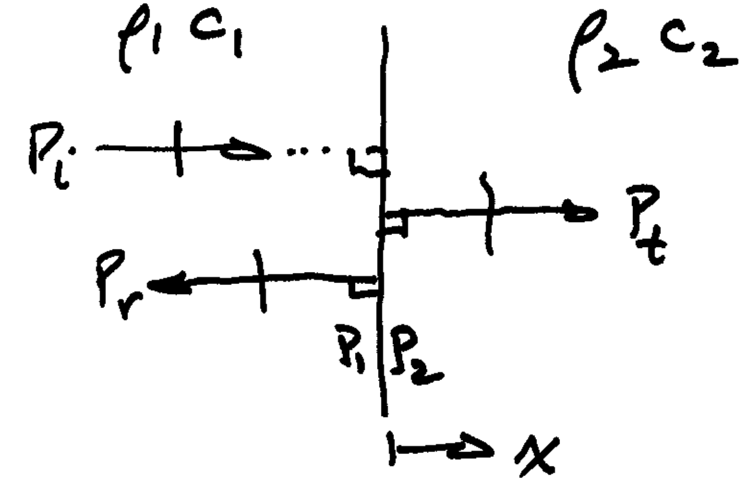
4. Fundamentals of Reflection and Transmission

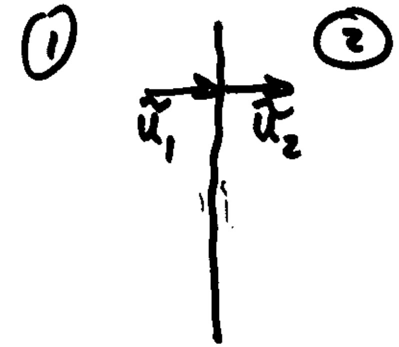
4.2 Normal Incidence



$$\frac{P_r}{P_i} = R = \qquad \frac{P_t}{P_i} = T =$$

$$\tilde{P}_1(0) = \tilde{P}_2(0)$$ To avoid infinite accele at the interface

(ii) Velocity continuity



$\hat{u}_{1_n}(0) = \hat{u}_{2_n}(0)$ for the two fluids to remain in contact

harmonic

$$j\omega \hat{\xi}_{1_n}(0) = j\omega \hat{\xi}_{2_n}(0)$$

displacement is also continuous

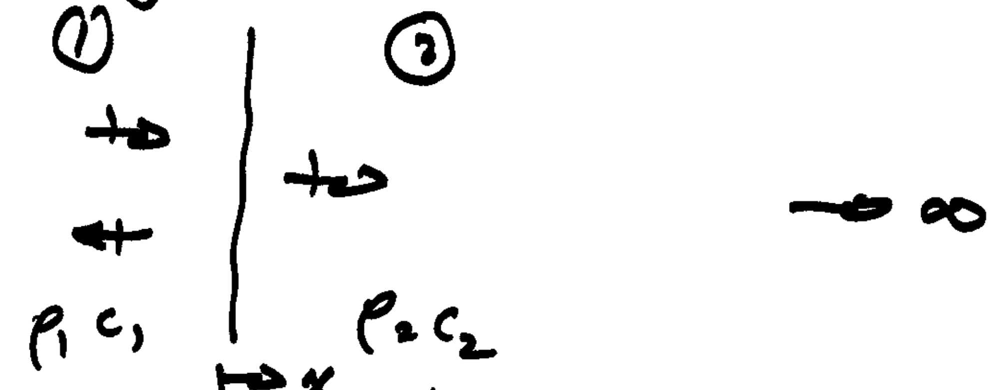
Notes:

1. since
$$\frac{\hat{\tilde{p}}_1(0) = \hat{\tilde{p}}_2(0)}{\hat{\tilde{u}}_{1n}(0) = \hat{\tilde{u}}_{2n}(0)}$$

$$\left.\frac{\hat{\tilde{p}}_1}{\hat{\tilde{u}}_{1n}}\right|_{x=0} = \left.\frac{\hat{\tilde{p}}_2}{\hat{\tilde{u}}_{2n}}\right|_{x=0} = \hat{\tilde{z}}_1\Big|_{x=0} = \hat{\tilde{z}}_2\Big|_{x=0}$$

normal specific acoustic impedance is continuous

Application of Boundary Conditions



$$\tilde{p}_1 = P_i e^{-ik_1 x} + P_r e^{+ik_1 x}$$

$$\tilde{p}_2 = P_t e^{-ik_2 x}$$

$k_1 = \frac{\omega}{c_1}$ $\qquad$ $k_2 = \frac{\omega}{c_2}$

pressure b.c. at $x=0$

$$\tilde{p}_1(0) = \tilde{p}_2(0)$$

$$\boxed{P_i + P_r = P_t} \qquad (1)$$

$$\tilde{u}_1 = \frac{P_i}{\rho_1 c_1} e^{-ik_1 x} - \frac{P_r}{\rho_1 c_1} e^{+ik_1 x}$$

$$\tilde{u}_2 = \frac{P_t}{\rho_2 c_2} e^{-ik_2 x}$$

$$\hat{\tilde{u}}_1(0) = \hat{\tilde{u}}_2(0)$$

$$\boxed{\frac{p_i}{\rho_1 c_1} - \frac{p_r}{\rho_1 c_1} = \frac{p_t}{\rho_2 c_2}} \qquad (2)$$

Normalize wrt p_i

$$R = \frac{p_r}{p_i} \qquad T = \frac{p_t}{p_i}$$

<u>pressure</u> reflection and transmission coefficients

Divide (1) & (2) by P_i

$Z_1 = \rho_1 c_1$
$Z_2 = \rho_2 c_2$] characteristic impedance

(1) $$1 + R = T$$

(2) $$1 - R = \left(\frac{\rho_1 c_1}{\rho_2 c_2}\right) T = \frac{1}{\zeta_{21}} T$$

$$\zeta_{21} = \frac{Z_2}{Z_1} = \left(\frac{\rho_2 c_2}{\rho_1 c_1}\right)$$

$$T = \frac{2\zeta_{21}}{\zeta_{21} + 1}$$

$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1}$$

Only 1 significant parameter

ζ_{21}

(i) If there is no dissipation in regions
① & ②

$$\frac{Z_2}{Z_1} = \zeta_{21} = \text{real}$$

$$T = \frac{2\zeta_{21}}{\zeta_{21}+1}$$

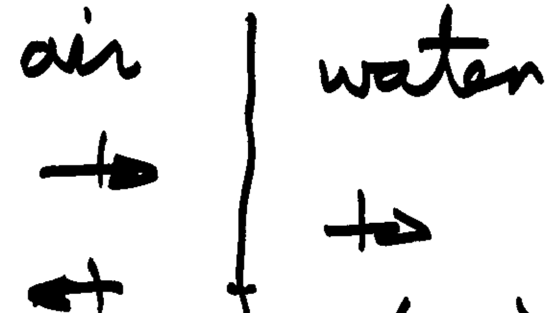
$$R = \frac{\zeta_{21}-1}{\zeta_{21}+1}$$

Then R & T are purely real

R can be +ve or -ve
depending on whether
$\zeta_{21} > 1$ or $\zeta_{21} < 1$

Reflection ~~is~~ in-phase (+ve)
" " out-of-phase (-ve)

(ii) $\zeta_{21} \to \infty$



$(\rho_1 c_1)_{air} \approx 415$ MKS Rayls

$(\rho_2 c_2)_{water} \simeq 1.5 \times 10^6$ MKS Rayls

$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1} \simeq +1$$ in-phase reflection

$$\hat{p}_1(0) = P_i + P_r = P_i\,(1 + \cancelto{1}{R})$$

$$= 2 P_i$$

when $\zeta_{21} \gg 1$ pressure is doubled at the surface

(iii) $\zeta_{21} >> 1$

air | water

$p_2 = p_1$ = twice the incident pressure

p_1 = twice the incident pressure

$$T = \frac{2\zeta_{21}}{\zeta_{21} + 1} \simeq 2$$

$R = 1 \qquad T = 2$

$R + T \neq 1$ in general

$$I_2 = \frac{\left(p_{rms}^2\right)_2}{\rho_2 c_2} \qquad I_{1_i} = \frac{\left(p_{rms}^2\right)_{1_i}}{\rho_1 c_1}$$

$$\simeq 0 \quad \text{when } S_{21} \gg 1$$

Essentially no energy transmission across the interface

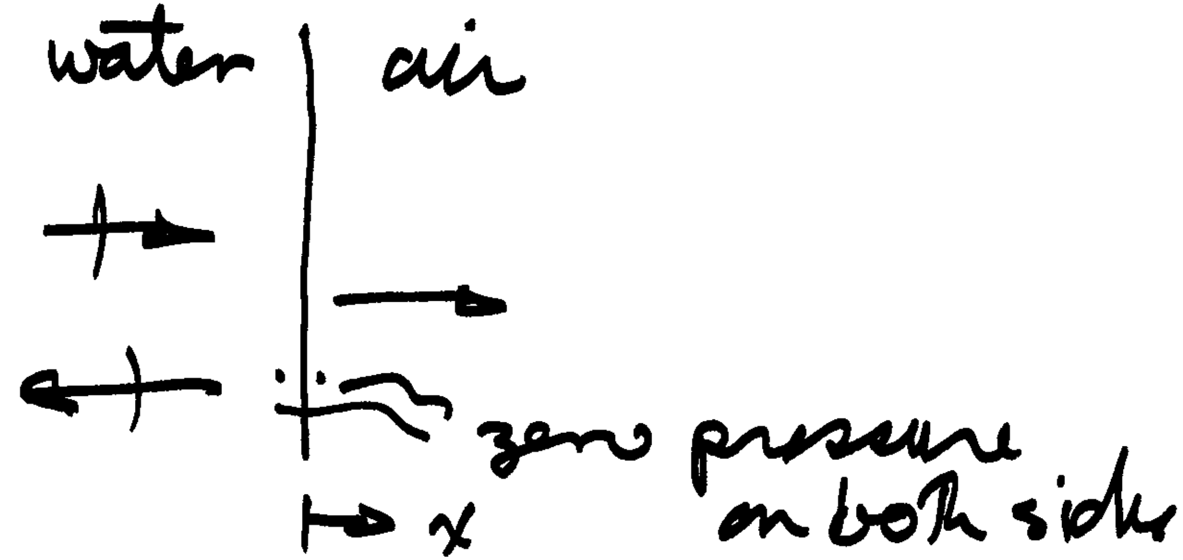
(iv) $$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1}$$

$R \to 0$ when $\zeta_{21} \to 1$ $\qquad T \to 1$

$$\rho_1 c_1 = \rho_2 c_2$$

$$\frac{\rho_1}{\rho_2} = \frac{c_2}{c_1}$$ if this true - perfect transmission + zero reflection

(v) $\zeta_{21} \ll 1$



$$R = \frac{\zeta_{21} - 1}{\zeta_{21} + 1} \simeq -1$$

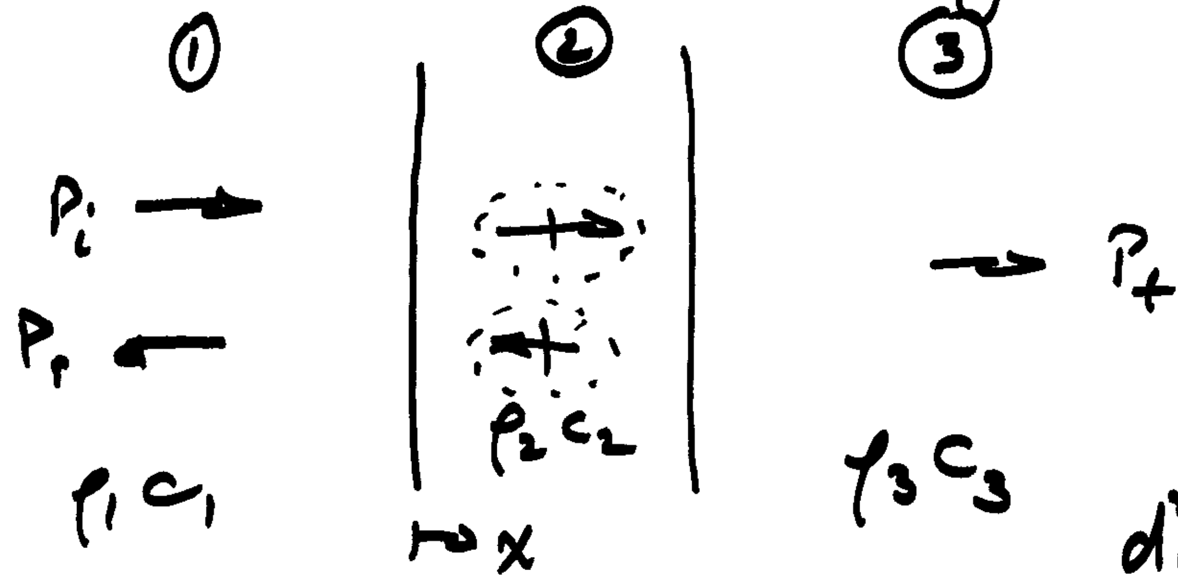
out-of-phase reflection

$$T = \frac{2\zeta_{21}}{\zeta_{21} + 1} \simeq 0$$

$$\hat{p}_1(0) = P_i + P_r = P_i\,(\underbrace{1 + \overset{\simeq -1}{R}}_{})$$

$\simeq 0$ "pressure release boundary"

4.2.2 Normal Incidence sound transmission through a fluid layer



$$\int_{-\infty}^{\infty} f(t)\, e^{-j2\pi f t}\, dt$$

$$\frac{d^2\tilde{p}_3}{dx^2} + k_3^2\,\tilde{p}_3 = 0$$

$$\frac{d^2\tilde{p}_1}{dx^2} + k_1^2\,\tilde{p}_1 = 0 \qquad \frac{d^2\tilde{p}_2}{dx^2} + k_2^2\,\tilde{p}_2 = 0$$

$$\tilde{p}_1(x) = P_i\, e^{-jk_1 x} + P_r\, e^{+jk_1 x}$$

$$\tilde{p}_2(x) = P_A\, e^{-jk_2 x} + P_B\, e^{+jk_2 x}$$

$$\tilde{p}_3(x) = P_t\, e^{-jk_3 x}$$