

- "Limp"
- no flexural stiffness
 - no free wave prop possible in the panel
 - responds only at the point where the force is applied
 - surface of local reaction

accel at any point on the panel
 = pressure diff across the
 panel at that point

$F = ma$ / per unit area basis

$$(\hat{p}_1 - p_2)|_{x=0} = m_s \frac{\partial^2 \tilde{w}}{\partial t^2} \Big] \text{ at each point}$$

EOM