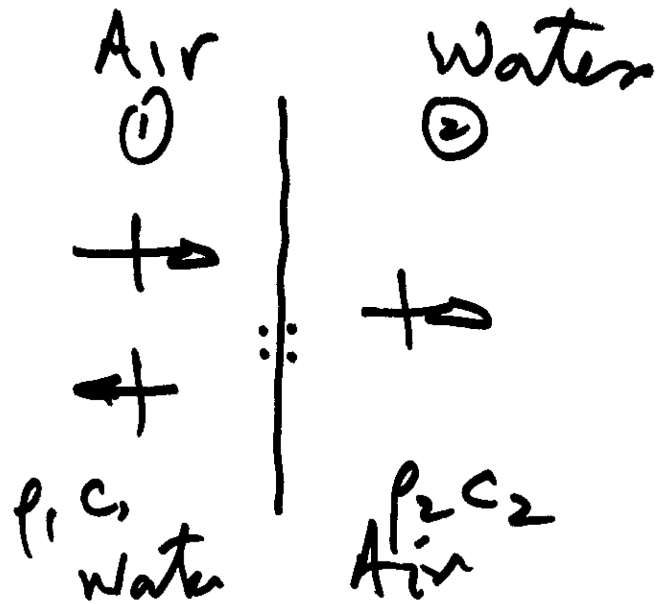
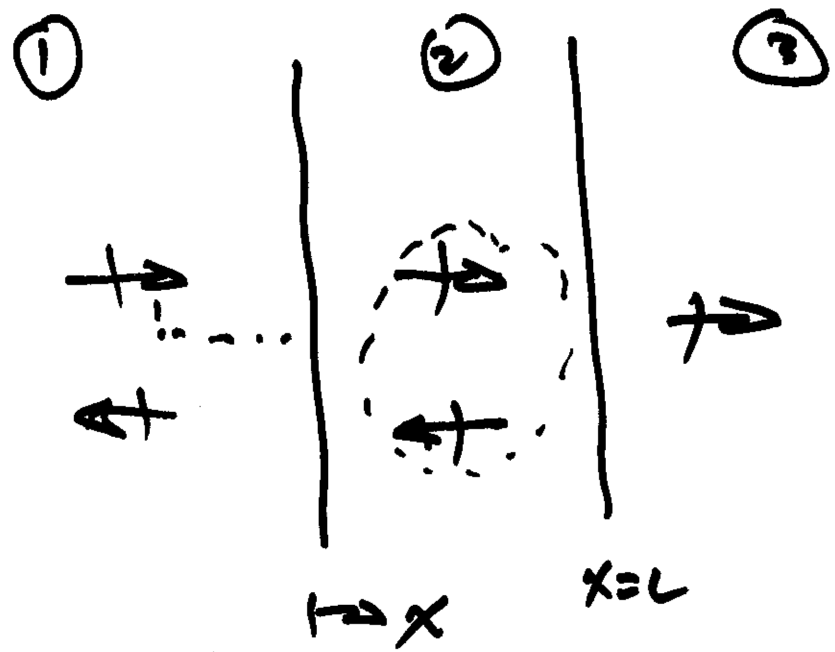


$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1} \qquad T = \frac{2\zeta_{21}}{\zeta_{21} + 1}$$

$$\zeta_{21} = \left(\frac{\rho_2 c_2}{\rho_1 c_1}\right)$$



Boundary conditions

at $x=0$ $\quad \hat{p}_1(0) = \hat{p}_2(0) \quad (1)$

$\hat{u}_{1_n}(0) = \hat{u}_{2_n}(0) \quad (2)$

at $x=L$ $\quad \hat{p}_2(L) = \hat{p}_3(L) \quad (3)$

$\hat{u}_{2n}(L) = \hat{u}_{3_n}(L) \quad (4)$

(1) $$P_i + P_r = P_A + P_B \quad \div P_i$$

$$\boxed{1 + R = A + B} \checkmark \qquad R = \frac{P_r}{P_i} \qquad A = \frac{P_A}{P_i} \qquad B = \frac{P_B}{P_i}$$

(2) $$1 - R = \frac{1}{\zeta_{21}}(A - B) \checkmark$$

$$T = \frac{P_t}{P_i}$$

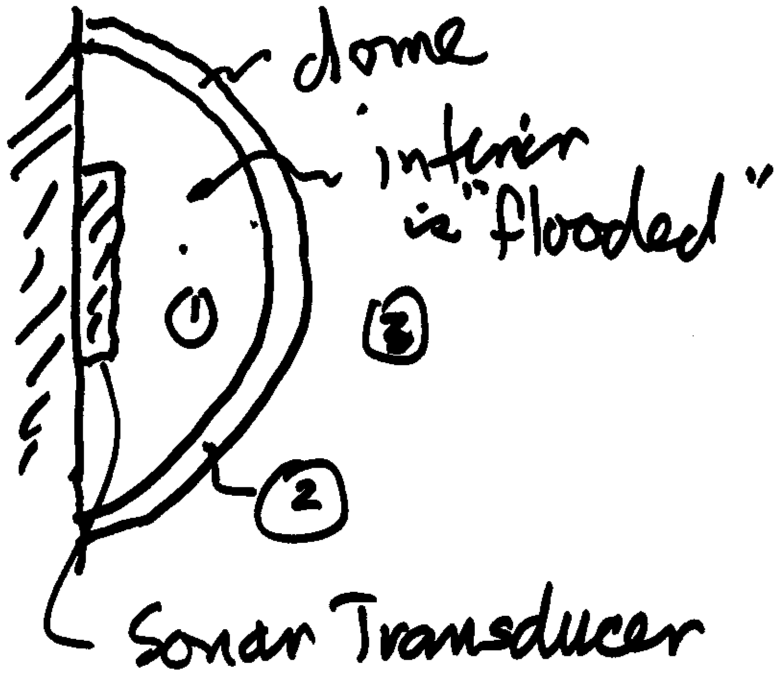
(3) $$A e^{-jk_2 L} + B e^{+jk_2 L} = T e^{-jk_3 L} \checkmark$$

(4) $$A e^{-jk_2 L} - B e^{+jk_2 L} = \frac{T}{\zeta_{32}} e^{-jk_3 L} \checkmark$$

$$\zeta_{21} = \frac{\rho_2 c_2}{\rho_1 c_1} \qquad \zeta_{32} = \frac{\rho_3 c_3}{\rho_2 c_2}$$

$$R = \frac{\left(1 - \frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos \underline{\underline{k_2 L}} + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} - \frac{\rho_1 c_1}{\rho_2 c_2}\right)\sin \underline{\underline{k_2 L}}}{\left(1 + \frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos k_2 L + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} + \frac{\rho_1 c_1}{\rho_2 c_2}\right)\sin k_2 L}$$

$$k_2 L = \frac{2\pi}{\lambda_2} L = 2\pi\left(\frac{L}{\lambda_2}\right)$$ non-dimensional layer depth



Sonar Dome problem

$R = 0 \qquad (T = 1)$

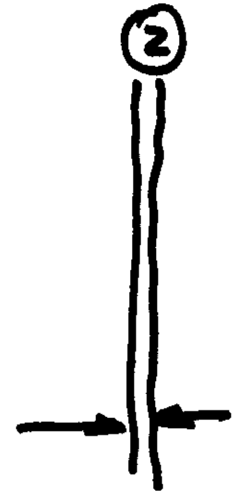
objective → perfect transmission

choose $k_2 L = n\pi \rightarrow L = n\left(\frac{\lambda_2}{2}\right)$ dome thickness must be an integral number of half wavelengths

$\left.\begin{array}{l} \sin k_2 L \rightarrow 0 \\ \cos k_2 L \rightarrow 1 \end{array}\right] + \quad \rho_1 c_1 = \rho_3 c_3$

$R = 0 \qquad |T| = 1$

Thin, heavy barrier case



1 $\qquad$ 3

$\rho_1 c_1$ $\qquad$ $\rho_3 c_3 = \rho_1 c_1$

very thin compared to a wavelength, limp, heavy

$k_2 L \ll 1$

$\rho_2 c_2 \gg \rho_1 c_1$

$\sin k_2 L \rightarrow k_2 L \qquad \cos k_2 L \rightarrow 1$

when $k_2 L \ll 1$

$$R \to \frac{\overbrace{\left(1 - \cancel{\frac{\rho_1 c_1}{\rho_3 c_3}}^{\,1}\right)}^{0} 1 + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} - \cancel{\frac{\rho_1 c_1}{\rho_2 c_2}}^{\,0}\right) k_2 L}{\underbrace{\left(1 + \cancel{\frac{\rho_1 \rho_1}{\rho_3 c_3}}^{\,1}\right) 1} + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} + \cancel{\frac{\rho_1 c_1}{\rho_2 c_2}}^{\,0}\right) k_2 L}$$

$\underbrace{\qquad}_{2}$

$$R \simeq \frac{j \dfrac{\rho_2 c_2}{\rho_3 c_3} \overbrace{\dfrac{\omega}{c_2} L}^{k_2 L}}{2 + j \dfrac{\rho_2 c_2}{\rho_3 c_3} \dfrac{\omega}{c_2} L}$$

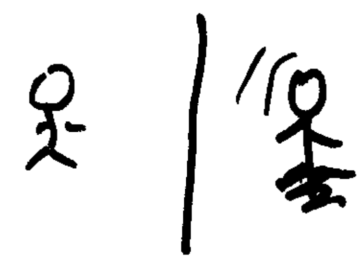
$\rho_2 L$ = mass/unit area of the "barrier"

$= m_s$

$$\rho_1 c_1 = \rho_3 c_3 = \rho_0 c$$] characteristic impedance of the ambient fluid

$$\boxed{R = \frac{j\omega m_s}{2\rho_0 c + j\omega m_s}}$$

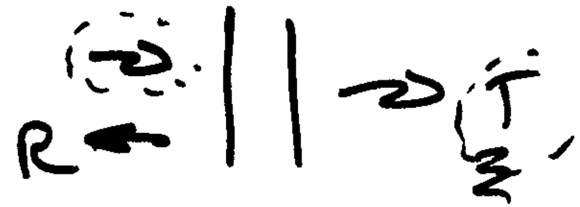
for a thin, heavy barrier



$\omega \to 0 \qquad R \to 0 \quad (T \to 1)$

$\omega \to \infty \qquad R \to 1$

Normal Incidence Transmission Coefficient



$$T = \frac{2\, e^{jk_3 L}}{\left(1 + \frac{\rho_1 c_1}{\rho_3 c_3}\right)\cos k_2 L + j\left(\frac{\rho_2 c_2}{\rho_3 c_3} + \frac{\rho_1 c_1}{\rho_2 c_2}\right)\sin k_2 L}$$

(i) Sonar Case:

$k_2 L = n\pi$ $\qquad$ $\sin k_2 L \to 0$

$\cos k_2 L \to 1$

$\rho_1 c_1 = \rho_3 c_3$

$$T = \frac{2e^{ik_3L}}{\pm 2}$$

$|T| = 1$ perfect transmission except for a phase change

(ii)
Thin, heavy barrier

$\rho_1 c_1 = \rho_3 c_3$ $\quad \dfrac{\rho_2 c_2}{\rho_1 c_1} \gg 1$

$k_2 L \ll 1$

$$T = \frac{2e^{jk_3L}}{2 + j\left(\frac{\rho_2 c_2}{\rho_3 c_3}\right)k_2 L}$$

$k_1 = k_3 = k$

$\rho_1 c_1 = \rho_3 c_3 = \rho_0 c$

$\rho_L L = m_s$

$$\boxed{T = \frac{2\rho_0 c\, e^{jkL}}{2\rho_0 c + j\omega m_s}}$$

$k = \frac{\omega}{c}$ air

Thin, limp barrier

$\omega \to 0 \quad |T| \to 1$

"high frequency" $\omega m_s \gg \rho_0 c$

$$\boxed{|T| \rightarrow \frac{2\rho_0 c}{\omega m_s}}$$

mass law

$$|T| \propto \frac{1}{\omega}$$

$$|T| \propto \frac{1}{m_s}$$

barrier performance increases directly with mass & frequency

limp → flexural stiffness is assumed to be negligible

sheet of AL 0.05"