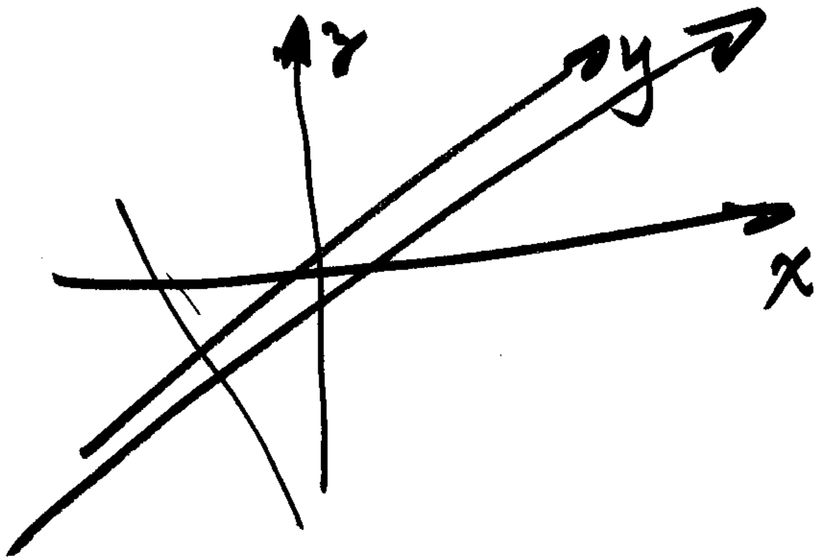
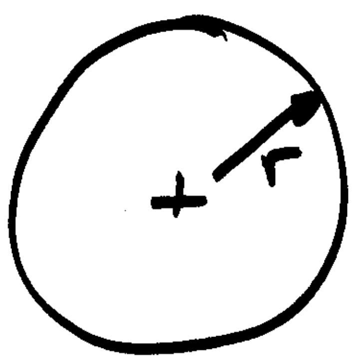
One-Dimensional solutions

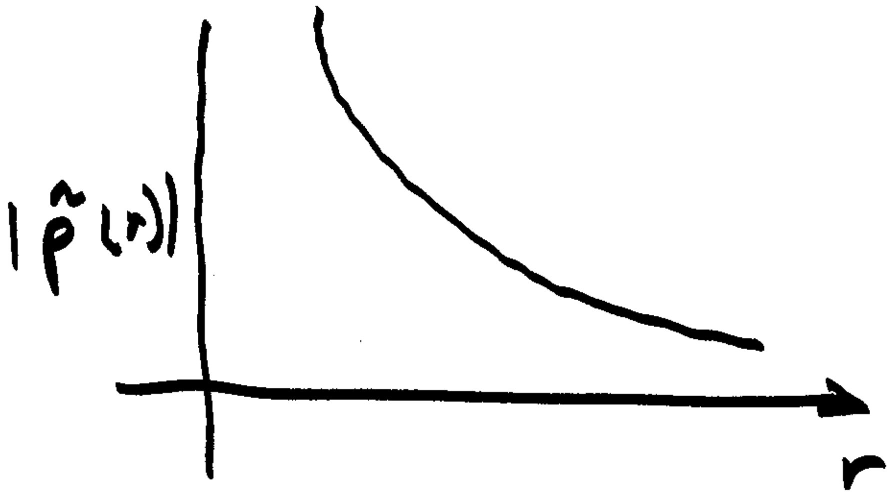


$$\tilde{p}(x,y,z,t) = A e^{i(\omega t \pm k_x x \pm k_y y \pm k_z z)}$$

$$k^2 = \underbrace{k_x^2 + k_y^2 + k_z^2} \qquad k = \frac{\omega}{c}$$



$$\tilde{p}(r) = \underbrace{\frac{A}{r} e^{-ikr}} + \underbrace{\frac{B}{r} e^{+ikr}}$$



Acoustic Particle Velocity

$$-\nabla p = \rho_0 \frac{d\bar{u}}{dt}$$

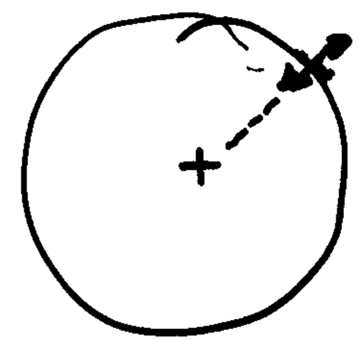
spherical symmetry case

$$\nabla = \left(\frac{\partial}{\partial r}\right)\bar{r}$$ ← unit vector in the radial direction

harmonic

$$p(r,t) = \tilde{p}(r)\, e^{j\omega t}$$

$$u_r(r,t) = \hat{u}_r(r)\, e^{j\omega t}$$



$$-\frac{d\tilde{p}}{dr} = j\omega\rho_0 \tilde{u}_r$$

$$\hat{u}_r = -\frac{1}{j\omega\rho_0}\frac{d\tilde{p}}{dr}$$

$$\tilde{p}_+ = \frac{A}{r} e^{-jkr}$$ outward

$$\frac{d\tilde{p}_+}{dr} = -\frac{A}{r}\left(\frac{1}{r} + jk\right) e^{-jkr}$$

$$\hat{\tilde{u}}_{r+} = \frac{\cancel{j}k}{j\omega\rho_0}\left(1 + \frac{1}{jkr}\right)\frac{A}{r}e^{-jkr}$$

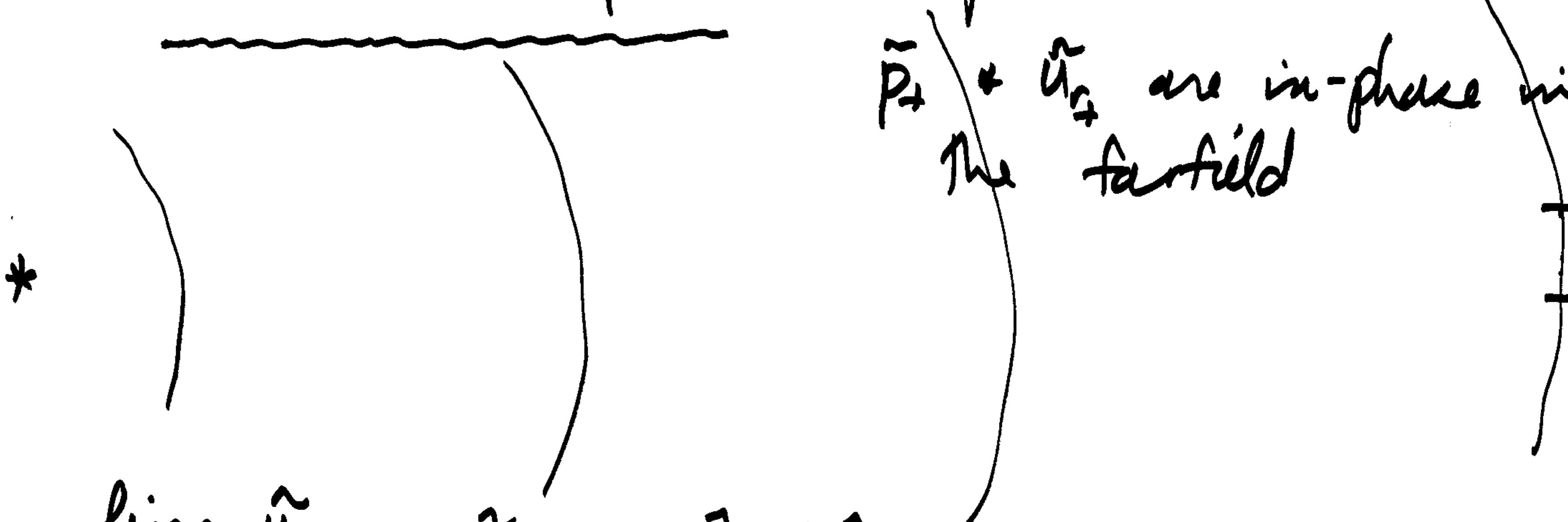
$$= \frac{1}{\rho_0 c}\left(1 + \underbrace{\frac{1}{jkr}}\right)\hat{\tilde{p}}_+$$

nearfield - significant ~~at~~ close (in terms of wavelengths) to the source

$$kr = 2\pi\left(\frac{r}{\lambda}\right)$$

$$\lim_{kr\to\infty} \hat{u}_{r+} \simeq \frac{\tilde{p}_+}{\rho_0 c}$$

same as the plane wave case.

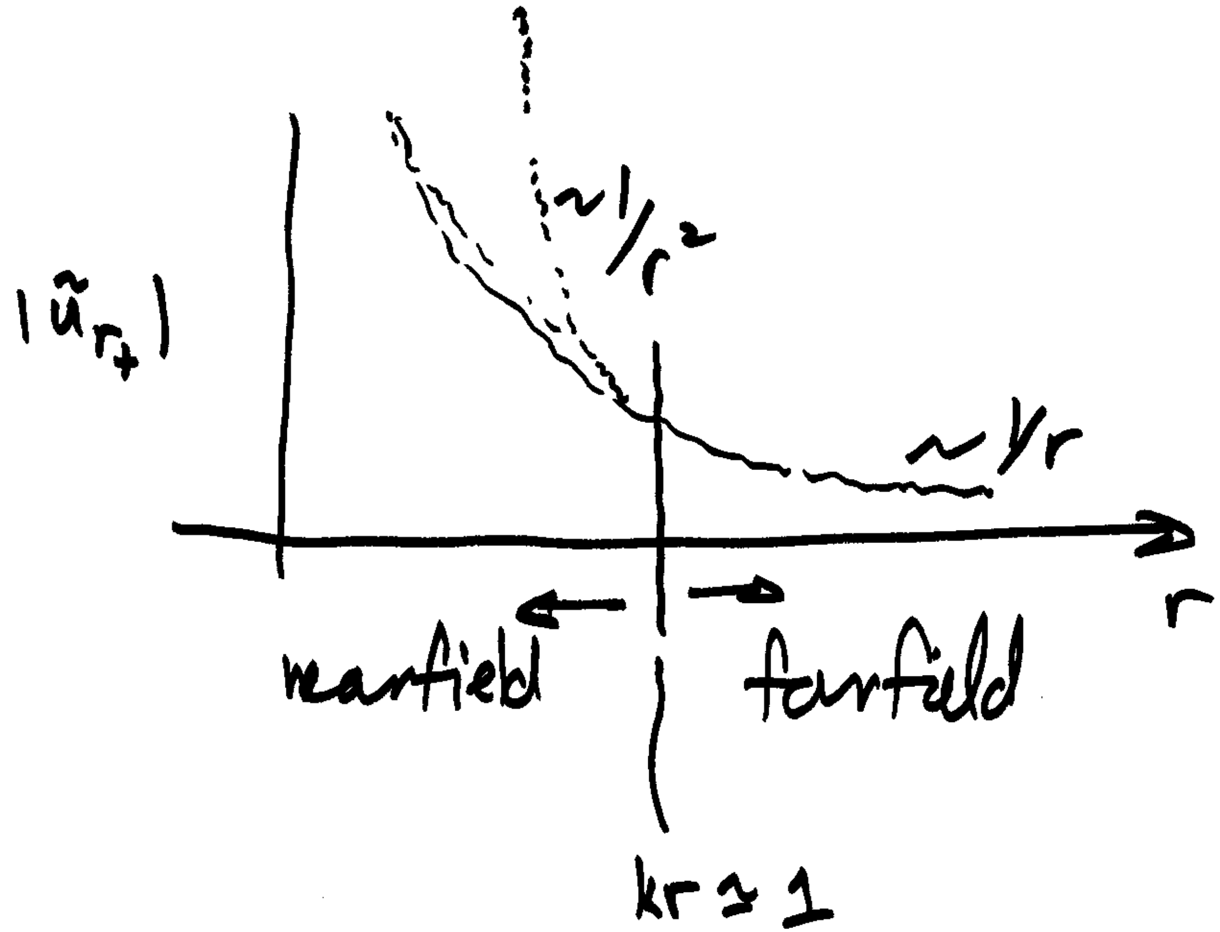


$\tilde{p}_+$ & $\hat{u}_{r+}$ are in-phase in the farfield

← locally plane

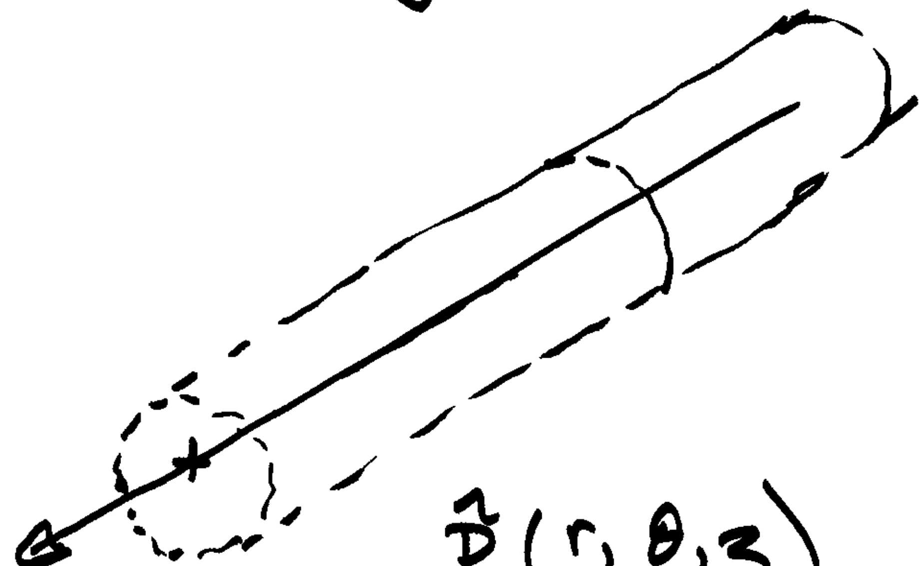
$$\lim_{kr\to 0} \hat{u}_{r+} = \frac{\tilde{p}_+}{jkr\rho_0 c}$$

$\tilde{p}_+$ & $\hat{u}_{r+}$ are 90° out of phase

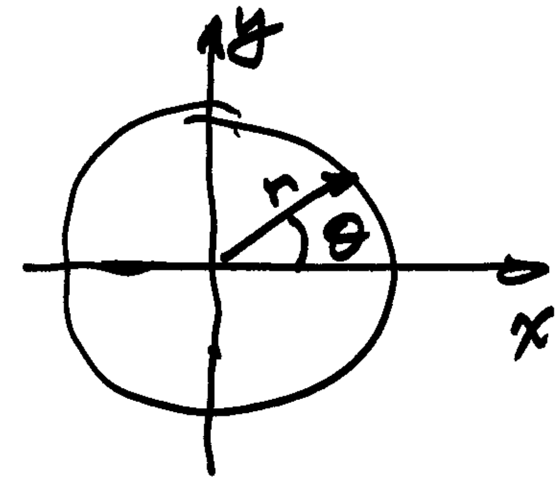


$|\tilde{u}_{r_+}|$
$\sim 1/r^2$
$\sim 1/r$
r
nearfield
farfield
$kr \approx 1$

3.3.3 Cylindrical Waves



$\tilde{p}(r, \theta, z)$



$$p(r,\theta,z,t) = \tilde{p}(r,\theta,z)\, e^{j\omega t}$$

$$p(r,\theta,z,t) = R(r)\, \cancel{\Theta(\theta)}\, \cancel{Z(z)}\, e^{j\omega t}$$

cylindrical symmetry

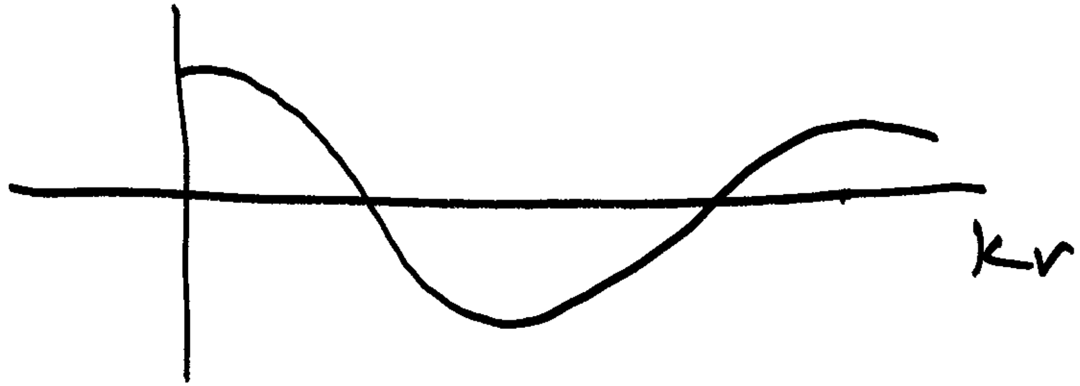
no - θ dependence   no - z-dependence

Outward propagating harmonic cylindrically symmetric sound field

$$p(r,t) = A\, H_0^{(2)}(kr)\, e^{j\omega t}$$

Hankel Function

$$H_0^{(2)}(kr) = \underbrace{J_0(kr) - j\, Y_0(kr)}_{\text{Bessel Functions}}$$

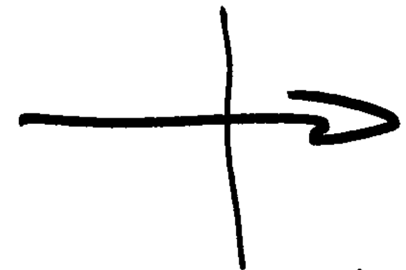


when $kr >> 1$ farfield

$$\tilde{p}(r) = A\left(\frac{2}{\pi k r}\right)^{1/2} e^{j(\omega t - kr + \frac{\pi}{4})}$$

$$|\tilde{p}(r)| \propto \frac{1}{\sqrt{r}}$$ for a cylindrical wave in the far field

plane wave



$|\tilde{p}_+|$ independent of position

spherical wave

$$|\tilde{p}_+| \propto \frac{1}{r}$$

Cylindrical Wave

$$|\tilde{p}_+| \propto \frac{1}{r^{1/2}}$$

3.4 Specific Acoustic Impedance

per unit area

$$\tilde{z} = \frac{\text{acoustic pressure}}{\text{acoustic particle velocity}}$$

$$= \frac{\tilde{p}}{\tilde{u}} \quad \text{harmonic case}$$

Plane +ve x-direction

$$\tilde{p}_+ = A e^{-ikx}$$

$$\tilde{\hat{u}}_+ = -\frac{1}{j\omega\rho_0}\frac{d\tilde{\hat{p}}_+}{dx} = \frac{A}{\rho_0 c}e^{-ikx} = \frac{\tilde{p}_+}{\rho_0 c}$$

$$\tilde{\hat{z}}_+ = \frac{\tilde{\hat{p}}_+}{\tilde{u}_+} = \rho_0 c \quad \text{characteristic impedance}$$

415 Rayls

For a negative-going plane wave

$$\hat{z}_- = \frac{\tilde{p}_-}{\tilde{u}_-} = -\rho_0 c$$

In general

$\longrightarrow$

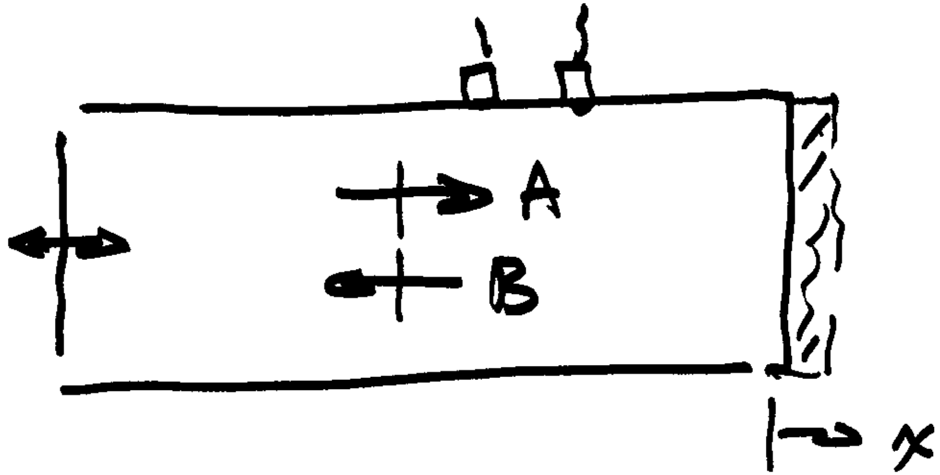
$\longleftarrow$

$$\hat{p}(x) = Ae^{-jkx} + Be^{+jkx}$$

$$\tilde{u}(x) = -\frac{1}{j\omega\rho_0}\frac{dp}{dx} = \frac{A}{\rho_0 c}e^{-jkx} - \frac{B}{\rho_0 c}e^{+jkx}$$

$$\tilde{Z} = \frac{\hat{p}(x)}{\hat{u}(x)} = \rho_0 c\,\frac{Ae^{-jkx} + Be^{+jkx}}{Ae^{-jkx} - Be^{+jkx}}$$

function of position



$\left(\frac{B}{A}\right)$ = plane wave reflection coefficient

$$\tilde{z} = \rho_0 c \, \frac{e^{-jkx} + \left(\frac{B}{A}\right) e^{+jkx}}{e^{-jkx} - \left(\frac{B}{A}\right) e^{+jkx}}$$

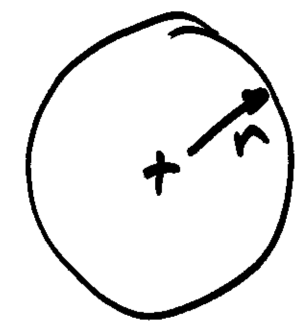
$$0 \leq \left|\frac{B}{A}\right| \leq 1$$

Special cases

(i) $\frac{B}{A} \to 0 \qquad \tilde{z} \to \rho_0 c$

(ii) $\frac{B}{A} \to 1 \qquad \hat{z} \to \rho_0 c \cot kx$

Spherically Symmetric Case



$$\frac{A}{r}e^{-jkr}$$

$$\tilde{u}_r = -\frac{1}{j\omega\rho_0}\frac{\partial \tilde{p}}{\partial r}$$

$$\tilde{u}_{r_+} = \frac{1}{\rho_0 c}\left(1 + \frac{1}{jkr}\right)\tilde{p}_+$$

$$\tilde{z}_+ = \frac{\tilde{p}_+}{\tilde{u}_{r_+}} = \frac{\rho_0 c}{1 + \frac{1}{jkr}}$$

$j\omega m$

nearfield $\tilde{z} \to jkr\rho_0 c$ (with $k = \omega/c$)

$kr \ll 1$

$\tilde{p}$ & $\tilde{u}_r$ out of phase

<u>mass-like</u>

- imaginary
- positive
- linearly prop. to ω

$kr \gg 1$ $\tilde{z} \to \rho c$

farfield