One-Dimensional Solutions

$$\tilde{p}_+(r) = \frac{A}{r} e^{-jkr} \qquad \tilde{u}_{r+} = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dr}$$

$$\tilde{u}_{r+}(r) = \frac{1}{\rho_0 c}\left(1 + \frac{1}{jkr}\right)\frac{Ae^{-jkr}}{r} \qquad \tilde{z} = \frac{\tilde{p}}{\tilde{u}}$$

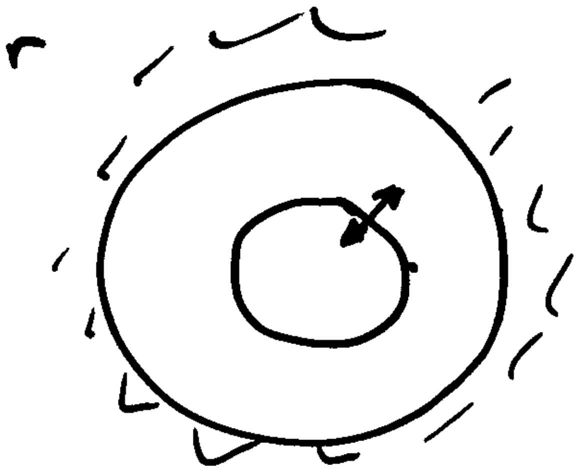
↑ nearfield

cylindrical sources

$$|\tilde{p}(r)| \propto \frac{1}{r^{1/2}}$$

Specific Acoustic Impedance

spherical case $\quad \tilde{z}_+ = \dfrac{\tilde{p}_+}{\tilde{u}_{r+}} = \dfrac{\rho_0 c}{1 + \frac{1}{jkr}}$



General spherical Case

$$\tilde{p}(r) = \frac{\tilde{A}}{r} e^{-jkr} + \frac{\tilde{B}}{r} e^{+jkr}$$

$$\tilde{u}_r(r) = -\frac{1}{j\omega\rho_0} \frac{d\tilde{p}}{dr} = 4 \text{ terms}$$

$$\tilde{z} = \frac{\tilde{p}}{\tilde{u}_r} = \text{relatively complicated}$$

Notes:

(i) Impedance is usually expressed in terms of

$$\tilde{z} = r + jx$$

r — specific acoustic resistance

x — s.a. reactance

(ii) ρc is the characteristic impedance

$\neq$ s.a. impedance

(iii) outward-going $\begin{pmatrix}\text{spherical} \\ \text{cylindrical}\end{pmatrix}$ waves

$kr \gg 1$ farfield $\qquad \tilde{z} \to \rho c$

3.5 Acoustic Intensity

In mechanics

Force × Distance = Work (Joules)
Force × Velocity = Power (Watts)

In acoustics

pressure × velocity = Power/unit area (Watts/m^2)

(pressure × velocity) = Intensity [Watts/m^2]

Instantaneous Intensity

$$= p(t)\,\bar{u}(t) = \bar{I}_t(t)$$

Intensity is a vector quantity

Intensity - means time-averaged acoustic intensity
- time-averaged rate of energy flow through a unit area

for a periodic signal

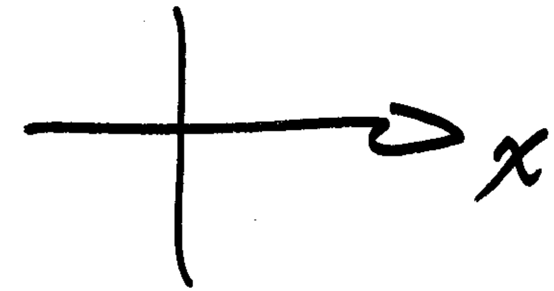
$$\bar{I} = \frac{1}{T}\int_0^T p_t(t)\,\bar{u}(t)\,dt$$

(under p_t and $\bar{u}$: real)

Intensity is real

T - period

+ve going plane wave



$$\frac{p_+}{u_+} = \rho_0 c \qquad u_+ = \frac{p_+}{\rho_0 c}$$

$$I_x = \frac{1}{T}\int_0^T \frac{p_+^2}{\rho_0 c}\, dt$$

$$= \frac{1}{\rho_0 c}\left[\frac{1}{T}\int_0^T p_+^2(t)\, dt\right]$$

mean square pressure

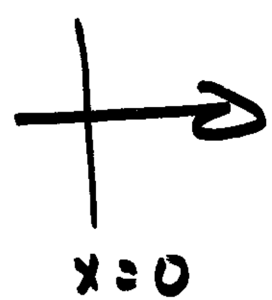
p_{rms}^2

$$I = \frac{p_{rms}^2}{\rho_0 c}$$ +ve going plane wave

Consider the harmonic case

$$\tilde{p} = A e^{j(\omega t - kx)}$$

$$\tilde{u}_x = \frac{A}{\rho_0 c} e^{j(\omega t - kx)}$$



$x = 0$

$$\hat{\tilde{p}}_+ = (A_r + jA_i)(\cos\omega t + j\sin\omega t)$$

$$\hat{\tilde{u}}_+ = \frac{(A_r + jA_i)}{\rho_0 c}(\cos\omega t + j\sin\omega t)$$

$$\mathrm{Re}\{\hat{\tilde{p}}_+\} = A_r\cos\omega t - A_i\sin\omega t$$

$$\mathrm{Re}\{\hat{\tilde{u}}_+\} = \frac{A_r\cos\omega t - A_i\sin\omega t}{\rho_0 c}$$

$$I = \frac{1}{T}\int_0^T \mathrm{Re}\{\tilde{p}_+\}\,\mathrm{Re}\{\tilde{u}_+\}\,dt$$

$$\vdots$$

$$= \frac{1}{\rho_0 c}\,\frac{1}{T}\left[T\,\frac{A_r^2}{2} + T\,\frac{A_i^2}{2}\right]$$

$$\tilde{p}_+ = Ae^{j\omega t}$$

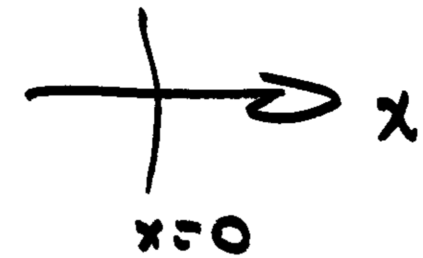
$$= \frac{1}{2\rho_0 c}\underbrace{\left[A_r^2 + A_i^2\right]}_{|A| = |\tilde{p}_+|^2}$$

$$I = \frac{|\tilde{p}_+|^2}{2\rho_0 c}$$ ← mean square pressure

$$= \frac{p_{rms}^2}{\rho_0 c}$$

Complex harmonic signal

$$I = \frac{1}{2} \mathrm{Re} \{ \tilde{p}_+ \hat{u}_+^* \}$$



$$\boxed{\frac{1}{2}\mathrm{Re}\left\{\tilde{p}_+ \tilde{u}_+^*\right\}}$$

$$= \frac{1}{2}\mathrm{Re}\left\{A e^{j\omega t} \frac{A^* e^{-j\omega t}}{\rho_0 c}\right\}$$

$$\left(e^{j\omega t}\right)^*$$

$$e^{-j\omega t}$$

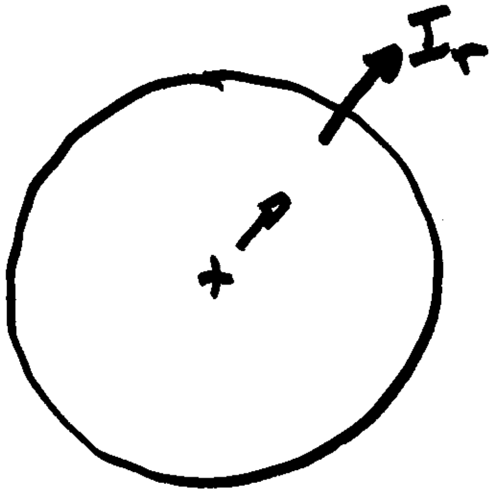
$$= \frac{1}{2}\mathrm{Re}\left\{\frac{AA^*}{\rho_0 c}\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{|A|^2}{\rho_0 c}\right\}$$

$$= \frac{1}{2}\frac{|A|^2}{\rho_0 c} \qquad |A| = |\hat{p}_+|$$

$$= \frac{1}{2}\frac{|\hat{p}_+|^2}{\rho_0 c} \qquad p^2_{rms}$$

For complex harmonic signals

$$\bar{I} = \frac{1}{2} \mathrm{Re}\{\tilde{p}\,\tilde{\vec{u}}^*\}$$



spherically symmetric waves

- radiating into free space

$$I_r = \frac{1}{2} Re\{ \tilde{p}_+ \tilde{u}_{r_+}^* \}$$

$$\tilde{u}_{r_+} = \frac{1}{\rho_0 c}\left(1 + \frac{1}{jkr}\right)\hat{p}_+$$

$$\tilde{u}_{r_+}^* = \frac{1}{\rho_0 c}\left(1 - \frac{1}{jkr}\right)\tilde{p}_+^*$$

$$\tilde{p}_+ \tilde{u}_{r_+}^* = \frac{\tilde{p}_+ \hat{p}_+^*}{\rho_0 c}\left(1 - \frac{1}{jkr}\right)$$

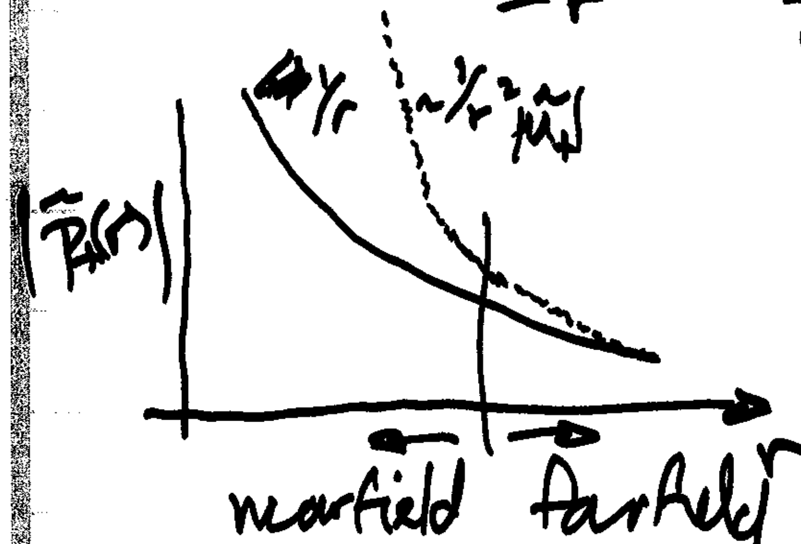
$$I_r = \frac{1}{2} Re\{ \tilde{p}_+ \tilde{u}_{r_+}^* \} = \frac{|\tilde{p}_+|^2}{2\rho_0 c} \quad p_{rms}^2$$

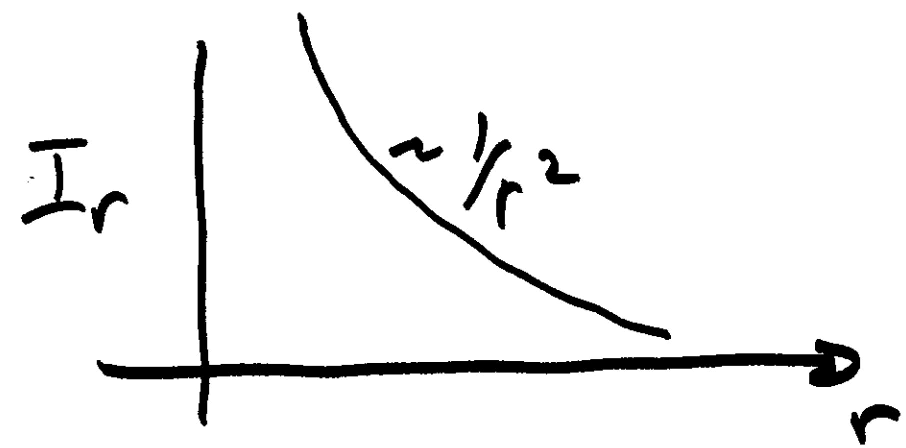
$$I_r = \frac{|\tilde{p}_+|^2}{2\rho_0 c}$$

$$\hat{p}_+ = \frac{A}{r} e^{-ikr}$$

$$|\tilde{p}_+| \propto \frac{1}{r}$$

$$I_r \propto \frac{1}{r^2}$$





* no intensity nearfield