Exam 23rd Session 27

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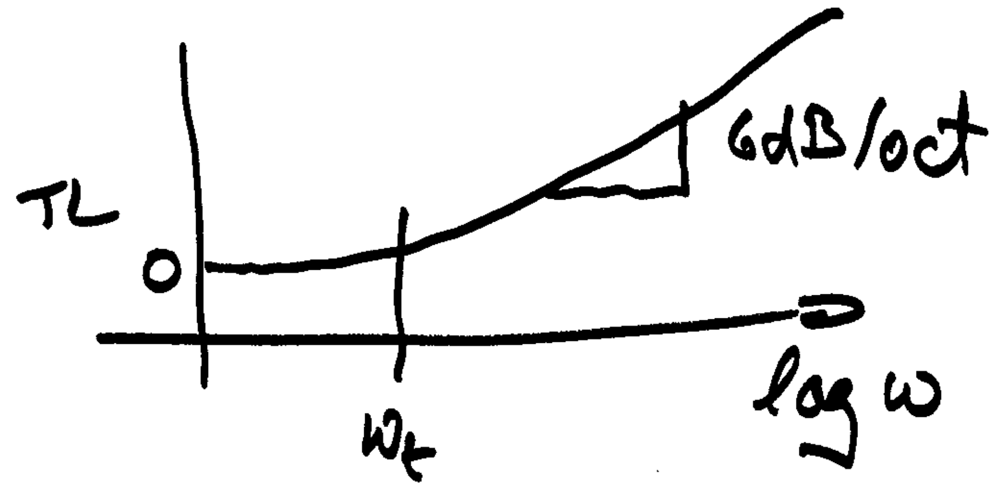
50 minutes

Thin heavy barrier



mass law

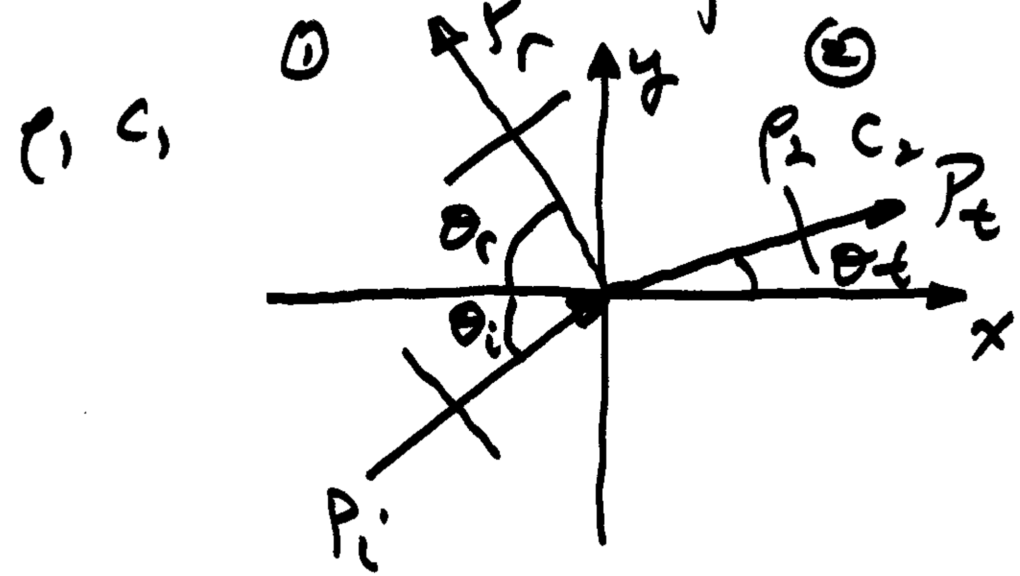
$$T = \frac{2\rho_0 c\, e^{jkL}}{2\rho_0 c + j\omega m_s}$$

$TL = TL_1 + TL_2 + 6\,dB$

$$R_I = \frac{(S_{11} - 1)^2}{(S_{21} + 1)^2}$$

4.3 Oblique Incidence Reflection & Transmission

4.3.1 Two-Fluid problem



<u>2-D problem</u>
- no propagation in the z-direction

$$\nabla^2 \tilde{p}_1 + k_1^2 \tilde{p}_1 = 0 \qquad \nabla^2 \tilde{p}_2 + k_2^2 \tilde{p}_2 = 0$$

$$k_1 = \frac{\omega}{c_1} \qquad k_2 = \frac{\omega}{c_2}$$

$$\tilde{p}_1 = \tilde{p}_i + \tilde{p}_r$$

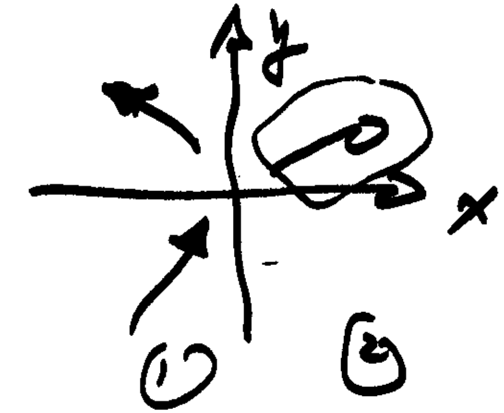
$$= P_i e^{-j(k_{ix}x + k_{iy}y)} + P_r e^{+j(k_{rx}x - k_{ry}y)}$$

$$k_{ix} = k_1 \cos\theta_i \qquad k_{iy} = k_1 \sin\theta_i \qquad k_z\text{'s} = 0$$

$$k_{rx} = k_1 \cos\theta_r \qquad k_{ry} = k_1 \sin\theta_r$$

$$k_{ix}^2 + k_{iy}^2 = k_1^2 \qquad k_{rx}^2 + k_{ry}^2 = k_1^2$$

need particle velocity normal to the interface



$$\tilde{u}_{1x} = -\frac{1}{j\omega\rho_1}\frac{\partial \tilde{p}_1}{\partial x}$$

$$= \frac{P_i}{\rho_1 c_1}\cos\theta_i \, e^{-j(k_{ix}x + k_{iy}y)} - \frac{P_r}{\rho_1 c_1}\cos\theta_r \, e^{+j(k_{rx}x - k_{ry}y)}$$

$$\tilde{p}_2 = P_t \, e^{-j(k_{tx}x + k_{ty}y)}$$

$$k_{tx} = k_2 \cos\theta_t \qquad k_{tx}^2 + k_{ty}^2 = k_2^2$$

$$k_{ty} = k_2 \sin\theta_t$$

Normal velocity in region (2)

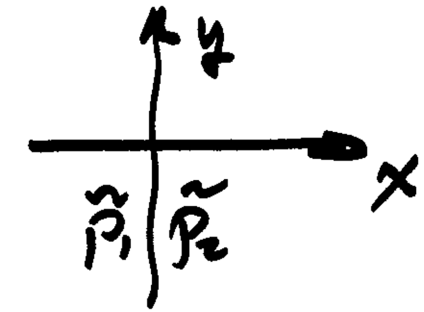
$$\tilde{u}_{2x} = -\frac{1}{j\omega\rho_2}\frac{\partial \tilde{p}_2}{\partial x}$$

$$= \frac{P_t}{\rho_2 c_2}\cos\theta_t\, e^{-i(k_{tx}x + k_{ty}y)}$$

P_i , P_r , P_t

$\div P_i$ $\quad$ $\underset{\sim}{R}$ $\quad$ $\underset{\sim}{T}$

Apply the b.c.'s



$$\tilde{p}_1(0,y) = \tilde{p}_2(0,y)$$

$$P_i \cancel{e^{-ik_{iy}y}} + P_r \cancel{e^{-ik_{ry}y}} = P_t \cancel{e^{-ik_{ty}y}}$$

b.c.'s must be independent of position in the y-direction

$$k_{iy} = k_{ry} = k_{ty}$$ if the above is to be true

$$\cancel{k_1} \sin\theta_i = \cancel{k_1} \sin\theta_r$$

$$\boxed{\theta_i = \theta_r}$$

angle of reflection = angle of incidence

$$k_{ry} = k_{ty}$$

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$

$$\left.\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1}{k_2} = \left(\frac{c_2}{c_1}\right)\right]$$ Snell's Law

Use Snell's Law
to calculate

θ_t given c_1, c_2, & θ_i

pressure b.c. at $x=0$

$$\tilde{p}_1 = \tilde{p}_2 \quad \text{at } x=0$$

$$P_i + P_r = P_t \qquad \div P_i$$

$$1 + R = T \qquad (1)$$

$$T = \frac{P_t}{P_i} \qquad R = \frac{P_r}{P_i}$$

particle velocity b.c.

Velocity ⊥ to the interface must be continuous

$$\tilde{u}_{1x}(0,y) = \tilde{u}_{2x}(0,y)$$

$$\frac{P_i}{\rho_1 c_1}\cos\theta_i - \frac{P_r}{\rho_1 c_1}\cos\theta_r = \frac{P_t}{\rho_2 c_2}\cos\theta_t \quad \div P_i$$

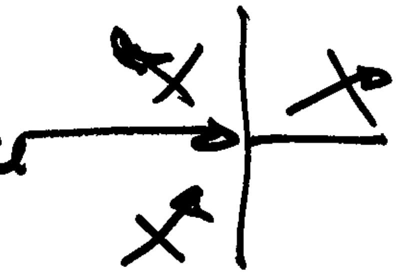
$$1 - R = \frac{1}{\zeta_{21}} \frac{\cos\theta_t}{\cos\theta_i} T \qquad (2)$$

$$\zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1}$$

solve (1) & (2)

$$R = \frac{\zeta_{21} - \frac{\cos\theta_t}{\cos\theta_i}}{\zeta_{21} + \frac{\cos\theta_t}{\cos\theta_i}}$$

normal incidence case



$\theta_i = \theta_t = 0$

$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1} \quad \theta_i = 0$$

$$T = \frac{2\zeta_{21}}{\zeta_{21} + \frac{\cos\theta_t}{\cos\theta_i}} \qquad \theta_i \to 0 \qquad T \to \frac{2\zeta_{21}}{\zeta_{21} + 1}$$

Write R & T in terms of θ_i

we know $\sin^2\theta_t + \cos^2\theta_t = 1$

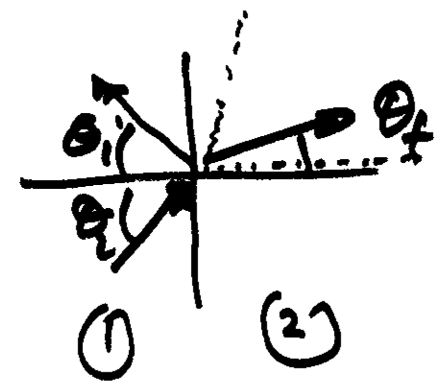
$$\cos\theta_t = \pm\sqrt{1 - \sin^2\theta_t}$$

Snell's law $\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i$

$$\cos\theta_t = \pm\sqrt{1 - \left(\frac{c_2}{c_1}\right)^2 \sin^2\theta_i}$$

sub into R & T

Notes:



① From Snell's Law

when $\frac{c_2}{c_1} < 1$ $\sin\theta_t < \sin\theta_i$

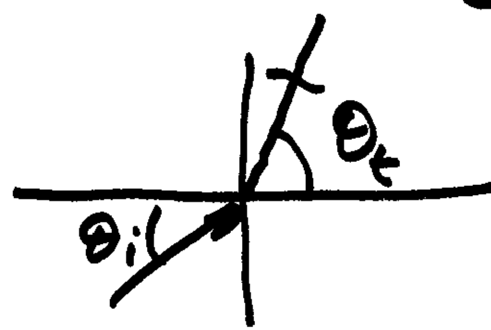
$\theta_t < \theta_i$

"sound refracts towards the normal"

when $c_2 < c_1$

② if $c_2 > c_1$

$$\sin\theta_t = \frac{c_2}{c_1}\sin\theta_i$$



$\theta_t > \theta_i$ sound refracts away from the normal

maximum possible <u>real</u> ~~angle~~ value of θ_t

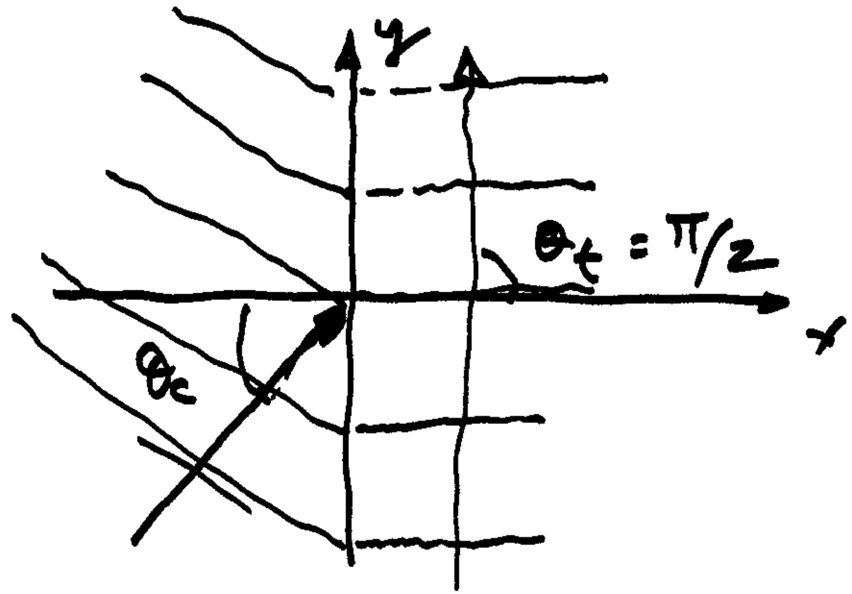
is $\pi/2$

critical incidence angle θ_c

causes $\theta_t \to \pi/2$

$$\sin\theta_t = 1 = \frac{c_2}{c_1}\sin\theta_c$$

$$\sin\theta_c = \frac{c_1}{c_2}$$] Definition of θ_c



$c_2 > c_1$

wave system when $\theta_i = \theta_c$

at θ_c sound field in region ②
propagates parallel to
the interface

$|R| \to 1$ $|T| \to 2$
Total Internal Reflection